The trapped-ion qubit tool box

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Physical Implementation of a quantum computer

David Divincenzo’s criteria:

1. Well defined qubits.

2. Initialization to a pure state


4. Qubit specific measurement.

5. Long coherence times (compared with gate & meas. time).
Universal Gate set

- For $N$ qubits, a general unitary transformation $U$ acts on a $2^N$-dimension Hilbert space.

- A *finite* set of unitary gates that spans any such $U$.

- The Deutsch-Toffoli gate

Universal Gate set

- For $N$-qubits, and unitary transformation $U$ on a $2^N$-dimension Hilbert space.

- A *finite* set of unitary gates that spans any such $U$.

- Rotations can be approximated to $\varepsilon$ by concatenating $k$ gates, from a finite set $\{V_i\}$, where $k < \text{polylog}(1/\varepsilon)$.

Physical Implementation of a quantum computer

David Divincenzo’s criteria:

How well???

Well enough to allow for a large scale computation: Fault tolerance

\[ F = \langle \Psi | \rho_\epsilon | \Psi \rangle \quad \epsilon = 1 - F \]
Fault-tolerant Quantum Computation

Noisy operations $\epsilon$

One level quantum error-correction codes $O((\epsilon/\epsilon_0)^2)$

Concatenation; threshold theorem

$k$-levels of fault-tolerant encoding $O((\epsilon/\epsilon_0)^{2^k}) \rightarrow 0$ if $\epsilon < \epsilon_0$

- $\epsilon_0$ Fault tolerance threshold.
- Heavy resource requirements when $\epsilon \simeq \epsilon_0$
- Depends on code, noise model, arch. constraints etc.
- Current estimates for $\epsilon_0 \simeq 10^{-2} - 10^{-4}$
Tutorial overview

1. The ion-qubit: different ion-qubit choices, Ion traps.
2. Qubit initialization.
3. Qubit measurement.
4. Universal set of quantum gates: single qubit rotations; two-ion entanglement gates
5. Memory coherence times

How well???
Benchmarked to current threshold estimates

Disclaimer: non exhaustive; focuses on laser-driven gates
Different ion-qubit choices

- One electron in the valence shell; “Alkali like” \(^2S_{1/2}\) ground state.

![Periodic Table](image)

*Lanthanide series

<table>
<thead>
<tr>
<th>Element</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Lanthanum</td>
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<td>Cerium</td>
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<tr>
<td>Praseodymium</td>
<td>Pr</td>
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<tr>
<td>Thulium</td>
<td>Tm</td>
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<tr>
<td>Ytterbium</td>
<td>Yb</td>
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**Actinide series

<table>
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<td>Flerovium</td>
<td>Fm</td>
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<tr>
<td>Mendelevium</td>
<td>Md</td>
</tr>
<tr>
<td>Noactinium</td>
<td>No</td>
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</tbody>
</table>
Electronic levels Structure

\[ \begin{align*}
\text{n}^2P_3/2 & \quad \text{fine structure} \\
\text{n}^2P_1/2 & \quad \text{w/o D} \\
\text{n}^2P & \\
\text{n}^2S_{1/2} & \quad \text{with D} \\
\end{align*} \]

\[ \begin{align*}
\text{Be}^+ & \quad 313 \text{ nm} \\
\text{Mg}^+ & \quad 280 \text{ nm} \\
\text{Zn}^+ & \quad 206 \text{ nm} \\
\text{Cd}^+ & \quad 226 \text{ nm} \\
\text{Ca}^+ & \quad 397 \text{ nm} \\
\text{Sr}^+ & \quad 422 \text{ nm} \\
\text{Ba}^+ & \quad 493 \text{ nm} \\
\text{Yb}^+ & \quad 369 \text{ nm} \\
\text{Hg}^+ & \quad 194 \text{ nm} \\
\end{align*} \]
$^2S_{1/2}$ Zeeman qubit

(Isotopes w/o nuclear spin)

e.g.

$^{24}\text{Mg}^+$
$^{64}\text{Zn}^+$
$^{114}\text{Cd}^+$
$^{40}\text{Ca}^+$
$^{88}\text{Sr}^+$
$^{138}\text{Ba}^+$
$^{174}\text{Yb}^+$
$^{202}\text{Hg}^+$

Advantages
- RF separation.
- Tunable.
- Infinite $T_1$.

Disadvantages
- Energy depends linearly on $B$.
- Transition photon carries no momentum.
- Momentum transfer with off-resonance lasers: photon scattering.
- Detection.

Turn on small $B$ field
$2.8 \text{ MHz}/G$

$m = 1/2 \uparrow$

$m = -1/2 \downarrow$
$^2S_{1/2}$ Hyperfine qubit

Hyperfine structure (order depends on the sign of $A_{hf}$)

Turn on small B field

**Advantages**
- MW energy separation.
- B-field independent qubit.
- Infinite $T_1$.
- State selective fluorescence Detection.

**Disadvantages**
- Few GHz energy separation.
- Transition photon carries no momentum.
- Off resonance photon scattering.
- Initialization to clock transition can be more tricky.

$^9$Be$^+$
$^{25}$Mg$^+$
$^{67}$Zn$^+$
$^{111}$Cd$^+$
$^{43}$Ca$^+$
$^{87}$Sr$^+$
$^{137}$Ba$^+$
$^{171}$Yb$^+$
$^{199}$Hg$^+$

$F = I - 1/2$

$F = I + 1/2$
Optical qubit

Innsbruck/Weizmann

with D

|Ca⁺| 729 nm
|Sr⁺| 674 nm
|Ba⁺| 1760 nm
|Yb⁺| 411 nm
|Hg⁺| 282 nm

Advantages
- Single optical photon (momentum).
- B-field independent qubit (if nuclear spin ≠ 0).
- Hardly any spontaneous decay during gates.
- State selective fluorescence Detection: excellent discrimination.

Disadvantages
- Finite (~ 1 sec) |↑⟩ lifetime.
- Coherence time is limited by laser linewidth.
Trapping

- Trap ions: a minimum/maximum to $\phi$, the electric potential.

- Impossible in all directions; Laplace’s equation:

$$\nabla^2 \Phi = 0$$
Trapping

Linear RF Paul trap

- Positive ion
- RF electrode
- High dc potential control electrode
- Low dc voltage control electrode
Dynamic trapping
(pondermotive forces)

Oscillating electric field:

\[ E(x, t) = E_0(x) \cos(\omega_{rf} t + \phi) \]

\[ x(t) = X(t) + \xi(t) \]

Large and slow
\[ X(t) \gg \xi(t) \]

small and fast
\[ \frac{\partial^2 X(t)}{\partial t^2} \ll \frac{\partial^2 \xi(t)}{\partial t^2} \]

Newton's E.O.M:

\[ m\left(\frac{\partial^2 X(t)}{\partial t^2} + \frac{\partial^2 \xi(t)}{\partial t^2}\right) = eE_0(x) \cos(\omega_{rf} t + \phi) \]
Dynamic trapping

\[ m \left( \frac{\partial^2 X(t)}{\partial t^2} + \frac{\partial^2 \xi(t)}{\partial t^2} \right) = eE_0(x) \cos(\omega_{rf} t + \phi) \]

Field expansion:

\[ E_0(x) = E_0(X) + \frac{\partial E_0(X)}{\partial X} \xi + O(\xi^2) \]

To 0th order:

\[ m \frac{\partial^2 \xi(t)}{\partial t^2} = eE_0(X) \cos(\omega_{rf} t + \phi) \]

\[ \xi(t) = -\frac{eE_0(X)}{m\omega^2_{rf}} \cos(\omega_{rf} t + \phi) \]

Next order:

\[ m \frac{\partial^2 X(t)}{\partial t^2} = -\frac{e^2 E_0(x) \partial E_0(X)}{m\omega^2_{rf}} \cos^2(\omega_{rf} t + \phi) \]

Average over one period:

\[ m \frac{\partial^2 X(t)}{\partial t^2} = -\frac{c^2}{4m\omega^2_{rf}} \frac{\partial E^2_0(X)}{\partial X} \]
Dynamic trapping

\[ m \frac{\partial^2 X(t)}{\partial t^2} = -\frac{e^2}{4m\omega_{rf}^2} \frac{\partial E_0^2(X)}{\partial X} \]

For electric quadruple:

\[ E_0(x) = \alpha \left[ \frac{V_{rf}}{d} \right] \frac{x}{d} \]

Pseudo-potential:

\[ U_{eff}(x) = \left[ \frac{\alpha^2 e^2 V_{rf}^2}{4m\omega_{rf}d^4} \right] x^2 \]

Harmonic frequency:

\[ \omega_{trap} \sim \frac{eV_{rf}}{m\omega_{rf}d^2} \]
Trapping

Linear RF Paul trap

Potential:
\[ \Phi = \sum_i \left( \alpha_i + \beta_i V_0 \cos(\omega_{rf} t) \right) X_i^2 \]

Solve E.O.M:
\[ MX'' = F = -e \nabla \Phi \]

(Mathieu eq: \( \frac{d^2 X_i}{d\tau^2} + \left[ a_i + 2 q_i \cos(2\tau) \right] X_i = 0 \))

Stable solution:
\[ \omega_{trap} \sim \frac{eV_{RF}}{m \omega_{rf} d^2} \]

\( d \) – the distance between the ion and the electrodes

Drive frequency \( \sim 20-30 \) MHz
RF amplitude \( \sim 200-300 \) V
Secular frequency
Radial \( \sim 2-3 \) MHz
Axial \( \sim 1 \) MHz
Weizmann trap.

- Laser machined Alumina

\[ n = 1 - 2 \text{ MHz (axial)} \]
\[ n = 2 - 3 \text{ MHz (radial)} \]

\[ 200 \text{ V}_{pp} \text{ AC} \]
\[ 21 \text{ MHz} \]

\[ 10 \text{ V DC} \]

\[ 1.2 \text{ mm} \]

\[ \sim 2-5 \mu m \]

Nitzan Akerman
Scale up?

One (out of many) problem: isolating one mode of motion for gates
The ion vision: Multiplexed trap array

interconnected multi-trap structure
subtraps completely decoupled

routing of ions by controlling
electrode voltages

Subtrap for different purpose:
Gates, readout, etc.

D. J. Wineland, et al.,
J. Res. Nat. Inst. Stand. Technol. 103, 259 (1998);
D. Kielpinski, C. Monroe, and D. J. Wineland,
Multi-zone ion trap

Gold on alumina construction

RF quadrupole realized in two layers

Six trapping zones

Both loading and experimental zones

One narrow separation zone

Closest electrode ~140 μm from ion
Ions can be moved between traps. Electrode potentials varied with time.

Ions can be separated efficiently in separation zone.

Small electrode’s potential raised.

Motion (relatively) fast.

Shuttling (adiabatic):
several 10 μs
Separating: few 100 μs
Surface-electrode traps

Elbows and tee-junctions possible:

Field lines:

RF electrodes  Control electrodes

NIST

Micro-fabricated traps


Amini; NIST
Shappert; Georgia Tech
Allcock; Oxford
Trapped ions on the tabletop

- Vacuum ~ $10^{-11}$ Torr
- Room temperature (or a bit above)
- RF created with coax/helical resonator
- Atoms created by oven.
- Ions created by photo ionization.
- Approx. F1 imaging optics to EMCCD/PMT (res = 0.8 um).

**Diagram:**
- EMCCD
- PMT
- Flip Mount
- RF resonator
- Microwaves
- f/1 lens
- B-field
- Cooling, detection and excitation beams
Harmonic oscillator levels

- Doppler cooling. \( \langle n \rangle \sim 3 - 20 \)
- Resolved side band cooling. \( \langle n \rangle \sim 0 \)
Sideband Spectroscopy

Scan the laser frequency across the $S \rightarrow D$ transition

$|\downarrow\rangle$ $|\uparrow\rangle$ $4^{2D}_{5/2}$

$D_n < 80 \text{ Hz}$

674 nm

$5^{2S}_{1/2}$

carrier

Red sidebands

Blue sidebands

$\Delta \nu < 80 \text{ Hz}$
Resolved-sideband Cooling

\[ |P\rangle \rightarrow |D\rangle \quad 1033 \text{ nm} \]
\[ |S\rangle \rightarrow |D\rangle \quad 674 \text{ nm} \]

\[ |D\rangle \rightarrow |S\rangle \quad \sim 4 \text{ MHz} \]

Red sideband

\[ n = 0 \quad n = 1 \quad n = 2 \quad n = 3 \]
Sideband cooling to the ground state

\[ \langle n \rangle < 0.05 \]

- \( T \approx 2 \ \mu \text{K} \)
- Uncertainty in ion position = ground state extent \( \sqrt{\frac{\hbar}{2m\omega_{ho}}} = 6nm \)
Anomalous heating

- Fluctuating charges
- $f^{-1.5}$ noise
- Thermally activated
- Due to monolayer of C on electrodes: gone after Ar+ ion cleaning

Qubit Initialization

Zeeman qubit

- Optical pumping into a dark state.
- CPT possible into any superposition.

\[ \begin{align*}
\ket{\downarrow} & \rightarrow \ket{\downarrow} \\
\ket{\uparrow} & \rightarrow \ket{\uparrow}
\end{align*} \]

\[ \begin{align*}
\epsilon_{\text{init}} &= \frac{\epsilon_{\pi}}{1 + \delta_{\pi}^2 / \gamma' r^2} + \frac{\epsilon_{\sigma^-}}{1 + \delta_{\sigma^-}^2 / \gamma' r^2} \\
\gamma' &= \frac{\gamma}{2} \sqrt{1 + s_0} \\
\epsilon &\sim 10^{-6} - 10^{-3}
\end{align*} \]

Error sources:
- Polarization purity.
Qubit Initialization

Zeeman qubit

Process tomography of optical pumping

Limited to $10^{-3}$ due to stress-induced birefringence in vacuum chamber optical viewports
Qubit Initialization

Zeeman qubit: the D level option

\[ g = 0.4 \text{ Hz} \]

- Limited by off-resonance coupling of \( \downarrow \) to D
- Power broadening and in-coherent noise
- \( \varepsilon = 10^{-4} \)

Qubit Initialization

Hyperfine qubit

- Optical pumping into a dark state.

Estimated:

\[ \epsilon \leq 2 \times 10^{-5} \]

Qubit Initialization

Optical qubit

- $|\downarrow\rangle$ state initialization: same as previous.

- $|\uparrow\rangle$ state initialization:
  - Rapid adiabatic passage.
    - $\varepsilon \sim 10^{-2}$
    (Wunderlich et. al. Journal of Modern Optics 54, 1541 (2007))
  - $\pi$-pulse.
    - $\varepsilon \sim 10^{-2}$