Transfer matrix of a general scatterer

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We consider one-dimensional problem of light scattering on a general object, see Fig. 1. The scattering is characterized by the transfer matrix T that can be conveniently expressed in the basis of right-propagating (E^+) and left-propagating (E^-) waves

$$E(z) = \begin{cases} E^+_{\text{left}} \mathrm{e}^{\mathrm{i}q_l z} + E^-_{\text{left}} \mathrm{e}^{-\mathrm{i}q_l z} & (z < 0) \\ E^+_{\text{right}} \mathrm{e}^{\mathrm{i}q_r (z-L)} + E^-_{\text{right}} \mathrm{e}^{-\mathrm{i}q_r (z-L)} & (z > L)\,, \end{cases} \tag{1}$$

where $q_{r,l}$ are light wave vectors from the left and from the right of the scatterer. The 2×2 matrix T relates the electric field amplitudes by

$$\begin{pmatrix} E^+_{\text{right}} \\ E^-_{\text{right}} \end{pmatrix} = T \begin{pmatrix} E^+_{\text{left}} \\ E^-_{\text{left}} \end{pmatrix} . \tag{2}$$

Goal: Express the transfer matrix elements via the complex reflection coefficients $r_{\hookleftarrow}$, $r_{\hookrightarrow}$ and transmission coefficients $t_{\rightarrow}$, $t_{\rightarrow}$ corresponding to the initial wave incidence from the left and right sides, as illustrated in Fig. 1.

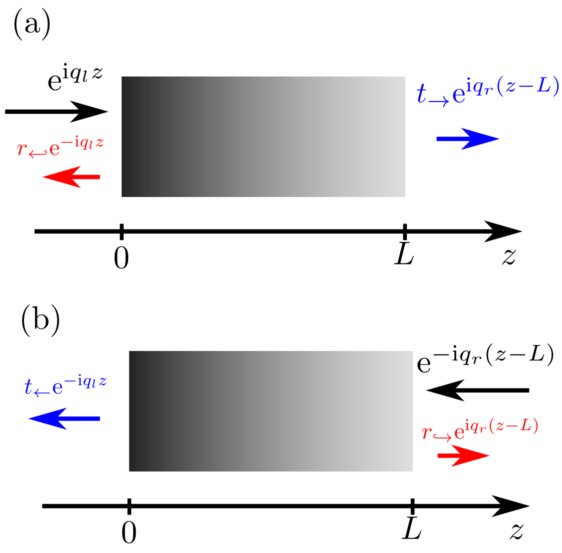

FIG. 1 Definition of reflection coefficients $r_{\hookleftarrow}$, $r_{\hookrightarrow}$ and transmission coefficients $t_{\rightarrow}$, $t_{\leftarrow}$ of light, incident upon the scatterer with length L from left (a) and right (b) half-spaces, respectively.