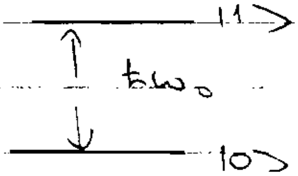
The Qubit:

a two-level Quantum System:



a general state:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = e^{i\varphi_\alpha}|\alpha||0\rangle + e^{i\varphi_\beta}|\beta||1\rangle$$

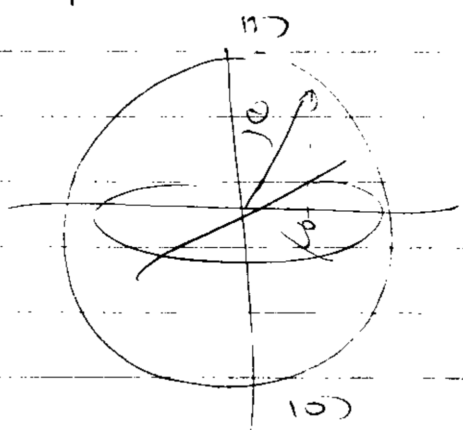
⇒ up to a global phase

$$|\alpha|^2 + Re\beta^2 + Im\beta^2 = 1$$

2D sphere embedded in 3D.

directions on Bloch sphere ←

⇒ not necessarily in lab.



Bloch sphere

→ states that are orthogonal on Bloch sphere are not orthogonal in Hilbert space.

$$\cos\left(\frac{\theta}{2}\right)|1\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|0\rangle$$

$$0 < \varphi < 2\pi$$

$$0 < \theta < \pi$$

- when we measure : collapse the qubit onto the measurement basis.

- Amount of info needed to describe 1 qubit : ∞
- Amount of info retrieved after 1 qubit meas. : 1 bit.

- to precisely determine the state of a qubit
  $\Rightarrow$ ∞ measurements.

1 copy $\rightarrow$ ∞ copies.

- No cloning theorem.

$U|\psi\rangle|S\rangle \Rightarrow |\psi\rangle|\psi\rangle$ U is unitary.
clones.

assume for any $|\psi\rangle$ and some $|\varphi\rangle$ and $|\chi\rangle$
$|\chi\rangle$

$U|\psi\rangle|\varphi\rangle = |\psi\rangle|\psi\rangle$

$U|\chi\rangle|\varphi\rangle = |\chi\rangle|\chi\rangle$

$\langle\psi|\langle\psi|\chi\rangle|\chi\rangle = |\langle\psi|\chi\rangle|^2$ can't work for any state.

classically:

$\langle\varphi|\langle\psi|U^{\dagger}U|\chi\rangle|\varphi\rangle = \langle\psi|\chi\rangle$

CNOT is a clone
$\Rightarrow$ entangled

$\Rightarrow \langle\psi|\chi\rangle = \begin{cases} 0 & \text{either the same} \\ 1 & \text{or orthogonal} \end{cases}$

Operations on a pure state ⇒ rotations

in spin Hilbert space 2x2 SU matrices. $[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$

3 Generators! $\sigma_k^2 = \hat{I}$ $\quad tr(\sigma_k) = 0 \quad \sigma_i \sigma_j = i\epsilon_{ijk}\sigma_k$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}; \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$R(\theta, \vec{n}) = e^{i\frac{\theta}{2}\vec{n}\cdot\vec{\sigma}} = \cos\left(\frac{\theta}{2}\right)\hat{I} + i\,\vec{\sigma}\cdot\vec{n}\,\sin\left(\frac{\theta}{2}\right)$$

notice that when $\theta = 180$ $\sin = 1$ $\cos = 0$

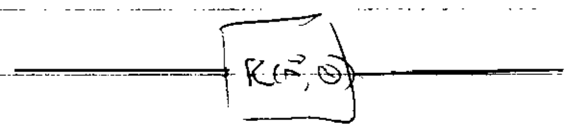
$$\Rightarrow i\,\vec{\sigma}\cdot\vec{n} = R(\vec{n}, 180)$$

σ: Both hermitian and unitary.

- usually in QIP no the evolution ⇒ operation

(no diff eq ⇒ linear algebra)

Circuit:



ρ

analog ⇒ infinitely many.

Density matrix:

statistical mixture of states: $\hat{\rho} = \sum_i p_i |\alpha_i\rangle\langle\alpha_i|$

$Tr(\hat{\rho}) = \sum_i p_i \, Tr(|\alpha_i\rangle\langle\alpha_i|) = \sum_i p_i = 1$

Similarly:

$\langle A \rangle = \cancel{Tr(\hat{A}\hat{\rho})} \; Tr[\hat{\rho}\cdot\hat{A}]$

By ~~construction~~ $\hat{\rho}$ is hermitian $\rho_{ij} = \rho_{ji}^*$

for a single qubit: # of free parameters

$$\begin{bmatrix} 1 & 2 \\ & 1 \end{bmatrix} = \underset{\text{hermiticity}}{4} - \underset{Tr=1}{1} = 3$$

since $I, \sigma_x, \sigma_y, \sigma_z$ form a basis for 2x2 hermitian

$$\hat{\rho} = \alpha\hat{I} + \beta\hat{\sigma}_x + \gamma\hat{\sigma}_y + \delta\hat{\sigma}_z$$

$$\langle\sigma_x\rangle = Tr[\hat{\rho}\,\sigma_x] = \underbrace{Tr[\alpha I\sigma_x]}_{0} + \underbrace{Tr[\beta\sigma_x\sigma_x]}_{2\beta} + \underbrace{Tr[\gamma\sigma_y\sigma_x]}_{0}$$

$$+ \underbrace{Tr[\delta\sigma_z\sigma_x]}_{0}$$

$$\beta = \langle\sigma_x\rangle\frac{1}{2}$$

$$\Rightarrow \hat{\rho} = \frac{1}{2}\left[\hat{I} + \sum_i \langle \sigma_i \rangle \hat{\sigma}_i\right]$$

ρ

state tomography

Geometrically $\hat{\rho}$ is represented by a 3D vector

with projections $\langle \sigma_i \rangle$ in the i direction

$$-1 < \langle \sigma_i \rangle < 1$$

the volume of the Bloch sphere.

E.g. fully mixed state $\hat{\rho} = \frac{1}{2}\hat{I}$

$\langle \sigma_x \rangle = \langle \sigma_y \rangle = \langle \sigma_z \rangle = 0$ center of sphere at origin

measure $|\pm i\rangle$ n times.

$$\langle \sigma_i \rangle = \frac{n_+ - n_-}{n}$$ std in $n_+/n_- = \sqrt{npq}$ binomial dist.

$\langle \sigma \rangle$ known within $\frac{\sqrt{pq}}{\sqrt{n}}$

if $p=1, q=0$ $\frac{\sqrt{pq}}{\sqrt{n}} = 0$ in $p=q=\frac{1}{2} \Rightarrow \frac{\sqrt{pq}}{\sqrt{n}} = \frac{1}{2\sqrt{n}}$