Collective light-matter interaction.

Problems for a minicourse

Alexander N. Poddubny*

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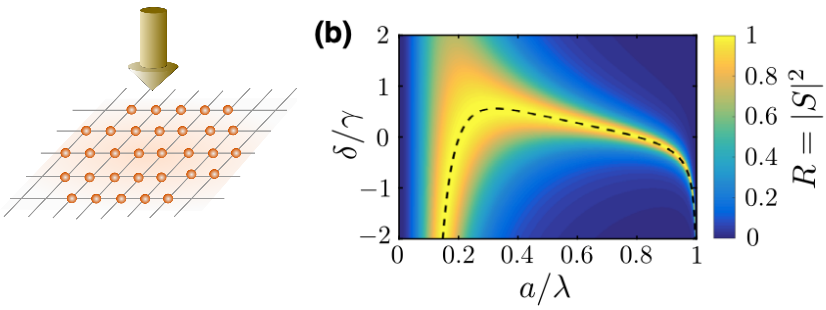
Contents

*Electronic address: poddubny@weizmann.ac.il

I. REFLECTION FROM A 2D METASURFACE. HARD, ANALYTICS+ NUMERICS

The goal of this task is to demonstrate collective enhancement of the light reflection from a two-dimensional array of atoms.

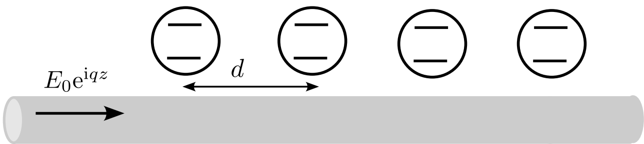
- Reproduce Eq. (4) from Ref. (Shahmoon *et al.*, 2017)
- Reproduce Eq. 19 for $\tilde{\Gamma}_{\rm QD}$ from Ref. (Ivchenko and Kavokin, 1992) (can be downloaded here)
- Demonstrate the connection between these two expressions for the array with the period much smaller than the wavelength.
- Reproduce Fig. 2b from Ref. (Shahmoon *et al.*, 2017). Hint: for lattice summation you can use Floquet summation or Ewald summation, as described in Eq. 63-65 here. here. If this turns out to be too hard, I will provide the MATLAB code.

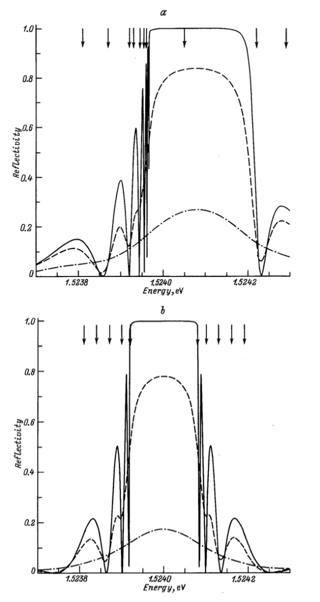


II. POLARITON EIGENMODES IN A FINITE ARRAY. NOT THAT HARD, ANALYTICS+ NUMERICS

The goal of this problem is to calculate collective modes of a one-dimensional array of emitters coupled to the waveguide.

- Reproduce Eqs. 17 from Ref. (Vladimirova *et al.*, 1998).
- Reproduce Fig. 4 with the reflection specta from Ref. (Vladimirova *et al.*, 1998) (note, that the arrow positions are taken from Eqs. 18)
- Starting from Eqs. 17 in Ref. (Vladimirova *et al.*, 1998), derive Eq. 4 for the decay rates of the subradiant modes from Ref. (Zhang and Mølmer, 2019).

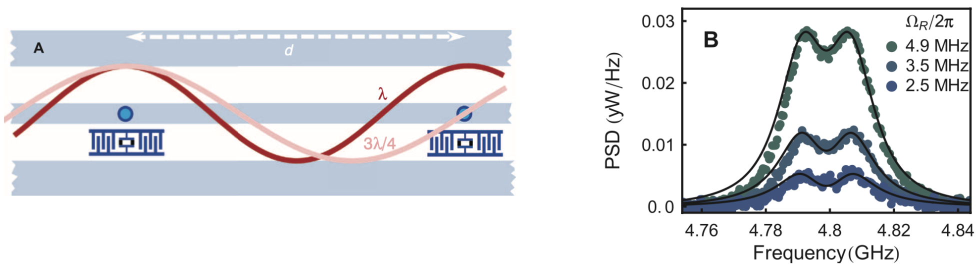




III. REFLECTION OF COHERENT LIGHT FROM TWO EMITTERS.NOT VERY HARD.

The goal of this problem is to calculate reflection of coherent light from two emitters, coupled to the waveguide. Contrary to all other problems, this one requires knowledge of quantum optics and the Lindblad master equation. Namely, the reflecton coefficient will change as the coherent drive power increases, which is a nonlinear effect.

- Reproduce Fig. 4b from Ref. (van Loo *et al.*, 2013).
- Recalculate Fig. 4b for $d/\lambda = 0.2$. Explain the difference with the case $d/\lambda = 0.25$.
- In the regime of vanishing drive power, verify that the normalized reflection spectra calculated from the quantum master equation approach agree with the reflection coefficient from the two emitters calculated classically. This one will be derived during the course, see also e.g. Eq. 26 in Ref. (Sheremet *et al.*, 2023).)



IV. BAND STRUCTURE IN 3D PHOTONIC CRYSTALS. ANALYTICS + NUMERICS/HARD

The goal of this problem is to calculate band structure of a 3D array of atoms interacting with light.

- Reproduce Eq.(6) and Fig. 1 from Ref. (van Coevorden *et al.*, 1996)

  Hint: for 3D summation of the Green function, you can use Ewald summation, as described in Eq. 65 here. If this turns out to be too hard, I will provide the MATLAB code.

- Check explicitly that obtained dispersion $\omega(k)$ is real. Why?

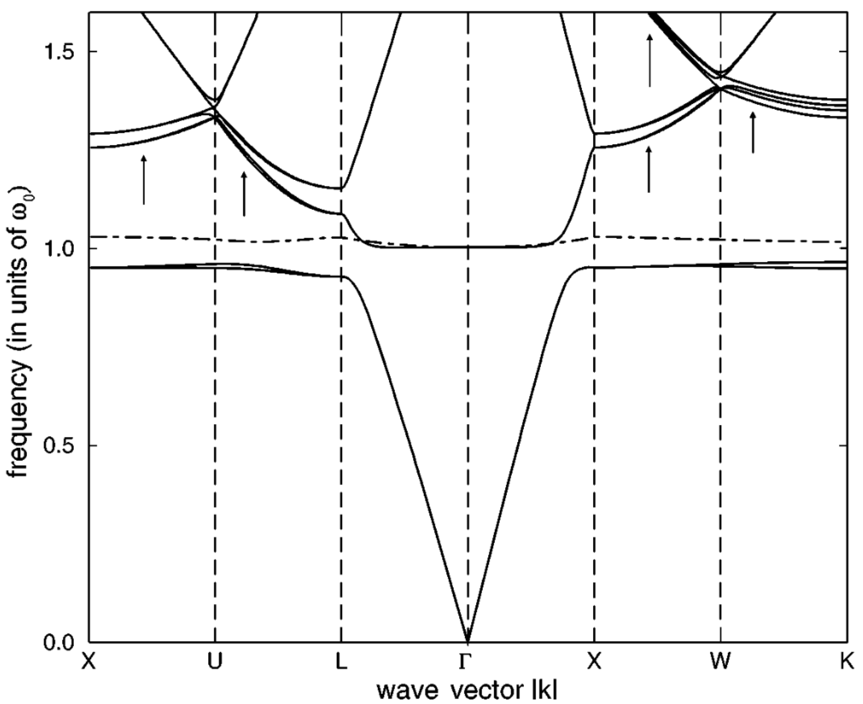


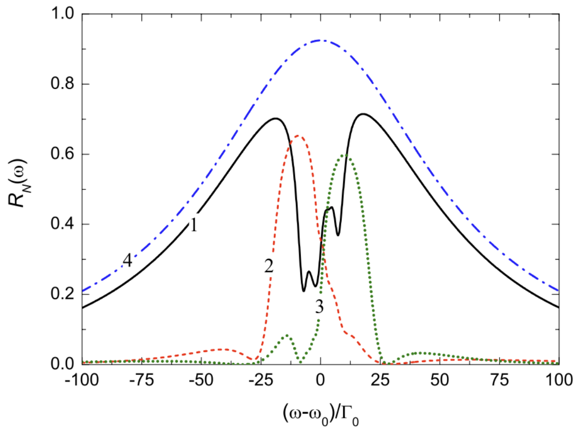
FIG. 1. Photonic band structure for a fcc lattice of resonant dipoles. The resonance wavelength λ_0 for this case equals $\lambda_0 = 1.26a$ ($P = 1.29$ and $Q = 30$ in Fig. 4). Horizontally, we have parametrized a path along the symmetry points X, U, L, Γ, X, W, K in the Brillouin zone. The dot-dashed line is a longitudinal polarization mode; the arrows indicate the vacuum modes that is unaltered by the interaction (see text).

V. SUPERRADIANT MODE IN QUASICRYSTALLINE FIBONACCI STRUCTURE. EASY/NUMERICS

The goal of this problem is to demonstrate that collective enhancement of the spontaneous emission can occur not only in the ordered periodic arrays, such as λ-spaced ones. but also in quasicrystalline arrays — ordered nonperiodic arrays.

- Try to reproduce numerically Fig. 2 from Ref. (Poddubny *et al.*, 2008).

FIG. 1. Scheme of the Fibonacci QW structure $\mathcal{F}_6$ with N=9 QWs.

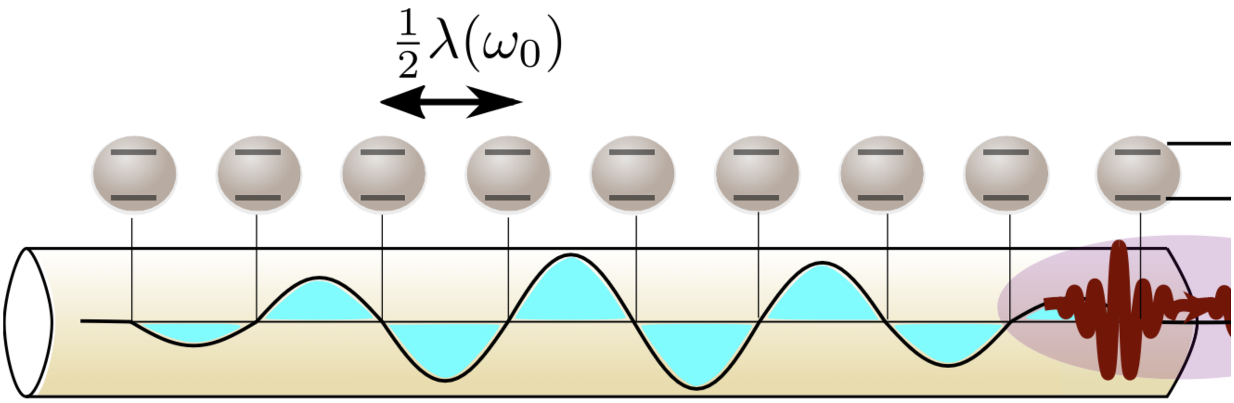


VI. ANOMALOUS TRANSMISSION THROUGH ABSORBING MEDIUM. NUMERICS/EASY

The goal of this task is to demonstrate that some light can get through an ordered array even if the optical density is very high.

Calculate transmission , reflection, and absorption spectra for a periodic arrays of emitters with different $N = 1 \ldots 50$ with $\gamma/\gamma_{1D} = 25$ and $\gamma_{1D}/\omega_0 = 2.5 \times 10^{-2}$. Consider two cases: (a) $\omega_0 d/c = \pi$ and (b) $\omega_0 d/c = \pi/2$.

If everything is done correctly, you will see a peak in the transmission at $\omega = \omega_0$. Try to explain the peak origin. This can be most easily understood by calculating and plotting the distributions of electric field inside the array in the peak region and outside the peak region. How is this transmission peak related to the results in Ref. (Borrmann, 1950)?



VII. TRANSMISSION THROUGH A RESONANT NUCLEI STRUCTURE. HARD/ANALYTICS

The purpose of this problem, as well as the next one, is to introduce another type of resonant systems — arrays of nuclei, that have resonances for γ-rays. Such systems can also host collective resonances, and can demonstrate collective enhancement of the spontaneous emission linewidth.

- Reproduce Eq. 3.8 from Ref. (Kagan *et al.*, 1979).
- How does the dynamics of emission at short times scale with the length of the array?
- How do results correspond to superradiance?

Hint: you can also use Ref. (Smirnov, 1999). Also, this paper (Lynch *et al.*, 1960) might help, although it does not have a full answer.

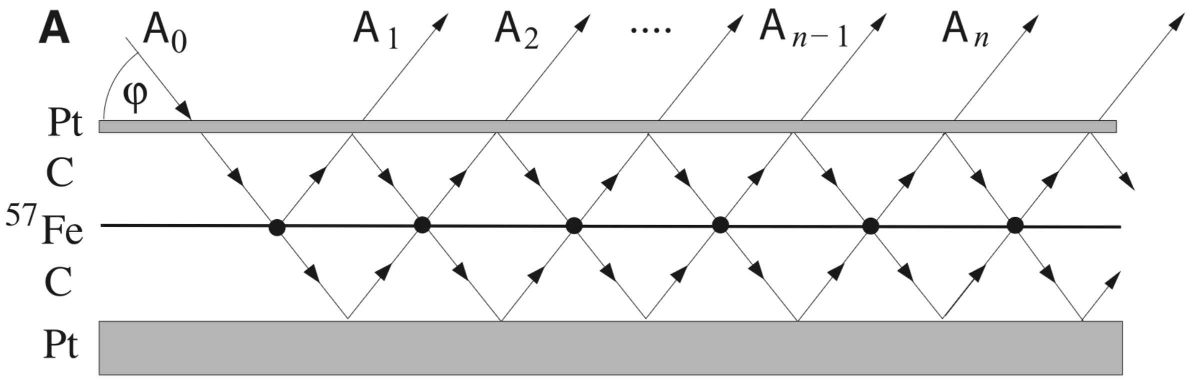
This is a hard and rather technical problem, suitable for fans of contour integrals in the complex plane.

$$a'(t) = \frac{1}{2\pi i} \oint d\omega \frac{e^{i\omega t}}{\omega - \omega_0 - \frac{1}{2} i\lambda} \times \exp[ib(\omega - \omega_0{}' - \tfrac{1}{2} i\lambda)^{-1}] \equiv \frac{1}{2\pi i} \oint G d\omega = \frac{1}{2\pi i}\left[\oint_{\omega_0 + \frac{1}{2} i\lambda} G d\omega + \oint_{\omega_0{}' + \frac{1}{2} i\lambda} G d\omega\right].$$

VIII. COLLECTIVE LAMB SHIFT FOR NUCLEI. VERY EASY/ANALYTICS

The purpose of this task, as well as the previous one, is to introduce another type of resonant systems — arrays of nuclei, that have resonances for γ-rays. Such systems can also host collective resonances, and can demonstrate collective enhancement of the spontaneous emission linewidth and collective shift of the resonance frequency . This problem is supposed to be very easy — just to show, how the resonance position is modified when the array of nuclei is put in a planar cavity.

- Reproduce Eq. (4) from Ref. (Röhlsberger *et al.*, 2010).



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