The trapped-ion qubit tool box

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http://www.weizmann.ac.il/complex/ozeri/Course2

Roee Ozeri

Weizmann Institute of Science
Rehovot, 76100, Israel
ozyeri@weizmann.ac.il
Physical Implementation of a quantum computer

David Divincenzo’s criteria:

1. Well defined qubits.

2. Initialization to a pure state


4. Qubit specific measurement.

5. Long coherence times (compared with gate & meas. time).
Universal Gate set

- For $N$ qubits, a general unitary transformation $U$ acts on a $2^N$-dimension Hilbert space.

- A *finite* set of unitary gates that spans any such $U$.

- The Deutsch-Toffoli gate

Universal Gate set

- For $N$-qubits, and unitary transformation $U$ on a $2^N$-dimension Hilbert space.

- A **finite** set of unitary gates that spans any such $U$.

- Rotations can be approximated to $\varepsilon$ by concatenating $k$ gates, from a finite set $\{V_i\}$, where $k < \text{polylog}(1/\varepsilon)$.

Physical Implementation of a quantum computer

David Divincenzo’s criteria:

How well???

Well enough to allow for a large scale computation: Fault tolerance

\[ F = \langle \Psi | \rho_\epsilon | \Psi \rangle \quad \epsilon = 1 - F \]
Fault-tolerant Quantum Computation

Noisy operations $\epsilon$

One level quantum error-correction codes $O((\epsilon/\epsilon_0)^2)$

concatenation; threshold theorem

$k$-levels of fault-tolerant encoding $O((\epsilon/\epsilon_0)^{2^k}) \rightarrow 0$ if $\epsilon < \epsilon_0$

- $\epsilon_0$ Fault tolerance threshold.
- Heavy resource requirements when $\epsilon \sim \epsilon_0$
- Depends on code, noise model, arch. constraints etc.
- Current estimates for $\epsilon_0 \approx 10^{-2} - 10^{-4}$
Tutorial overview

1. The ion-qubit: different ion-qubit choices, Ion traps.

2. Qubit initialization.

3. Qubit measurement.

4. Universal set of quantum gates:
   - single qubit rotations; two-ion entanglement gates

5. Memory coherence times
   
   How well???
   
   Benchmarked to current threshold estimates

Disclaimer: non exhaustive; focuses on laser-driven gates
Different ion-qubit choices

- One electron in the valence shell; “Alkali like” $^2S_{1/2}$ ground state.

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* Lanthanide series

** Actinide series
Electronic levels Structure

$n^2P_{3/2}$
$n^2P$
$n^2P_{1/2}$

Fine structure

with $D$
- Ca$^+$ 397 nm
- Sr$^+$ 422 nm
- Ba$^+$ 493 nm
- Yb$^+$ 369 nm
- Hg$^+$ 194 nm

w/o $D$
- Be$^+$ 313 nm
- Mg$^+$ 280 nm
- Zn$^+$ 206 nm
- Cd$^+$ 226 nm

$n^2S_{1/2}$
$^2S_{1/2}$ Zeeman qubit

(Isotopes w/o nuclear spin)

**e.g.**

- $^{24}\text{Mg}^+$
- $^{64}\text{Zn}^+$
- $^{114}\text{Cd}^+$
- $^{40}\text{Ca}^+$
- $^{88}\text{Sr}^+$
- $^{138}\text{Ba}^+$
- $^{174}\text{Yb}^+$
- $^{202}\text{Hg}^+$

**Advantages**
- RF separation.
- Tunable.
- Infinite $T_1$.

**Disadvantages**
- Energy depends linearly on $B$.
- Transition photon carries no momentum.
- Momentum transfer with off-resonance lasers: photon scattering.
- Detection.

$$m = 1/2 \quad \uparrow$$

$$n \, ^2S_{1/2}$$

$$m = -1/2 \quad \downarrow$$

Turn on small $B$ field

$2.8 \, \text{MHz/G}$
$^2S_{1/2}$ Hyperfine qubit

- MW energy separation.
- B-field independent qubit.
- Infinite $T_1$.
- State selective fluorescence Detection.

**Advantages**

- Few GHz energy separation.
- Transition photon carries no momentum
- Off resonance photon scattering.
- Initialization to clock transition can be more tricky

**Disadvantages**

$F = I - 1/2$

$F = I + 1/2$

$m = 0$

$|\uparrow\rangle$

$|\downarrow\rangle$

$1-40$ GHz

$^9$Be$^+$
$^{25}$Mg$^+$
$^{67}$Zn$^+$
$^{111}$Cd$^+$
$^{43}$Ca$^+$
$^{87}$Sr$^+$
$^{137}$Ba$^+$
$^{171}$Yb$^+$
$^{199}$Hg$^+$
**Optical qubit**

- **Advantages**
  - Single optical photon (momentum).
  - B-field independent qubit (if nuclear spin ≠ 0).
  - Hardly any spontaneous decay during gates.
  - State selective fluorescence Detection: excellent discrimination.

- **Disadvantages**
  - Finite (~ 1 sec) $|\uparrow\rangle$ lifetime.
  - Coherence time is limited by laser linewidth.

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**Innsbruck/Weizmann**

- $\mathrm{Ca}^+ \quad 729 \, \text{nm}$
- $\mathrm{Sr}^+ \quad 674 \, \text{nm}$
- $\mathrm{Ba}^+ \quad 1760 \, \text{nm}$
- $\mathrm{Yb}^+ \quad 411 \, \text{nm}$
- $\mathrm{Hg}^+ \quad 282 \, \text{nm}$

$|\uparrow\rangle$ $|\downarrow\rangle$ $^{2}\mathrm{D}_{5/2}$ $^{2}\mathrm{S}_{1/2}$
Trapping

- Trap ions: a minimum/maximum to $\phi$, the electric potential.

- Impossible in all directions; Laplace’s equation:

$$\nabla^2 \Phi = 0$$
Trapping

Linear RF Paul trap

Positive ion
RF electrode
High dc potential control electrode
Low dc voltage control electrode
Dynamic trapping
(pondermotive forces)

Oscillating electric field:
\[ E(x, t) = E_0(x) \cos(\omega_{rf} t + \phi) \]

\[ x(t) = X(t) + \xi(t) \]

Large and slow

\[ X(t) \gg \xi(t) \]

Small and fast

\[ \frac{\partial^2 X(t)}{\partial t^2} \ll \frac{\partial^2 \xi(t)}{\partial t^2} \]

Newton’s E.O.M:
\[ m \left( \frac{\partial^2 X(t)}{\partial t^2} + \frac{\partial^2 \xi(t)}{\partial t^2} \right) = e E_0(x) \cos(\omega_{rf} t + \phi) \]
Dynamic trapping

\[ m \left( \frac{\partial^2 X(t)}{\partial t^2} + \frac{\partial^2 \xi(t)}{\partial t^2} \right) = e E_0(x) \cos(\omega_{rf} t + \phi) \]

Field expansion:

\[ E_0(x) = E_0(X) + \frac{\partial E_0(X)}{\partial X} \xi + O(\xi^2) \]

To 0\(^{th}\) order:

\[ m \frac{\partial^2 \xi(t)}{\partial t^2} = e E_0(X) \cos(\omega_{rf} t + \phi) \]

\[ \xi(t) = -\frac{e E_0(X)}{m \omega_{rf}^2} \cos(\omega_{rf} t + \phi) \]

Next order:

\[ m \frac{\partial^2 X(t)}{\partial t^2} = -\frac{e^2 E_0(x)}{m \omega_{rf}^2} \frac{\partial E_0(X)}{\partial X} \cos^2(\omega_{rf} t + \phi) \]

Average over one period:

\[ m \frac{\partial^2 X(t)}{\partial t^2} = -\frac{e^2}{4m \omega_{rf}^2} \frac{\partial E_0^2(X)}{\partial X} \]
Dynamic trapping

\[ m \frac{\partial^2 X(t)}{\partial t^2} = -\frac{e^2}{4m\omega_{rf}^2} \frac{\partial E_0^2(X)}{\partial X} \]

For electric quadruple:

\[ E_0(x) = \alpha \left[ \frac{V_{rf}}{d} \right] \frac{x}{d} \]

Pseudo-potential:

\[ U_{eff}(x) = \left[ \frac{\alpha^2 e^2 V_{rf}^2}{4m\omega_{rf} d^4} \right] x^2 \]

Harmonic frequency:

\[ \omega_{trap} \sim \frac{eV_{rf}}{m\omega_{rf}d^2} \]
Trapping

RF Paul trap

Potential:
\[ \Phi = \sum_i \left( \alpha_i + \beta_i V_0 \cos(\omega_{rf} t) \right) X_i^2 \]

Solve E.O.M:
\[ M \ddot{X} = F = -e \nabla \Phi \]

(Mathieu eq:
\[ \frac{d^2 X_i}{d\tau^2} + [a_i + 2q_i \cos(2\tau)]X_i = 0 \])

Stable solution:
\[ \omega_{trap} \sim \frac{eV_{RF}}{m\omega_{rf}d^2} \]

- Drive frequency \( \sim 20-30 \text{ MHz} \)
- RF amplitude \( \sim 200-300 \text{ V} \)
- Secular frequency
  - Radial \( \sim 2-3 \text{ MHz} \)
  - Axial \( \sim 1 \text{ MHz} \)

\( d \) – the distance between the ion and the electrodes
Weizmann trap.

- Laser machined Alumina

$0.6 \text{ mm}$

$0.3 \text{ mm}$

$0.2 \text{ mm}$

$1.2 \text{ mm}$

$200 \text{ V}_{pp} \text{ AC}$

$21 \text{ MHz}$

$10 \text{ V} \text{ DC}$

$\nu = 1 - 2 \text{ MHz (axial)}$

$\nu = 2 - 3 \text{ MHz (radial)}$

$\sim 2 - 5 \mu m$

Nitzan Akerman
Scale up?

One (out of many) problem: isolating one mode of motion for gates
The ion vision: Multiplexed trap array

interconnected multi-trap structure
subtraps completely decoupled

routing of ions by controlling electrode voltages

Subtrap for different purpose:
Gates, readout, etc.

D. J. Wineland, et al.,
J. Res. Nat. Inst. Stand. Technol. 103, 259 (1998);
D. Kielpinski, C. Monroe, and D. J. Wineland,
Multi-zone ion trap

- Gold on alumina construction
- RF quadrupole realized in two layers
- Six trapping zones
- Both loading and experimental zones
- One narrow separation zone
- Closest electrode $\sim 140 \, \mu m$ from ion
Ions can be moved between traps.
Electrode potentials varied with time
Ions can be separated efficiently in sep. zone
Small electrode’s potential raised
Motion (relatively) fast
Shuttling (adiabatic): several 10 μs
Separating: few 100 μs

6-zone alumina/gold trap
(Murray Barrett, John Jost)
Surface-electrode traps

Field lines:

Trapping center

Filter capacitor

Microfabricated filter resistor

Control lead

Elbows and tee-junctions possible:

RF electrodes
Control electrodes

Micro-fabricated traps

Trapped ions on the tabletop

- Vacuum \( \sim 10^{-11} \) Torr
- Room temperature (or a bit above)
- RF created with coax/helical resonator
- Atoms created by oven.
- Ions created by photo ionization.
- Approx. F1 imaging optics to EMCCD/PMT (res = 0.8 \text{ um}).
Harmonic oscillator levels

- Doppler cooling. \( \langle n \rangle \sim 3 - 20 \)

- Resolved side band cooling. \( \langle n \rangle \sim 0 \)
Sideband Spectroscopy

Scan the laser frequency across the S → D transition

\[ 4^2D_{5/2} \]

\[ 5^2S_{1/2} \]

\[ \Delta \nu < 80 \text{ Hz} \]

Carrier

Red sidebands

Blue sidebands

axial

radial
Resolved-sideband Cooling

$|P\rangle$  
$\rightarrow$  
$1033 \text{ nm}$  
$|D\rangle$

$|S\rangle$  
$\rightarrow$  
$674 \text{ nm}$

Blue sideband

Red sideband

$n = 0$  
$n = 1$  
$n = 2$  
$n = 3$

$\sim 4 \text{ MHz}$
Sideband cooling to the ground state

- $T \approx 2 \, \mu K$
- Uncertainty in ion position = ground state extent
  \[
  \sqrt{\frac{\hbar}{2m\omega_{ho}}} = 6nm
  \]

- $\langle n \rangle < 0.05$
Anomalous heating

- Fluctuating charges
- $f^{-1.5}$ noise
- Thermally activated
- Due to monolayer of C on electrodes: gone after Ar+ ion cleaning