Different ion-qubit choices

- One electron in the valence shell; “Alkali like” $^2S_{1/2}$ ground state.
Electronic levels Structure

\[ n^2P_{3/2} \]
\[ n^2P \]
\[ n^2P_{1/2} \]

Fine structure

\[ n-1^2D_{5/2} \]
\[ n-1^2D \]
\[ n-1^2D_{3/2} \]

w/o D
Be\(^+\) 313 nm
Mg\(^+\) 280 nm
Zn\(^+\) 206 nm
Cd\(^+\) 226 nm

with D
Ca\(^+\) 397 nm
Sr\(^+\) 422 nm
Ba\(^+\) 493 nm
Yb\(^+\) 369 nm
Hg\(^+\) 194 nm
$^2S_{1/2}$ Zeeman qubit

(Isotopes w/o nuclear spin)

e.g.

- $^{24}\text{Mg}^+$
- $^{64}\text{Zn}^+$
- $^{114}\text{Cd}^+$
- $^{40}\text{Ca}^+$
- $^{88}\text{Sr}^+$
- $^{138}\text{Ba}^+$
- $^{174}\text{Yb}^+$
- $^{202}\text{Hg}^+$

Advantages
- RF separation.
- Tunable.
- Infinite $T_1$.

Disadvantages
- Energy depends linearly on $B$.
- Transition photon carries no momentum.
- Momentum transfer with off-resonance lasers: photon scattering.
- Detection.

Turn on small $B$ field

$2.8 \text{ MHz/G}$
$^2S_{1/2}$ Hyperfine qubit

Hyperfine structure (order depends on the sign of $A_{hf}$)

Turn on small B field

Advantages
- MW energy separation.
- B-field independent qubit.
- Infinite $T_1$.
- State selective fluorescence Detection.

Disadvantages
- Few GHz energy separation.
- Transition photon carries no momentum
- Off resonance photon scattering.
- Initialization to clock transition can be more tricky

$F = I - 1/2$

$F = I + 1/2$

$|\uparrow\rangle$

$m = 0$

$|\downarrow\rangle$

$1 - 40 \text{ GHz}$
Optical qubit

Innsbruck/Weizmann

with D

| Ca⁺ | 729 nm |
| Sr⁺ | 674 nm |
| Ba⁺ | 1760 nm |
| Yb⁺ | 411 nm |
| Hg⁺ | 282 nm |

Advantages
- Single optical photon (momentum).
- B-field independent qubit (if nuclear spin ≠ 0).
- Hardly any spontaneous decay during gates.
- State selective fluorescence Detection: excellent discrimination.

Disadvantages
- Finite (~ 1 sec) $|\uparrow\rangle$ lifetime.
- Coherence time is limited by laser linewidth.
Qubit Initialization

Zeeman qubit

- Optical pumping into a dark state.
- CPT possible into any superposition..

Error sources:
- Polarization purity.

\[
\begin{align*}
\epsilon_{\text{init}} &= \frac{\epsilon_\pi}{1 + \delta_\pi^2/\gamma'^2} + \frac{\epsilon_{\sigma^-}}{1 + \delta_{\sigma^-}^2/\gamma'^2} \\
\gamma' &= \frac{\gamma}{2} \sqrt{1 + S_0} \\
\epsilon &\sim 10^{-6} - 10^{-3}
\end{align*}
\]
Qubit Initialization

Zeeman qubit

Limited to $10^{-3}$ due to stress-induced birefringence in vacuum chamber optical viewports

Process tomography of optical pumping
Qubit Initialization

Zeeman qubit: the D level option

- Limited by off-resonance coupling of $\downarrow$ to D
- Power broadening and incoherent noise
- $\varepsilon = 10^{-4}$

$\gamma = 0.4 \text{ Hz}$

Qubit Initialization

Hyperfine qubit

- Optical pumping into a dark state.

Estimated:

$$\epsilon \leq 2 \times 10^{-5}$$

Qubit Initialization

- $|\downarrow\rangle$ state initialization: same as previous.

- $|\uparrow\rangle$ state initialization:
  - Rapid adiabatic passage.
    \[ \varepsilon \sim 10^{-2} \]
    (Wunderlich et. al. Journal of Modern Optics 54, 1541 (2007))
  - $\pi$-pulse.
    \[ \varepsilon \sim 10^{-2} \]
Measurement: state selective fluorescence

Optical qubit

$\text{n}^2P_{3/2}$

Fine structure

$\text{n}^2P_{1/2}$

After 200 $\mu$sec:

Error sources:
- Dark counts (10/S)
- Laser scatter (100/S)
- State decay.

Threshold test

$\text{n}^2S_{1/2}$
Zeeman Qubit Detection

\[ ^2\text{P}_{3/2} \]
\[ ^2\text{P}_{1/2} \]

Resonance fluorescence

422 nm

1092 nm

\[ ^2\text{S}_{1/2} \left| \downarrow \right\rangle \]

\[ ^2\text{D}_{5/2} \gamma = 0.4 \text{ Hz} \]

\[ ^2\text{D}_{3/2} \]

Double Shelving

\[ ^2\text{S}_{1/2} \left| \uparrow \right\rangle \]
$^{88}\text{Sr}^+$ Zeeman Qubit Detection

Expected distributions:
- Bright photon detection rate: 73.5 kHz
- Dark photon detection rate: 1.75 kHz
- D level lifetime: 390 ms
- Detection time: 285 μs

Measured for $|\uparrow\rangle$

Initialization and shelving error: $\varepsilon_\uparrow = 1 \times 10^{-3}$; $\varepsilon_\downarrow = 0.6 \times 10^{-3}$

$^{88}$Sr$^+$ Zeeman Qubit Detection

State discrimination error $(10^{-4})$

Minimal State discrimination error = $0.3 \times 10^{-3}$ @ $\tau_{det} = 285 \ \mu s$ and $n_{threshold} = 6$

Average Detection fidelity: 0.9989

Measurement: state selective fluorescence

Hyperfine qubit

Error Sources:
- Polarization purity (Bright -> dark optical pumping).
- Off resonance dark -> bright optical pumping.
- Dark counts (10/S).
- Laser scatter (100/S).

Benefit from:
- Large hyperfine splitting.
- Large angular momentum splitting between bright and dark states.

Threshold test: $\varepsilon \sim 8 \times 10^{-5}$

Measurement: Photon arrival time analysis

- Maximum likelihood test:  $\varepsilon \sim 8.7 \times 10^{-5}$

(Myerson et. al. Phys. Rev. Lett. 100, 200502 (2008); Oxford ions)

225 $\mu$sec

(As compared with $1.8 \times 10^{-4}$ threshold error in 420 $\mu$s)
Measurement with CCD

Antiferromagnetic ground-states in quantum magnetism (JQI, Maryland)

- Multiple ions: histograms overlap
- Which ion is bright?
- Slow readout
- Readout noise
- Cross-talk

Highest Camera fidelity (optical qubit; Oxford): 0.9991

Acton et. al., Quant. Inf. Comp. 6, 465, (2006)
Measurement: Other

- Photon detection efficiency:

  0.6 NA gives 0.99 in 10 µs and 0.9915 in 100 µs (hyperfine qubit, Duke)
  Noek et. al. arXiv1304.3511 (2013)

- Ancila qubits:

  • Entangled ancila for twice the fluorescence (hyperfine qubit; NIST)
    Schaetz et. al. PRL 94, 010501 (2005)

  • State transfer to a different species ion (optical qubit; NIST)
    Hume et. al. PRL 99, 137205 (2007)
Tutorial overview

1. The ion-qubit: different ion-qubit choices, Ion traps.
2. Qubit initialization.
3. Qubit measurement.
4. Universal set of quantum gates: 
   single qubit rotations; 
   two-ion entanglement gates
5. Memory coherence times

How well???
Benchmarked to current threshold estimates

Disclaimer: non exhaustive; focuses on laser-driven gates
Single qubit gates

Reviewed in many places e.g. :

- Coupling between the two qubit levels using e.m. traveling plane waves (far-field).

For a single ion:

\[ \hat{H}(t) = \hat{H}_0 + \hat{V}(t) \]

\[ \hat{H}_0 = \frac{1}{2} \hbar \omega_0 \hat{\sigma}_z + \hbar \omega_m (\hat{a}^\dagger \hat{a} + \frac{1}{2}) \quad \text{and} \quad \hat{V}(t) = \hbar \Omega_0 (\hat{\sigma}^+ + \hat{\sigma}^-) \cos (k \hat{x} - \omega t + \phi) \]

\[ \hat{\sigma}_- = \hat{\sigma}_x + i \hat{\sigma}_y = | ↓ \rangle \langle \uparrow | \]

\[ \hat{\sigma}_+ = \hat{\sigma}_x - i \hat{\sigma}_y = | \uparrow \rangle \langle \downarrow | \]

Where:

\[ x_0 = \sqrt{\frac{\hbar}{2M \omega_m}} \]

\[ k \hat{x} = k x_{eq} + k x_0 (\hat{a}^\dagger + \hat{a}) \equiv k x_{eq} + \eta (\hat{a}^\dagger + \hat{a}) \]

Typ. 0.05-0.2 for optical \( k \)
Single qubit gates

In the interaction representation and within the Rotating Wave Appr. (RWA)

\[ H_{\text{int}}(t) = \hbar \Omega_0 / 2 \hat{\sigma}_+ \exp(i\eta(\hat{a}e^{-i\omega_m t} + \hat{a}^\dagger e^{i\omega_m t}))e^{i(k_{eq}+\phi-\delta_t)} + H.C. \]

When \( \delta = s \omega_m \), only \( |\downarrow, n\rangle \) and \( |\uparrow, n + s\rangle \) will be resonantly coupled (another RWA).

\[ \Omega_{n,n+s} = \Omega_{n,s,n} = \Omega_0 |\langle n + s|e^{i\eta(\hat{a}+\hat{a}^\dagger)}|n\rangle| \equiv \Omega_0 D_{n+s,n} \]

\[ D_{n+s,n} = \exp\left(-\eta^2/2\right)\eta^{|s|} \left( \frac{n_{<}}{n_{>}} \right)^{1/2} L_{n_{<}}^{|s|}(\eta^2) \]

**Carrier:** \( s = 0 \)

\[ \hat{H}_{\text{carrier}} = \frac{\hbar \Omega_{n,n}}{2}(\hat{\sigma}_+ \exp (i\phi) + \hat{\sigma}_- \exp (-i\phi)) \]

**Red sideband (RSB):** \( s = -1 \)

\[ \hat{H}_{\text{RSB}} = \frac{\hbar \Omega_{n-1,n}}{2}(\hat{a}\hat{\sigma}_+ \exp (i\phi) + \hat{a}^\dagger \hat{\sigma}_- \exp (-i\phi)) \]

**Blue sideband (BSB):** \( s = +1 \)

\[ \hat{H}_{\text{BSB}} = \hat{H}_{\text{int}} = \frac{\hbar \Omega_{n+1,n}}{2}(\hat{a}^\dagger \hat{\sigma}_+ \exp (i\phi) + \hat{a}\hat{\sigma}_- \exp (-i\phi)) \]
Lamb – Dicke regime

\[ \eta \sqrt{\langle (\hat{a}^\dagger + \hat{a})^2 \rangle} \ll 1 \]

\[ \Omega_{n,n} \approx \Omega_0 \left[ 1 - (n + 1/2)\eta^2 \right], \]

\[ \Omega_{n-1,n} \approx \Omega_0 n^{1/2} \eta, \]

\[ \Omega_{n+1,n} \approx \Omega_0 (n + 1)^{1/2} \eta. \]

- Momentum conservation.

- Bosonic amplification.
Single qubit gates

**Carrier rotations:**

Since $H_{int}$ is $t$ independent:

$$|\psi(t)\rangle_{int} = e^{-i\tilde{H}_{int}t/\hbar} |\psi(0)\rangle_{int} = e^{-i\theta \hat{\sigma} \cdot \tilde{n}} |\psi(0)\rangle_{int} \equiv \hat{R}(\theta, \phi) |\psi(0)\rangle_{int}$$

$$\begin{align*}
\theta &= \Omega_0 t \\
\tilde{n} &= \begin{pmatrix} \cos(\phi) \\ i \sin(\phi) \\ 0 \end{pmatrix}
\end{align*}$$

$$\hat{R}(0, \phi, \theta) = \begin{bmatrix}
\cos(\theta/2) & -ie^{i\phi} \sin(\theta/2) \\
-ie^{-i\phi} \sin(\theta/2) & \cos(\theta/2)
\end{bmatrix}$$
Coherent qubit (carrier) rotations

\[ R(\theta, \phi) \]

\[ R(\pi, 0) \]

\[ R(\pi/2, \pi/2) \]

Any single qubit rotation can be composed of 1-3 pulses.
Single qubit gates

RF qubit (Zeeman or Hyperfine)

Magnetic dipole coupling

\[ V(t) = -\hat{\mu} \cdot B_0 \cos(k\mathbf{x} - \omega t + \phi) \]

\[ \hat{\mu} = \mu_B (g_S \hat{S} + g_L \hat{L} + g_I \hat{I}) \quad \Rightarrow \quad \Omega_0 = \langle \downarrow |\hat{\mu} \cdot \mathbf{B}_0| \uparrow \rangle \]

e.g. for a Zeeman qubit:

\[ \hat{\mu} = g_S \mu_B \hat{S} \]

\[ \approx 2 \quad \approx 1.4 \text{ MHz/G} \]

\[ \Omega_0 = 2\pi \times 2.8 B_0 \text{ MHz/G} \]

Advantages
- Very classical and controlled.

Disadvantages
- No momentum transfer ($\eta = 0$)
- No single qubit addressing.
  (... In the far field)
Single qubit gates
RF qubit (Zeeman or Hyperfine)

Magnetic dipole coupling

e.g. $^{88}\text{Sr}^+$ Zeeman qubit

$\varepsilon = 2 \times 10^{-3}$

Error sources:
- Fluctuations in RF power.
- Relative phase/frequency noise (e.g. B field noise).
Process Tomography of the Identity operation

\[ E_1 = \hat{1}, \quad E_2 = \hat{\sigma}_x, \quad E_3 = -i\hat{\sigma}_y, \quad E_4 = \hat{\sigma}_z \]

Process tomography Fidelity

\[ F_{proc} = Tr(\chi_{ideal} \chi_{proc}) = 0.997(1) \]

Single qubit gates

RF qubit (Zeeman or Hyperfine)

Two-photon Raman coupling

\[ \begin{align*}
{^2P}_{3/2} & \quad \downarrow \quad \Delta \\
{^2P}_{1/2} & \\
\end{align*} \]

\[ \begin{align*}
\{ |\downarrow\rangle \} & \quad |\downarrow\rangle |1\rangle \quad |\downarrow\rangle |0\rangle \quad \downarrow \quad \omega_0 \\
{^2S}_{1/2} & \\
\end{align*} \]
Single qubit gates

RF qubit (Zeeman or Hyperfine)

Two-photon Raman coupling
- Including the $^2P$ levels we have three (or more) level coupling.
- For large enough $\Delta$ excited states are “adiabatically eliminated”.
- Back to “effective” two level coupling.

$$\vec{E}_r = \hat{\epsilon}_r E_{r0} \cos(\vec{k}_r \cdot \hat{x} - \omega_r t + \phi_r) \quad \vec{E}_b = \hat{\epsilon}_b E_{b0} \cos(\vec{k}_b \cdot \hat{x} - \omega_b t + \phi_b)$$

For a single excited state:

$$\phi = \phi_b - \phi_r$$

$$\Omega_0 = \frac{E_{r0}E_{b0}}{4\hbar^2} \sum_i \frac{\langle \uparrow | \hat{d} \cdot \hat{e}_r | e_i \rangle \langle e_i | \hat{d} \cdot \hat{e}_b | \downarrow \rangle}{\Delta_i}$$

$$\vec{k} = \Delta \vec{k} = \vec{k}_b - \vec{k}_r$$
Single qubit Raman gates

Stark shifts

\[ \hat{H}_{int} = \frac{\hbar \Omega_0}{2} D_{n,n} (\hat{\sigma}_+ e^{i\phi} + \hat{\sigma}_- e^{-i\phi}) + (\Delta_\uparrow - \Delta_\downarrow) \hat{\sigma}_z \]

\[ \Delta_\uparrow = \frac{|E_r|^2}{4\hbar^2} \sum_i \frac{|\langle \uparrow | \hat{\mathbf{d}} \cdot \hat{\mathbf{e}}_r | e_i \rangle|^2}{\Delta_{i,r}} + \frac{|E_b|^2}{4\hbar^2} \sum_i \frac{|\langle \uparrow | \hat{\mathbf{d}} \cdot \hat{\mathbf{e}}_b | e_i \rangle|^2}{\Delta_{i,b}} \]

\[ \Delta_\downarrow = \frac{|E_r|^2}{4\hbar^2} \sum_i \frac{|\langle \downarrow | \hat{\mathbf{d}} \cdot \hat{\mathbf{e}}_r | e_i \rangle|^2}{\Delta_{i,r}} + \frac{|E_b|^2}{4\hbar^2} \sum_i \frac{|\langle \downarrow | \hat{\mathbf{d}} \cdot \hat{\mathbf{e}}_b | e_i \rangle|^2}{\Delta_{i,b}} \]

- Differential Stark shift can be tuned to zero with beam polarizations and detuning.
Single qubit gates
RF qubit (Zeeman or Hyperfine)

Raman carrier transitions: co-propagating beams.

$^9$Be$^+$, $|F=1, mf = -1\rangle$; $|F=2, mf = -2\rangle$

$^{43}$Ca$^+$, Clock transition

Single qubit gates

RF qubit (Zeeman or Hyperfine)

Two-photon Raman coupling

Randomizing gates:

\[ \varepsilon \text{ in a } \pi/2 \text{ gate} = 0.0048 \]

Error sources:
- Laser intensity and beam pointing noise.
- Inelastic spontaneous scattering of photons.
- B field noise.

Single qubit gates

Optical qubit

Electric Quadruple coupling

\[ |\uparrow\rangle \quad \text{Electric Quadruple coupling} \quad |\downarrow\rangle \]

\[ \Omega = \left| \frac{eE_0}{2\hbar} \langle S, m | (\mathbf{\epsilon} \cdot \mathbf{r}) (\mathbf{k} \cdot \mathbf{r}) | D, m' \rangle \right| \]

\[ {^2}\!S_{1/2} \quad {^2}\!D_{5/2} \]
Single qubit gates

Optical qubit: Electric Quadruple coupling

**Required:**
- Ground-state cooling
- Intensity noise-eater.
- Frequency auto-calibration every 100 s.

**Error sources:**
- Beam pointing~ 0.3%
- Frequency drift 0.3%
- Laser linewidth 0.2%
- Magnetic field noise 0.1%

Single qubit gates: Individual addressing

- Spatial: tightly focused laser beams (Innsbruck)
- Spatial: Large gradients in MW fields (NIST)
- Spectral: Large B field gradients (Siegen)
- Spectral: Inhomogeneous dressing field (Weizmann)