- course on Quantum Computers and not on Quantum Computing.
- A few words on QC.

- Quantum dynamics involves α^n (# of states) different amplitudes calculating the evolution of which is hard. [assume $\alpha = 2$; $n = 80$ and 8 bit numerical resolution per amplitude.]

$\Rightarrow 2^{80+8} = 2^{88}$ bits 1 Tbyte $\sim 10^{12} \times 8 \simeq 2^{[illegible]}$ ($8 = 2^3$)

10^{10} Tbyte just for memory at each moment in time. Not to mention # of operations.

- More formally: complexity classes P vs. NP.

P = # of steps is polynomial in input.

NP = "Nondeterministic polynomial". Given the right solution, it can be checked in polynomial time.

$\Rightarrow P \in NP$

$P = NP$? $\Rightarrow$ open question (1 of 7 millenium prize problems.)

NP complete - set of all problems that are "as hard" as any other problem in NP i.e. any other problem in NP can be reduced to it

in polynomial time.

$\Rightarrow$ if you solve an NP complete problem in polynomial time $\Rightarrow$ NP = P

- (Nearest / Neighbour) Ising model

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{ij}\, \sigma_i\, \sigma_j$$

for $J_{ij} \in$ positive / negative $\Rightarrow$ ground state ferromagnetic / antiferromagnetic

J_{ij} = changes sign randomly $\Rightarrow$ spin glass

finding GS for - nearest neighbours in 3D

- J_{ij} long range) connectivity,

finding the ground-state is NP complete

- Feynman (81) : maybe if we use a quantum computer simulating quantum dynamics would be easy

- Not necessarily : in the case of the Ising G.S.

in principle efficient in adiabatically following to g.s. but

- meas. will collapse onto the meas. basis. ← speculation

- some global properties → i.e. g.s. magnetization
spin correlation functions.

Since then: several algorithms

- Rely on QFT ⇒ Shor's algorithm for factorization
↳ not NP hard)

- Data Search ⇒ Grover's algorithm $2^n \to \sqrt{2^n}$

- Q Symulations

- Scott Aharonson : calc matrix permanent - NP complete
↳ using Q random walk

- complexity class BQP
↳ Bound probability.

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All these rely on a Quantum paralelism

+ b. Interference to get one (global!) answer.

Example : Deutsch - Jozsa

$\{0,1\}$ $f(x) \to \{0,1\}$ classically 2 queries to learn that function.

$$U_f |x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$$

unitary.

$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle|f(0)\rangle + |1\rangle|f(1)\rangle\right]$$

Get both queries in the same superposition.

Not enough ⇒ have to measure at least twice.

- Global question. $f(0) = f(1) = f$?

or $f(0) = \overline{f(1)}$

$$\frac{1}{\sqrt{2}}\left[|0\rangle + |1\rangle\right]\frac{1}{\sqrt{2}}\left[|0\rangle - |1\rangle\right] \Rightarrow$$

~~$\frac{1}{2}|0\rangle|f(0)\rangle - |0\rangle|f(1)\rangle$~~

$$\frac{1}{2}\Big[\,|0\,f(0)\rangle - |0,1\oplus f(0)\rangle + |1\,f(1)\rangle - |1,1\oplus f(1)\rangle\,\Big]$$

$$\left.\begin{array}{l} 1\oplus 0 = 1 \\ 1\oplus 1 = 0 \end{array}\right\} \Rightarrow 1\oplus x = \bar{x}$$

$$= \frac{1}{2}\Big[\,|0\,f_0\rangle - |0\,\bar{f}_0\rangle + |1\,f_1\rangle - |1\,\bar{f}_1\rangle\,\Big]$$

if $f_0 = f_1 = f \begin{cases} 0 \\ 1 \end{cases}$

$$= \frac{1}{2}\Big[\,|0\rangle\big[\,|f\rangle - |\bar{f}\rangle\big] + |1\rangle\big[\,|f\rangle - |\bar{f}\rangle\big]\Big] =$$

$$= \frac{1}{2}\big(|0\rangle + |1\rangle\big)\big(|f\rangle - |\tilde{f}\rangle\big) = \underset{f = \begin{cases} 0 \\ 1 \end{cases}}{\pm}\ \frac{1}{2}\big(|0\rangle + |1\rangle\big)\big(|0\rangle - |1\rangle\big)$$

if $f_0 = \tilde{f}_1 = f \begin{cases} 0 \\ 1 \end{cases}$

$$= \frac{1}{2}\Big[\,|0\,f\rangle - |0\,\tilde{f}\rangle + |1\,\tilde{f}\rangle - |1\,f\rangle\,\Big]$$

$$\frac{1}{2}\Big[\big(|0\rangle - |1\rangle\big)|f\rangle - \big(|0\rangle - |1\rangle\big)|\tilde{f}\rangle\Big] =$$

$$= \frac{1}{2}\big(|0\rangle - |1\rangle\big)\big(|f\rangle - |\bar{f}\rangle\big) = \pm\frac{1}{2}\big(|0\rangle - |1\rangle\big)\big(|0\rangle - |1\rangle\big)$$

can be extended to larger x

$f(x) \begin{cases} \text{constant} \\ \text{balanced} \end{cases}$?

A Universal Q computer.

~~Notes of the xxxx~~ - Register of qubits (spins)

"supermagnet" - initialize

- implement ANY unitary operation

- Measure

- without noise interfering.

A tough job:

Examples: arbit. unitary on n spins $2^n \times 2^n = 4^n$

4^n ind. generators. $\Leftarrow$ Pauli group $\underbrace{\sigma \otimes \sigma \otimes \sigma \ldots \otimes \sigma}_{n}$

how do you generate this?

- effect of noise better looked on using states.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\,|\underbrace{000\ldots0}_{n}\rangle + e^{i\varphi}|111\ldots1\rangle\,\right]$$

meas. 1 collapses the full superposition!

write in the basis $|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$

$\varphi = 0 \qquad |\psi\rangle = \frac{1}{\sqrt{2^{n-1}}} \sum |\text{even}\rangle$ i.e. even # of 1's

$\varphi = \pi \qquad |\psi\rangle = \frac{1}{\sqrt{2^{n-1}}} \sum |\text{odd}\rangle$ ie odd # of 1's

$\Rightarrow$ a single $|+\rangle \to |-\rangle$ bit flip completely dephases $|\psi\rangle$.

- A universal QC is a universal state generating machine enabling the construct of any structure in Hilbert space.

- Other applications: Metrology.

More importantly it pushes QM towards a non-explored end, Course ~~contx~~ topics:

Part 1.

- Qubit
-
- Qubit arrays + tensor products of Hilbert spaces.
- Entanglement
- Universal gate set.
- Effect of imperfections/noise ⇒ Quantum operations
- QPT, QST
- Distance measures in Hilbert space.
- Quantum Error Correction Codes.
- Fault tolerant QC.

Part 2.

- trapped ion QC
- Superconducting QC
- Photon QC.

At least for part 1: Nielsen & Chuang.

QECC: Ch7 of John Preskil's Book (online)

14 2 hour meetings. (No lecture on Dec. 19th). 3 exercises. exam on 30/1

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Remark: I am giving the traditional story.

Alternatives: - Adiabatic QC

e.g. - Dissipative QC

- One-way QC.

- Topological QC