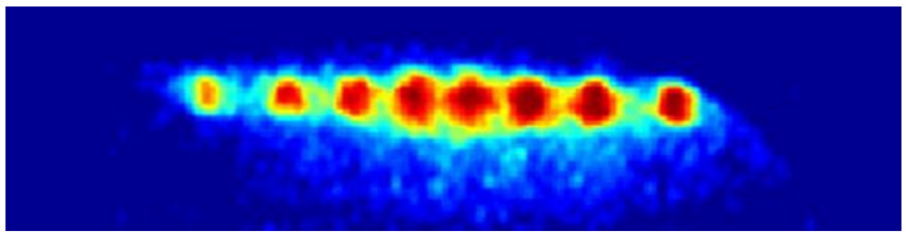
The trapped-ion qubit tool box

Contemporary Physics, 52, 531-550 (2011)
http://www.weizmann.ac.il/complex/ozeri/Course2



Roee Ozeri

Weizmann Institute of Science
Rehovot, 76100, Israel
ozeri@weizmann.ac.il

Physical Implementation of a quantum computer

David Divincenzo's criteria:

1. Well defined qubits.

2. Initialization to a pure state

3. Universal set of quantum gates.

4. Qubit specific measurement.

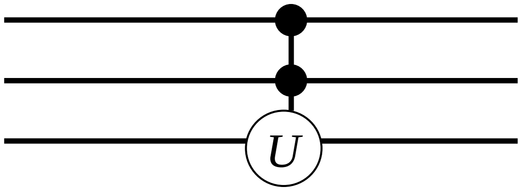
5. Long coherence times (compared with gate & meas. time).

Universal Gate set

- For N qubits, a general unitary transformation U acts on a 2^N- dimension Hilbert space.

- A *finite* set of unitary gates that spans any such U.

- The Deutsch-Toffoli gate



- D. Deutsch, *Proc. R. Soc. London*, Ser. A, **425**, 73, (1989).

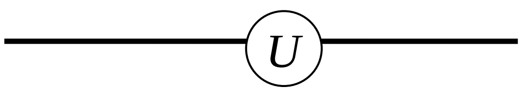
Universal Gate set

- For N-qubits, and unitary transformation U on a 2^N-dimension Hilbert space.

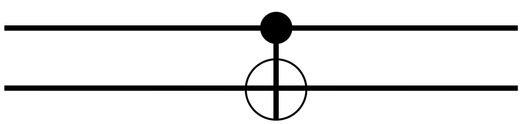
- A *finite* set of unitary gates that spans any such U.

Single qubit gate

$U \in SU[2]$



2-qubit C-not gate



- Rotations can be approximated to ε by concatenating k gates, from a finite set $\{V_i\}$, where $k < polylog(1/\varepsilon)$.

- Barenco *et. al. Phys. Rev. A*, **52**, 3457, (1995).

Physical Implementation of a quantum computer

David Divincenzo's criteria:

How well???

Well enough to allow for a large scale computation:
Fault tolerance

$$F = \langle \Psi | \rho_\epsilon | \Psi \rangle \qquad \epsilon = 1 - F$$

Fault-tolerant Quantum Computation

Noisy operations ϵ

One level quantum error-correction codes $O((\epsilon/\epsilon_0)^2)$

concatenation;
threshold theorem

k-levels of fault-tolerant encoding

$O((\epsilon/\epsilon_0)^{2^k}) \rightarrow 0$ if $\epsilon < \epsilon_0$

- ϵ_0 Fault tolerance threshold.
- Heavy resource requirements when $\epsilon \simeq \epsilon_0$
- Depends on code, noise model, arch. constraints etc.
- Current estimates for $\epsilon_0 \simeq 10^{-2} - 10^{-4}$

Tutorial overview

1. The ion-qubit: different ion-qubit choices, Ion traps.
2. Qubit initialization.
3. Qubit measurement.
4. Universal set of quantum gates:
   single qubit rotations; two-ion entanglement gates
5. Memory coherence times

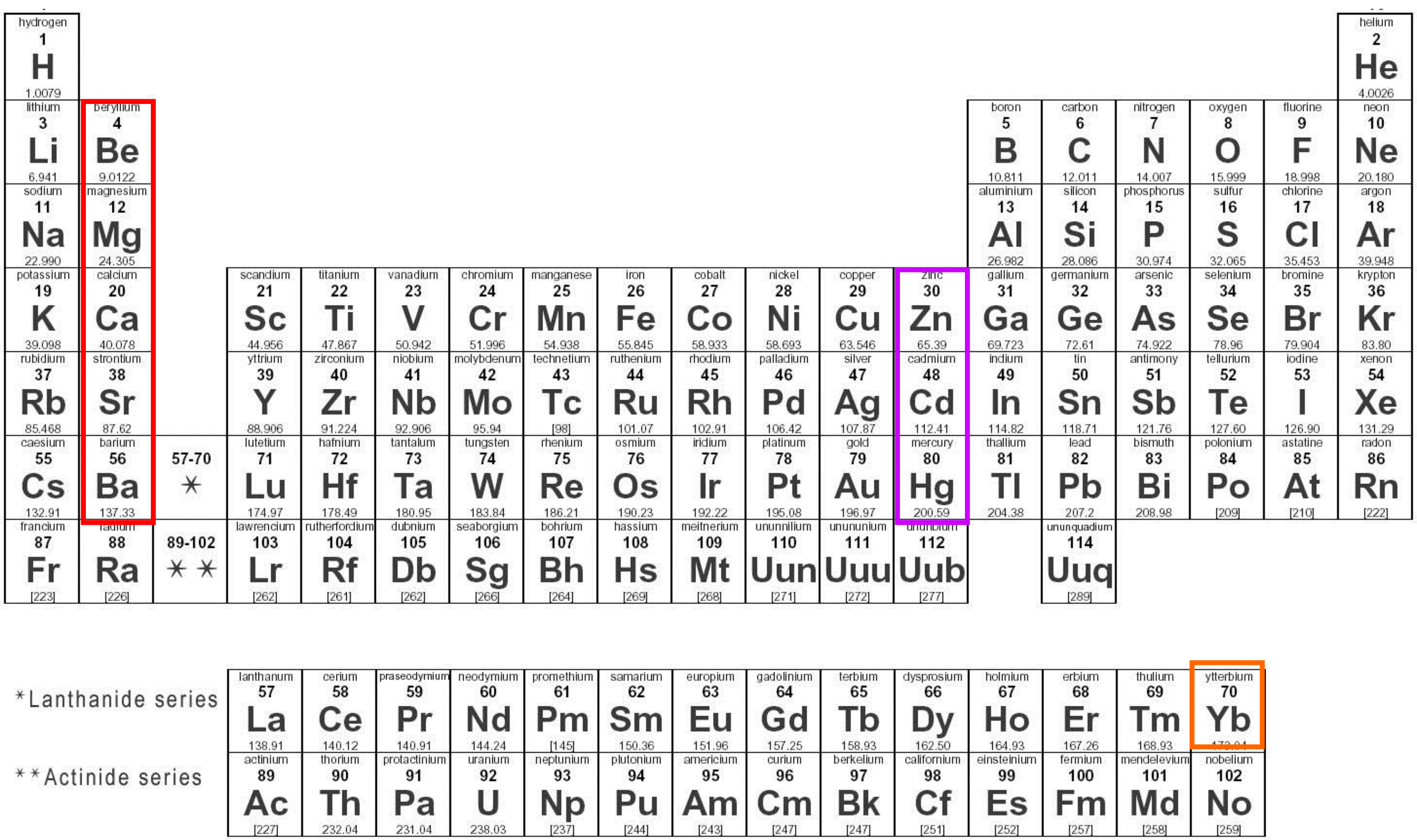
How well???

Benchmarked to current threshold estimates

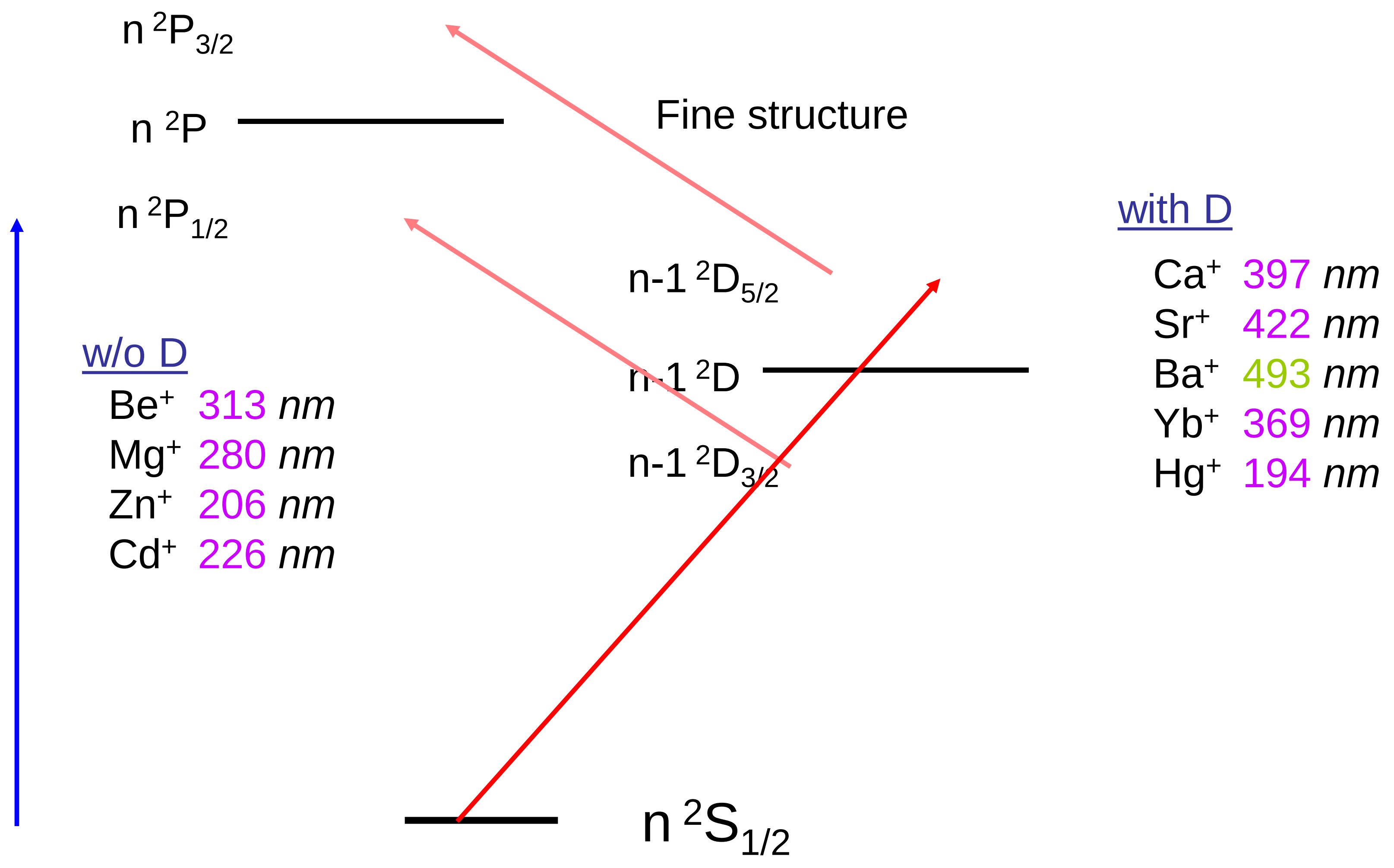
Disclaimer: non exhaustive; focuses on laser-driven gates

Different ion-qubit choises

- One electron in the valence shell; “Alkali like” $^2S_{1/2}$ ground state.



Electronic levels Structure



$^2S_{1/2}$ Zeeman qubit

(Isotopes w/o nuclear spin)

e.g.

$^{24}Mg^+$
$^{64}Zn^+$
$^{114}Cd^+$
$^{40}Ca+$
$^{88}Sr+$
$^{138}Ba+$
$^{174}Yb+$
$^{202}Hg+$

Advantages

- RF separation.
- Tunable.
- Infinite T_1.

Disadvantages

- Energy depends linearly on B.
- Transition photon caries no momentum
- Momentum transfer with off-resonance lasers: photon scattering.
- Detection.

m = 1/2 $|\uparrow\rangle$

$n\ ^2S_{1/2}$

m = -1/2 $|\downarrow\rangle$

Turn on small B field
2.8 *MHz/G*

$^2S_{1/2}$ Hyperfine qubit

e.g.

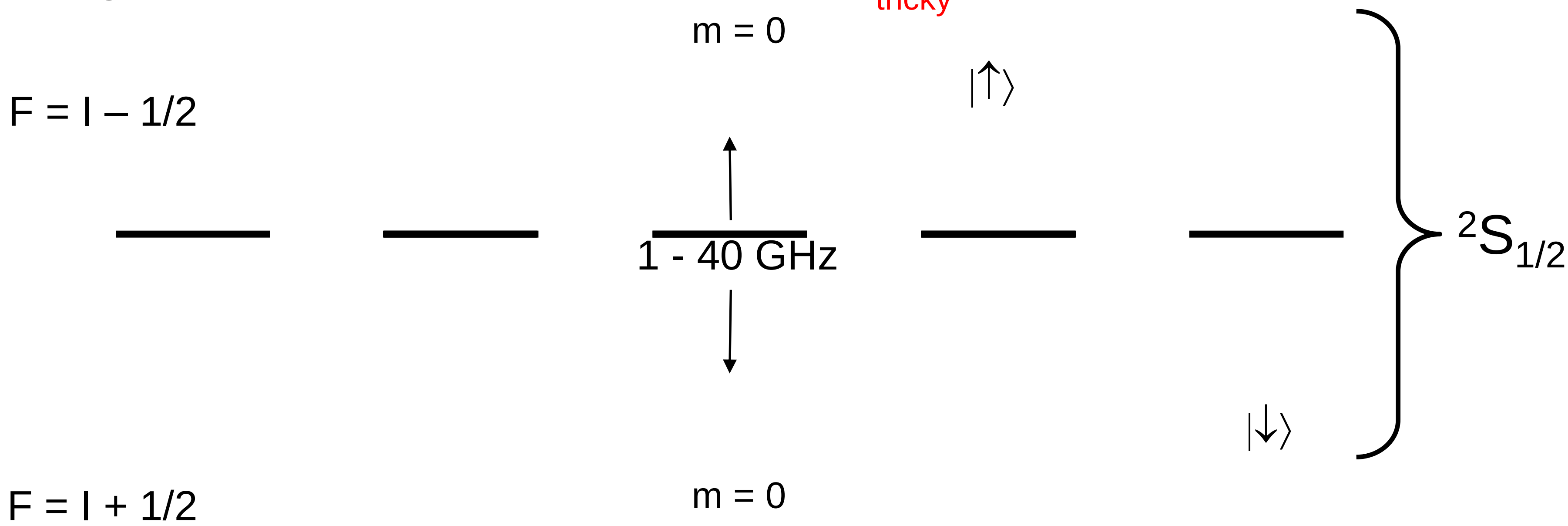
$^9Be^+$
$^{25}Mg^+$
$^{67}Zn^+$
$^{111}Cd^+$
$^{43}Ca+$
$^{87}Sr+$
$^{137}Ba+$
$^{171}Yb+$
$^{199}Hg+$

Hyperfine structure (order depends on the sign of A_{hf})

Turn on small B field

Advantages
- MW energy separation.
- B-field independent qubit.
- Infinite T_1.
- State selective fluorescence Detection.

Disadvantages
- Few GHz energy separation.
-Transition photon caries no momentum
- Off resonance photon scattering.
- initialization to clock transition can be more tricky



Optical qubit

$^2D_{5/2}$

Innsbruck/Weizmann

$|\uparrow\rangle$

with D

Ca^+	729 *nm*
Sr^+	674 *nm*
Ba^+	1760 *nm*
Yb^+	411 *nm*
Hg^+	282 *nm*

Advantages

- Single optical photon (momentum).
- B-field independent qubit (if nuclear spin ≠ 0).
- Hardly any spontaneous decay during gates.
- State selective fluorescence Detection: excellent discrimination.

Disadvantages

- Finite (~ 1 sec) $|\uparrow\rangle$ lifetime.
- Coherence time is limited by laser linewidth.

$^2S_{1/2}$

$|\downarrow\rangle$

Trapping

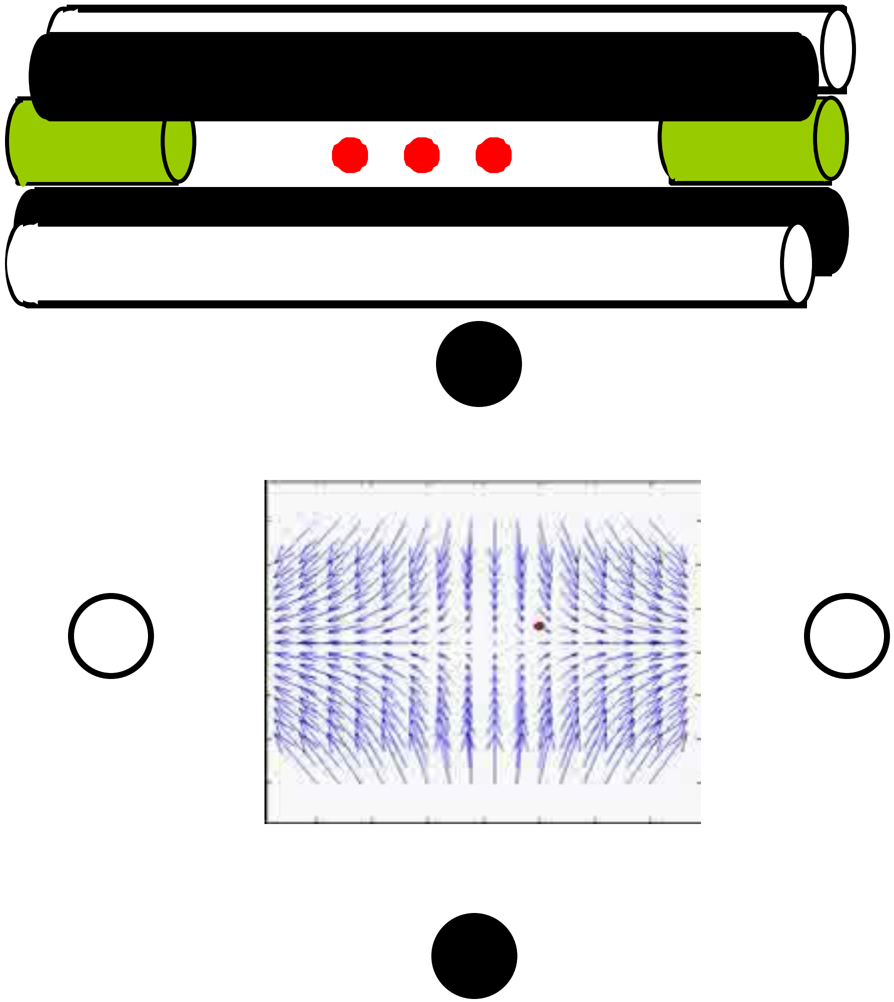
- Trap ions: a minimum/maximum to ϕ, the electric potential.

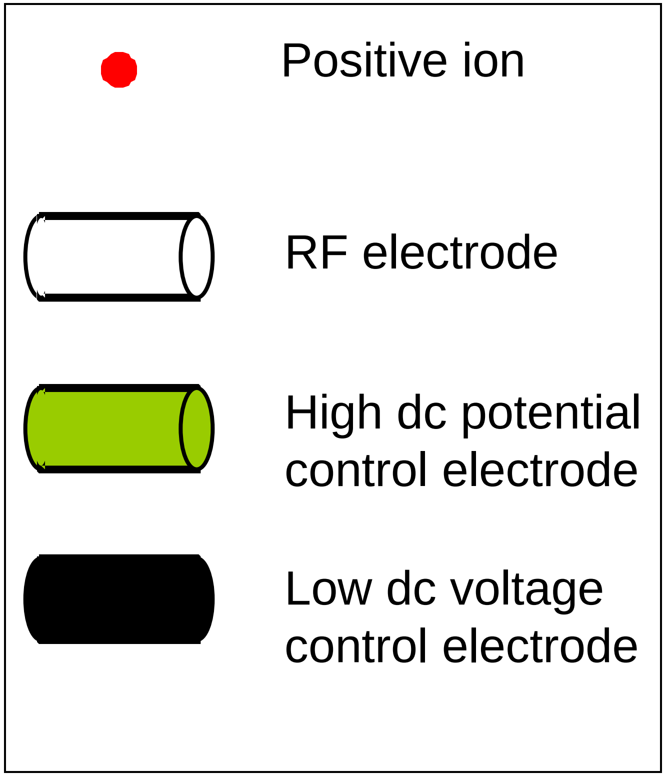
- Impossible in all directions; Laplace's equation:

$$\nabla^2\Phi = 0$$

Trapping

Linear RF Paul trap





Dynamic trapping (pondermotive forces)

Oscillating electric field:

$$E(x,t) = E_0(x)\cos(w_{rf} + \phi)$$

$$x(t) = X(t) + \xi(t)$$

Large and slow

$$X(t) \gg \xi(t)$$

small and fast

$$\frac{\partial^2 X(t)}{\partial t^2} \ll \frac{\partial^2 \xi(t)}{\partial t^2}$$

Newton's E.O.M:

$$m\left(\frac{\partial^2 X(t)}{\partial t^2} + \frac{\partial^2 \xi(t)}{\partial t^2}\right) = eE_0(x)\cos(\omega_{rf}t + \phi)$$

Dynamic trapping

$$m\left(\frac{\partial^2 X(t)}{\partial t^2}+\frac{\partial^2 \xi(t)}{\partial t^2}\right) = eE_0(x)\cos(\omega_{rf}t+\phi)$$

Field expansion:

$$E_0(x) = E_0(X) + \frac{\partial E_0(X)}{\partial X}\xi + O(\xi^2)$$

To 0th order:

$$m\frac{\partial^2 \xi(t)}{\partial t^2} = eE_0(X)\cos(\omega_{rf}t+\phi)$$

$$\xi(t) = -\frac{eE_0(X)}{m\omega_{rf}^2}\cos(\omega_{rf}t+\phi)$$

Next order:

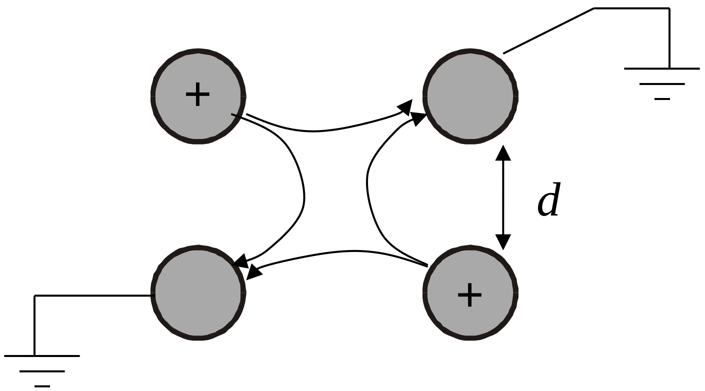
$$m\frac{\partial^2 X(t)}{\partial t^2} = -\frac{e^2E_0(x)}{m\omega_{rf}^2}\frac{\partial E_0(X)}{\partial X}\cos^2(\omega_{rf}t+\phi)$$

Average over one period:

$$m\frac{\partial^2 X(t)}{\partial t^2} = -\frac{e^2}{4m\omega_{rf}^2}\frac{\partial E_0^2(X)}{\partial X}$$

Dynamic trapping

$$m\frac{\partial^2 X(t)}{\partial t^2} = -\frac{e^2}{4m\omega_{rf}^2}\frac{\partial E_0^2(X)}{\partial X}$$



For electric quadruple:

$$E_0(x) = \alpha\left[\frac{V_{rf}}{d}\right]\frac{x}{d}$$

Sign of e

Pseudo-potential:

$$U_{eff}(x) = \left[\frac{\alpha^2 e^2 V_{rf}^2}{4m\omega_{rf}d^4}\right]x^2$$

Harmonic frequency:

$$\boxed{\omega_{trap} \sim \frac{eV_{rf}}{m\omega_{rf}d^2}}$$

Trapping

RF Paul trap

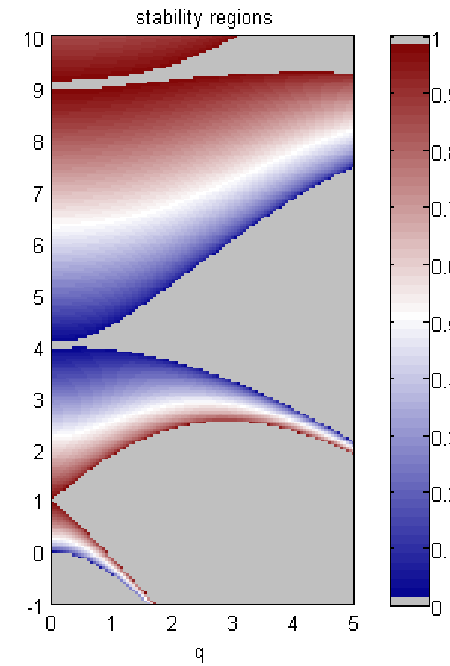
Potential :

$$\Phi = \sum_i \left(\alpha_i + \beta_i V_0 \cos(\omega_{rf} t)\right) X_i^{\,2}$$

Solve E.O.M :

$$M\ddot{\vec{X}} = F = -e\nabla\Phi$$

(Mathieu eq': $\frac{d^2 X_i}{d\tau^2} + [a_i + 2q_i \cos(2\tau)]X_i = 0$)



Stable solution:

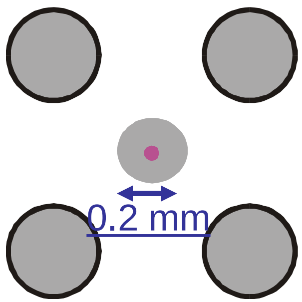
$$\boxed{\omega_{trap} \sim \frac{eV_{RF}}{m\omega_{rf} d^2}}$$

d – the distance between the ion and the electrodes

- Drive frequency ~ 20-30 MHz
- RF amplitude ~ 200-300 V
- Secular frequency
  - Radial ~ 2-3 MHz
  - Axial ~ 1 MHz

Weizmann trap.

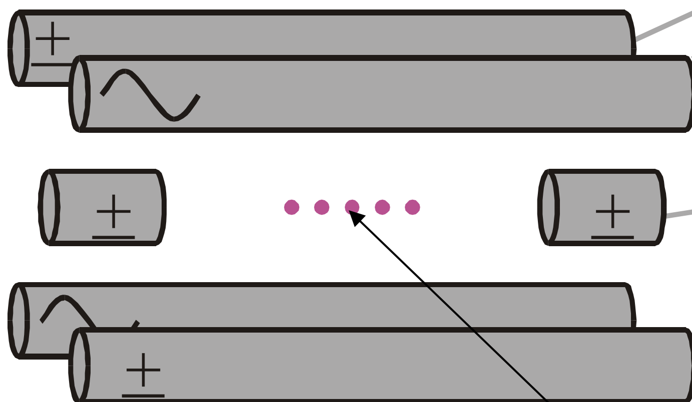
0.6 mm



0.3 mm

0.2 mm

- Laser machined Alumina

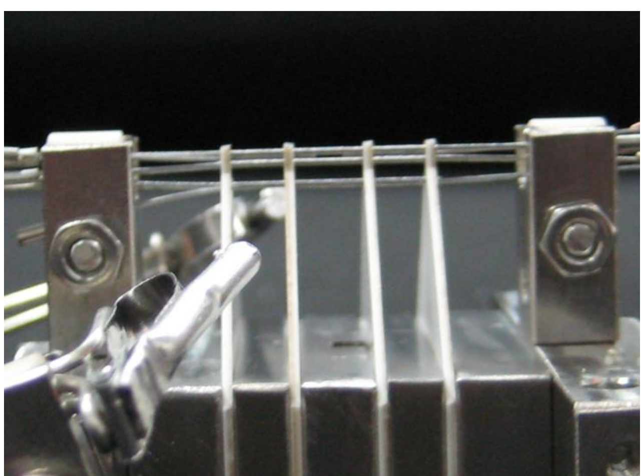


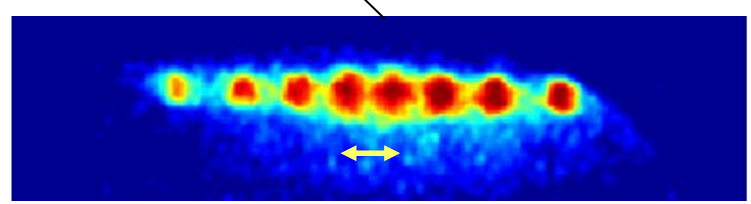
$200\ V_{pp}$ AC
21 MHz

10 V DC

$\nu_z = 1 – 2$ MHz (axial)
$\nu_r = 2 – 3$ MHz (radial)

1.2 mm

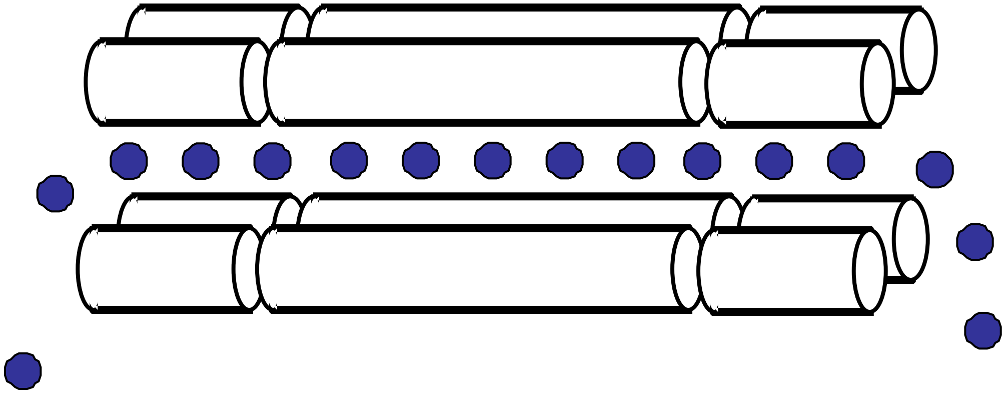




~ 2-5 μm

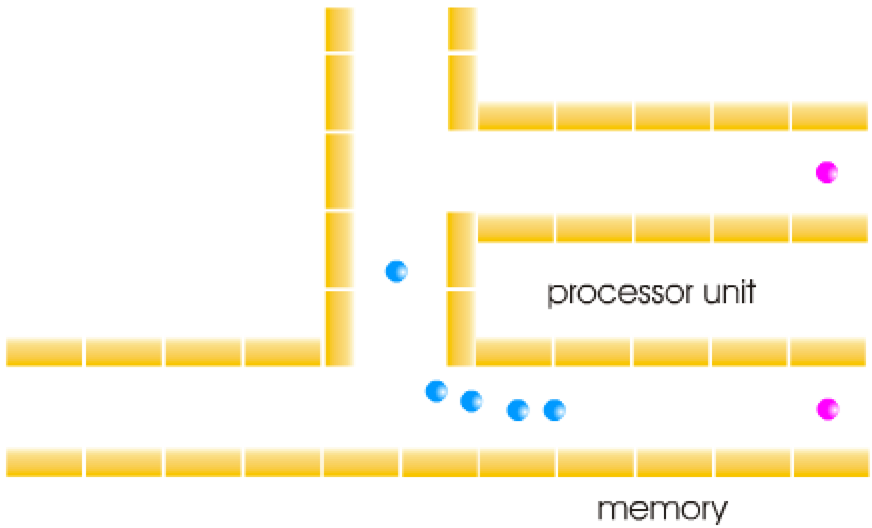
Nitzan Akerman

Scale up?



One (out of many) problem:
isolating one mode of motion for gates

The ion vision: Multiplexed trap array



interconnected multi-trap structure
subtraps completely decoupled

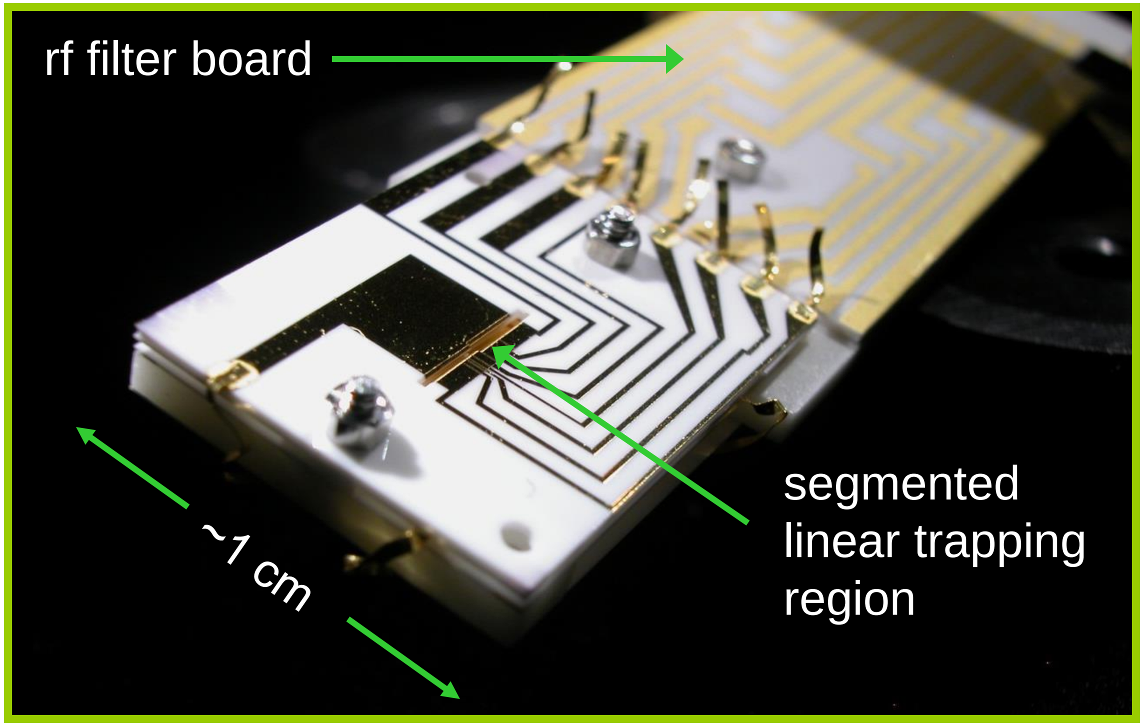
routing of ions by controlling electrode voltages

Subtrap for different purpose:
Gates, readout, etc.

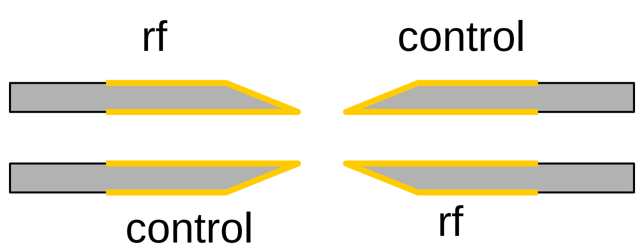
D. J. Wineland, *et al.*,
J. Res. Nat. Inst. Stand. Technol. **103**, 259 (1998);
D. Kielpinski, C. Monroe, and D. J. Wineland,
Nature **417**, 709 (2002).
Other proposals: DeVoe, Phys. Rev. A **58**, 910 (1998) .
Cirac & Zoller, Nature **404**, 579 (2000) .

Multi-zone ion trap

NIST



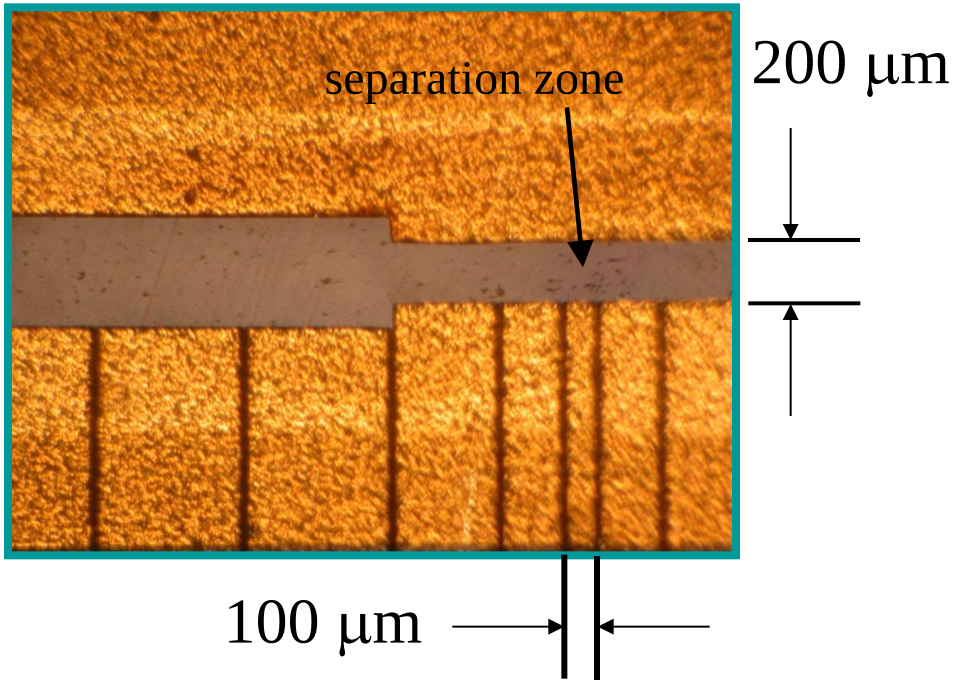
view along axis:



- Gold on alumina construction
- RF quadrupole realized in two layers
- Six trapping zones
- Both loading and experimental zones
- One narrow separation zone
- Closest electrode ~140 μm from ion

Ion transport

NIST



6-zone alumina/gold trap

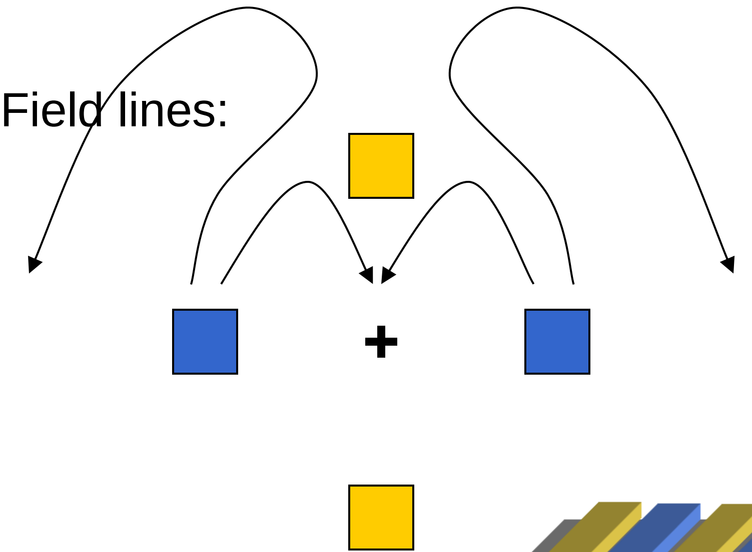
(Murray Barrett, John Jost)

- Ions can be moved between traps.
  - Electrode potentials varied with time
- Ions can be separated efficiently in sep. zone
  - Small electrode's potential raised
- Motion (relatively) fast
  - Shuttling (adiabatic): several 10 μs
  - Separating: few 100 μs

NIST

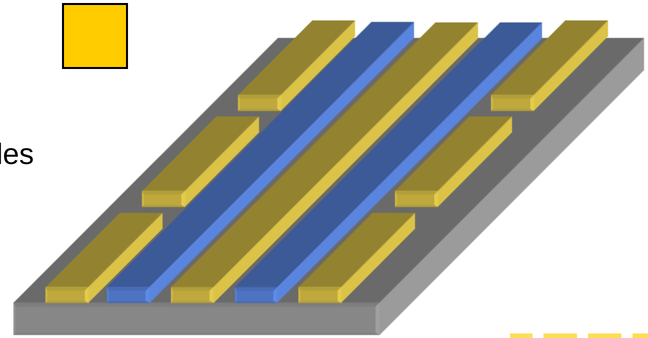
Surface-electrode traps

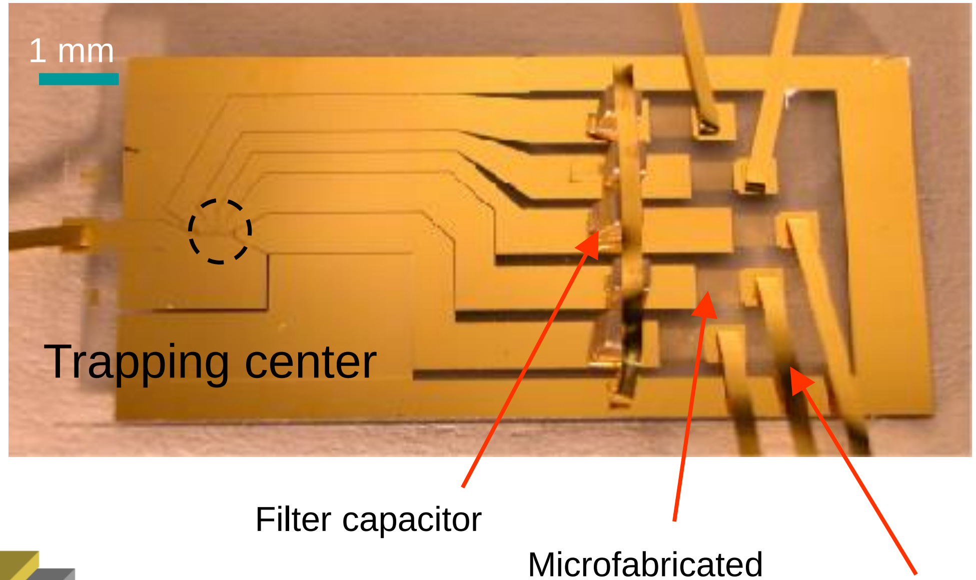
Field lines:



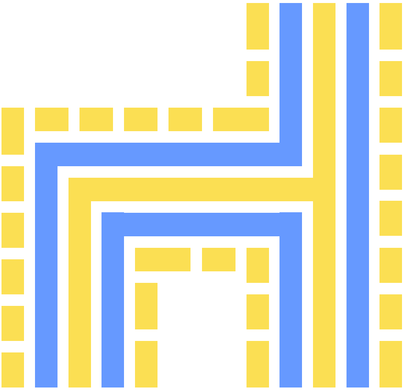
RF electrodes

Control electrodes





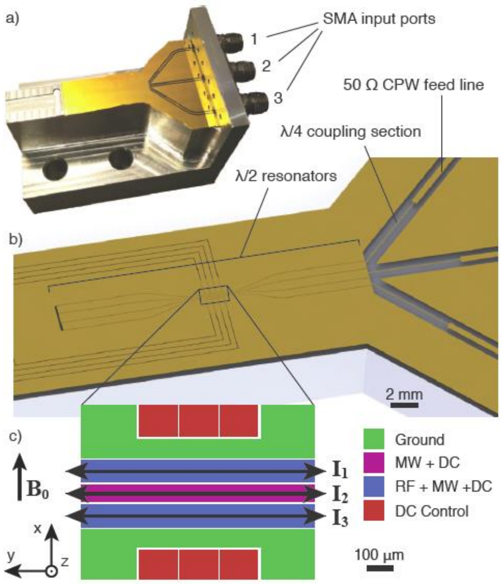
Elbows and tee-junctions possible:



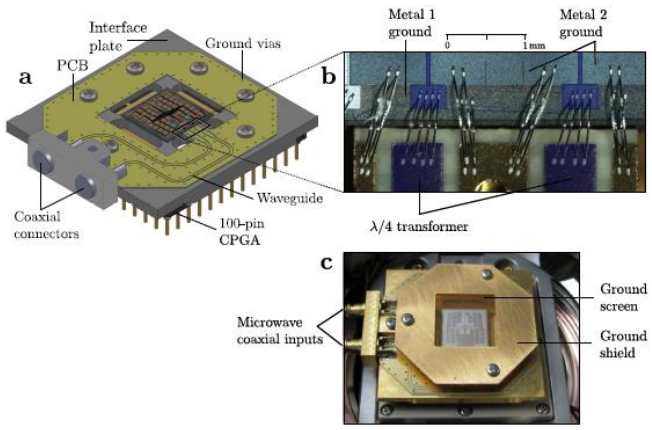
J. Chiaverini *et al*. Quant. Inform. Comp. 5, 419 (2005).

S. Seidelin *et al*. Phys. Rev. Lett. 96, 253003 (2006).

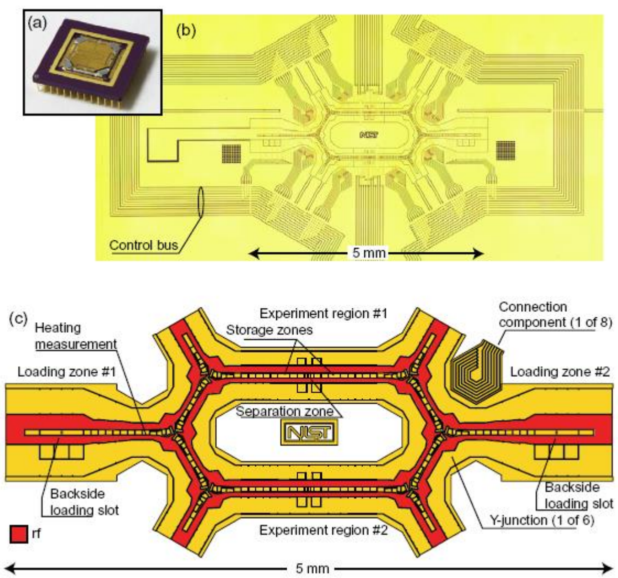
Micro-fabricated traps



Allcock; Oxford



Shappert; Georgia Tech



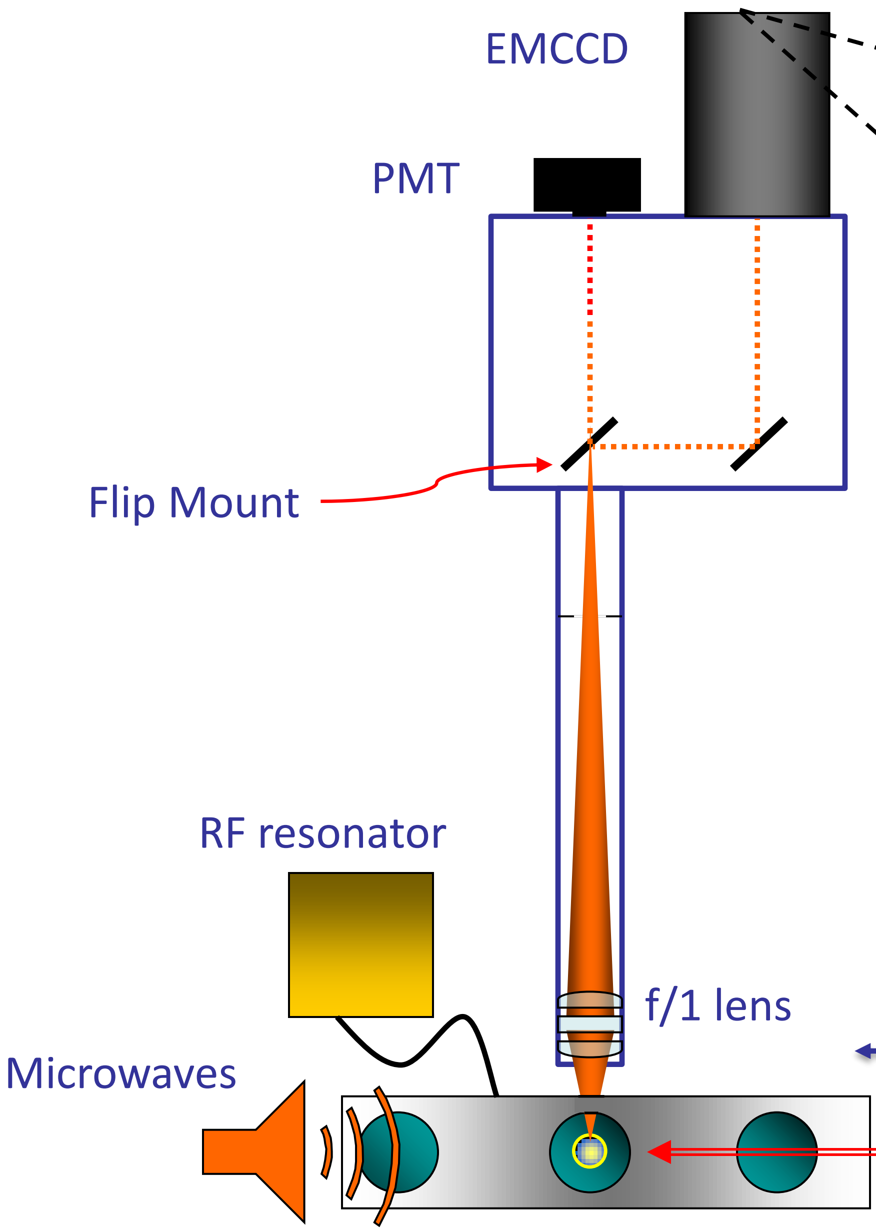
Amini; NIST

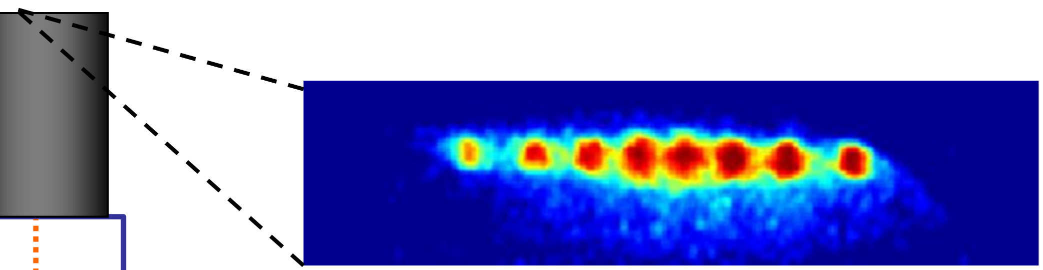
Amini *et al*. New J. Phys. 12, 033031 (2010).

Allcock *et al*. App. Phys. Lett. 102, 044103 (2012).

Shappert *et al*. arXiv quant-ph\1304. 6636 (2013).

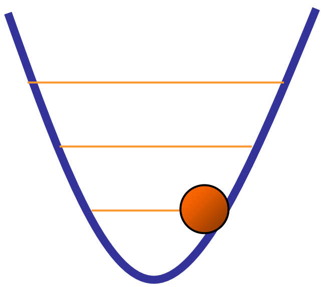
Trapped ions on the tabletop

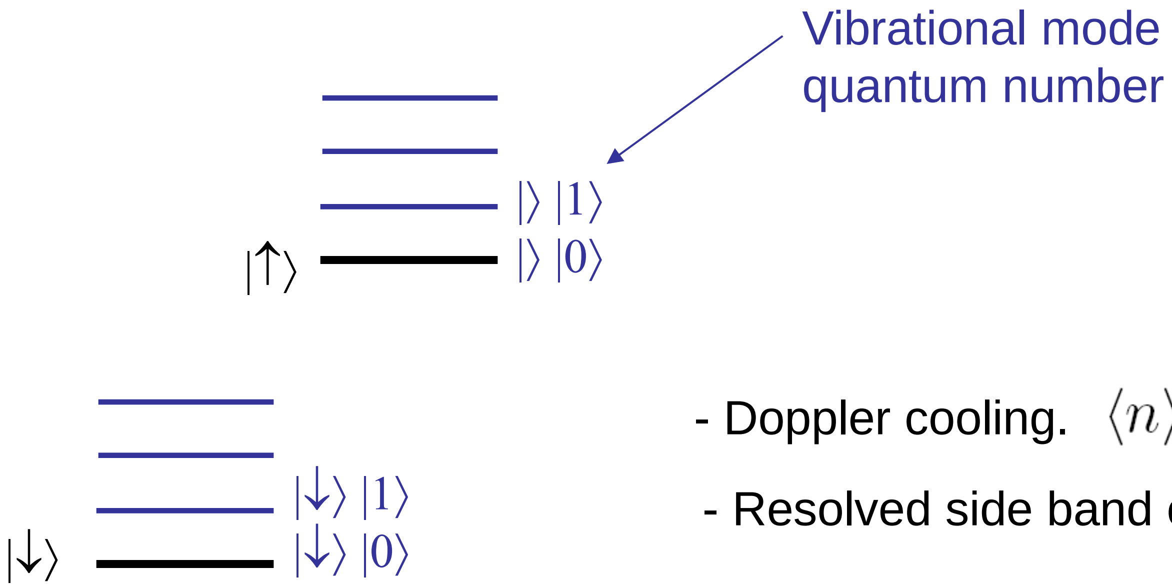




- Vacuum ~ 10^{-11} Torr
- Room temperature (or a bit above)
- RF created with coax/helical resonator
- Atoms created by oven.
- Ions created by photo ionization.
- Approx. F1 imaging optics to EMCCD/PMT (res = 0.8 um).

Harmonic oscillator levels

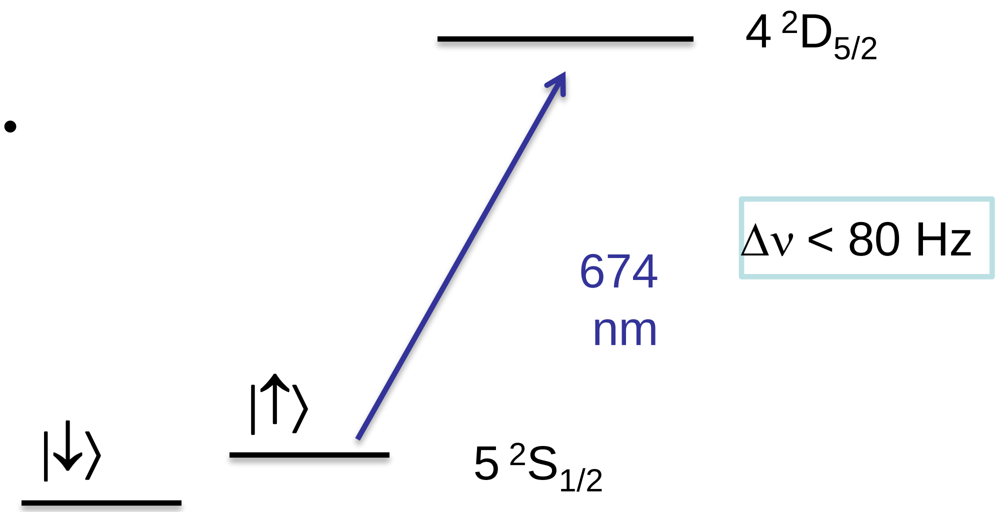


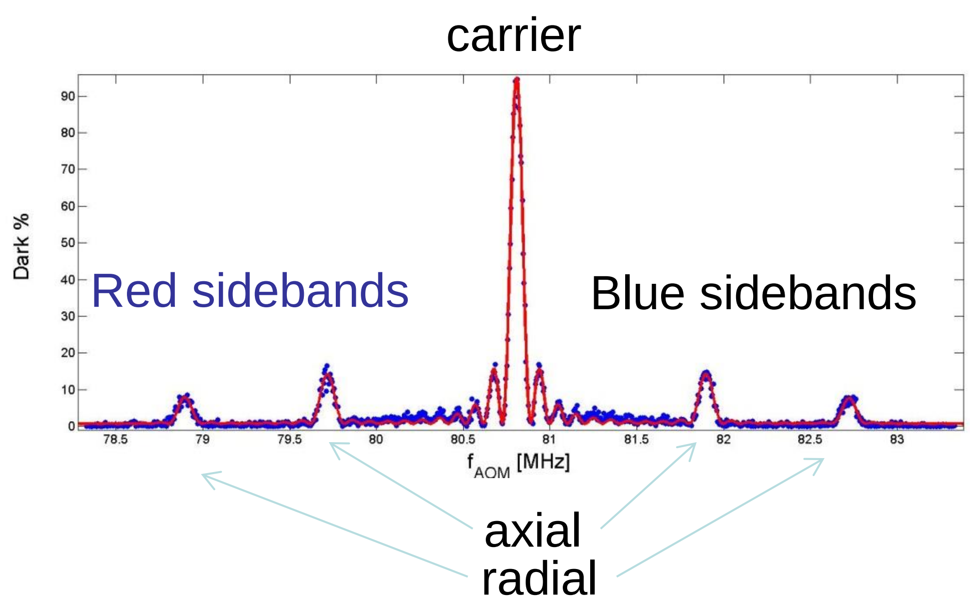


- Doppler cooling. $\langle n \rangle \sim 3 - 20$
- Resolved side band cooling. $\langle n \rangle \sim 0$

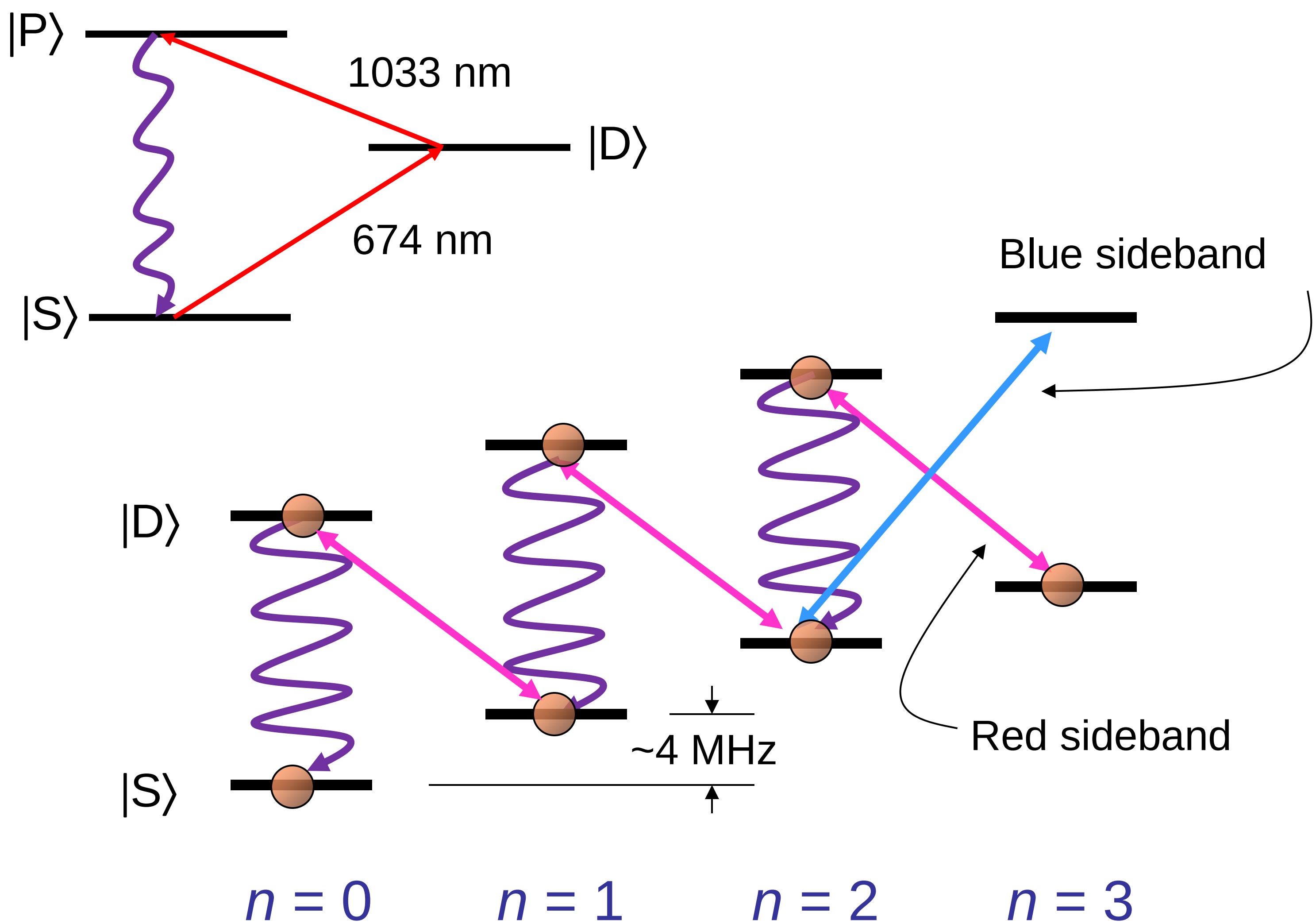
Sideband Spectroscopy

- Scan the laser frequency across the S →D transition

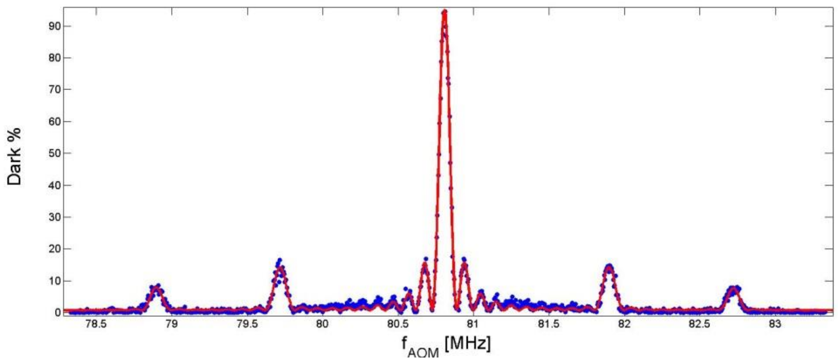


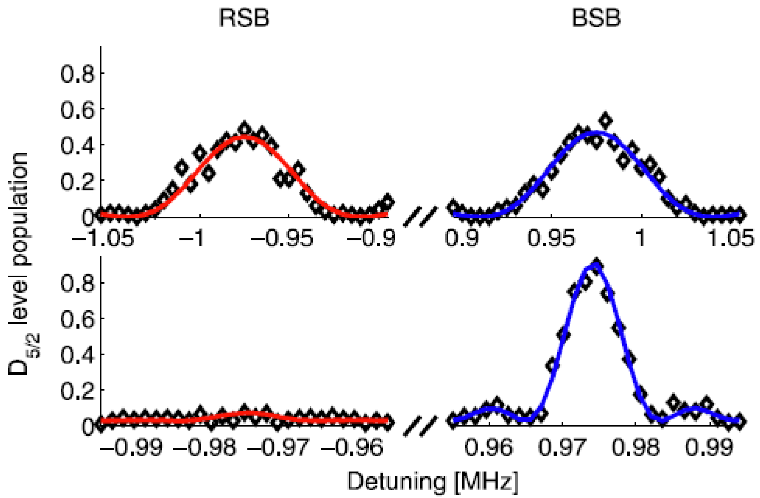


Resolved-sideband Cooling



Sideband cooling to the ground state

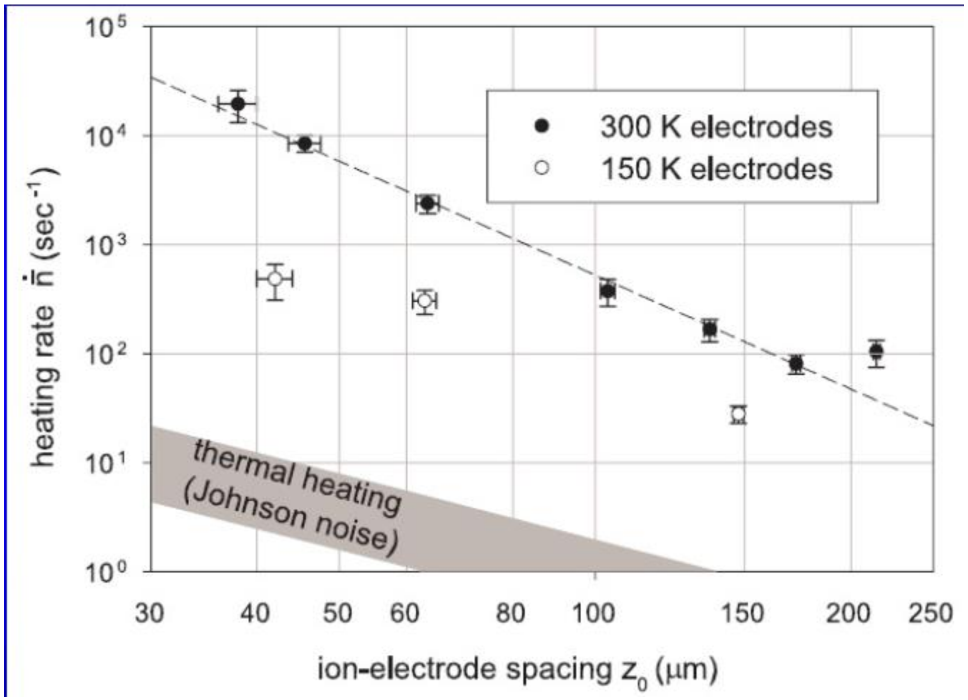




$$\langle n \rangle < 0.05$$

- T $\approx$ 2 μK
- Uncertainty in ion position = ground state extent $\sqrt{\dfrac{\hbar}{2m\omega_{ho}}} = 6nm$

Anomalous heating



$$\dot{n} \propto \frac{1}{z_o^{3.5}}$$

- Fluctuating charges
- $f^{-1.5}$ noise
- Thermally activated
- Due to monolayer of C on eletrodes: gone after Ar+ ion cleaning

Deslauriers *et al*. Phys. Rev. Lett. 97, 103007 (2006)

Hite *et al*. Phys. Rev. Lett. 109, 103001 (2012)