Reciprocity of scattering on array of resonant emitters in 1D

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We study scattering of an electromagnetic wave, propagating in a one-dimensional waveguide, on an array of identical resonant point light emitters located at the points z_n, see Fig. 1. We describe light-emitter interaction by a wave equation

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}E(z) + q^2 E(z) = -4\pi q^2 \sum_{n=1}^{N} p_n \delta(z - z_n), \tag{1}$$

where the dipole moment

$$p_n = \frac{1}{2\pi q}\frac{\gamma_{1\mathrm{D}}}{\omega_0 - \omega - \mathrm{i}\gamma}E(0)\ , \tag{2}$$

characterizes the resonant polarization of the emitter. Here, $E(z)$ is the amplitude of the electric field at the frequency ω, $q = \omega/c$ is the light wave vector, ω_0 is the resonant frequency of the emitter, γ is the phenomenological decay rate, characterizing the nonradiative processes withing the emitter and $\gamma_{1\mathrm{D}}$ is the radiative decay rate. An electromagnetic wave is incident upon the emitters from either from the left ($E_{0,\rightarrow}(z) = \mathrm{e}^{\mathrm{i}qz}$) or from the right ($E_{0,\leftarrow}(z) = \mathrm{e}^{-\mathrm{i}qz}$).

Goal: Prove that the amplitude transmission coefficients of the electromagnetic wave from left to right $t_{\rightarrow}$ and from the right to left $t_{\leftarrow}$ are equal.

Tip: A useful intermediate result is

$$t_{\rightarrow} = 1 + \sum_{n,m=1}^{N} \mathrm{e}^{\mathrm{i}q(z_m - z_n)}G_{nm}, \quad t_{\leftarrow} = 1 + \sum_{n,m=1}^{N} \mathrm{e}^{\mathrm{i}q(z_n - z_m)}G_{nm}\ , \tag{3}$$

where the matrix Green function G_{nm} is the inverse of the matrix

$$(\omega_0 - \omega - \mathrm{i}\gamma)\delta_{mn} - \mathrm{i}\gamma_{1\mathrm{D}}\mathrm{e}^{\mathrm{i}q|z_m - z_m|}\ .$$

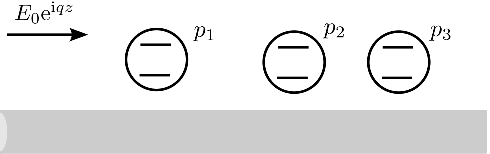

FIG. 1 Schematics of resonant light scattering on $N = 3$ emitters.