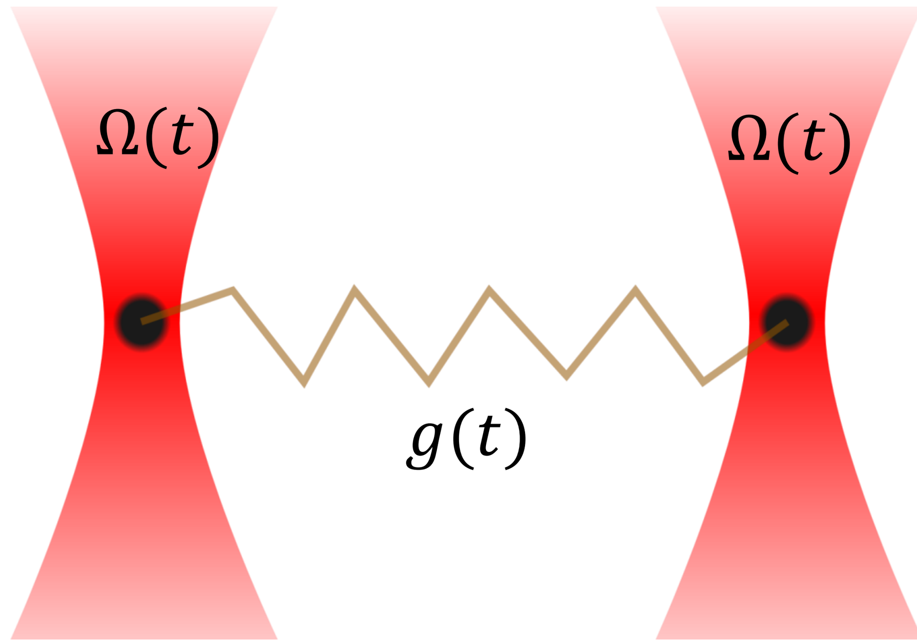
Controlled entanglement between levitated masses via parametric resonance

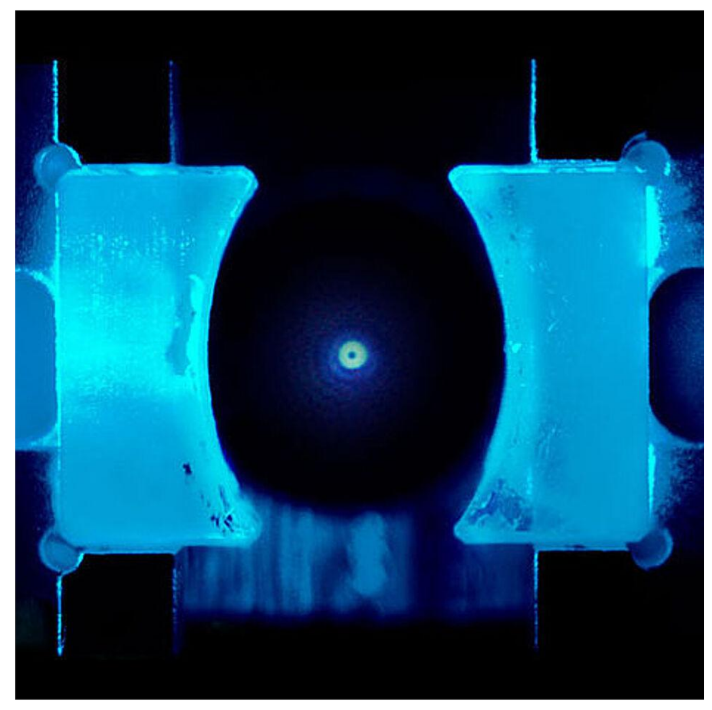
Gad Horovitz, Alexander Poddubny

Department of Physics of Complex Systems, Weizmann Institute of Science

Abstract

The quantum entanglement of relatively large-massed particles by optomechanics methods has become viable in recent years and garnered interest in various practical and theoretical fields. The Coulombic coupling of two optically levitating charged particles is a potential realization of this system. We focus on building a theoretical framework for such a system by utilizing continuous optical homodyne measurement control and parametric modulation of the particles — a novel and original approach in the field.





Theory – The system

- The system is comprised of two Coulombic interacting harmonically trapped particles.
- The active measurement and control of the system will give rise to the stochastic master equation

$$d\rho_c = -i[H,\rho]dt - \sum_{k=1,2}\left(\mathcal{D}_{cl}^k[\rho_c] + \mathcal{D}_{pr}^k[\rho_c]\right)dt + \sum_{k=1,2}\left(\mathcal{H}_{cl}^k[\rho_c]\right)dW_k.$$

- The quadratic master equation ensures that an initial Gaussian state will remain as such. We can therefore focus on the first and second quadrature moments

$$d\boldsymbol{X}^c = \boldsymbol{A}(t)\boldsymbol{X}^c dt + \boldsymbol{B}(t)\boldsymbol{u}dt + \Sigma^c\,\mathbf{C}^{\mathrm{T}}(t)\mathbf{W}^{-1}d\boldsymbol{w}$$

$$\frac{d\Sigma^c}{dt} = \boldsymbol{A}(t)\Sigma^c + \Sigma^c\boldsymbol{A}(t)^T + \boldsymbol{V}(t) - \Sigma^c\mathbf{C}^{\mathrm{T}}(t)\mathbf{W}^{-1}\mathbf{C}(t)\Sigma^c,$$

where

$$\boldsymbol{A} = \begin{pmatrix} 0 & \Omega(t) & 0 & 0 \\ -\Omega(t)-2g(t) & -\gamma & 2g(t) & 0 \\ 0 & 0 & 0 & \Omega(t) \\ 2g(t) & 0 & -\Omega(t)-2g(t) & -\gamma \end{pmatrix}.$$

Theory – Parametric resonance

- One can diagonalize the system into its two eigenfrequencies

$$\Omega_+ = \Omega$$
$$\Omega_- = \sqrt{\Omega^2 + 4g\Omega}.$$

- The system can be parametrically driven by modulating the following parameters:

1. The trap's frequency,

$$\Omega(t) = \Omega_0 + 2\Omega_1\cos(\Omega_{trap}t).$$

2. The Coulombic coupling strength

$$g(t) = -|g_0| + 2g_1\cos(\Omega_{coupling}t).$$

- Different modulation values can increase/decrease the entanglement of the two particles.
- The entanglement is characterized by the logarithmic negativity of the system.

Theory – Floquet Theorem

- The presence of periodic time modulation leads to the emergence of exponentially diverging\decaying Floquet modes, governed by the linear equation

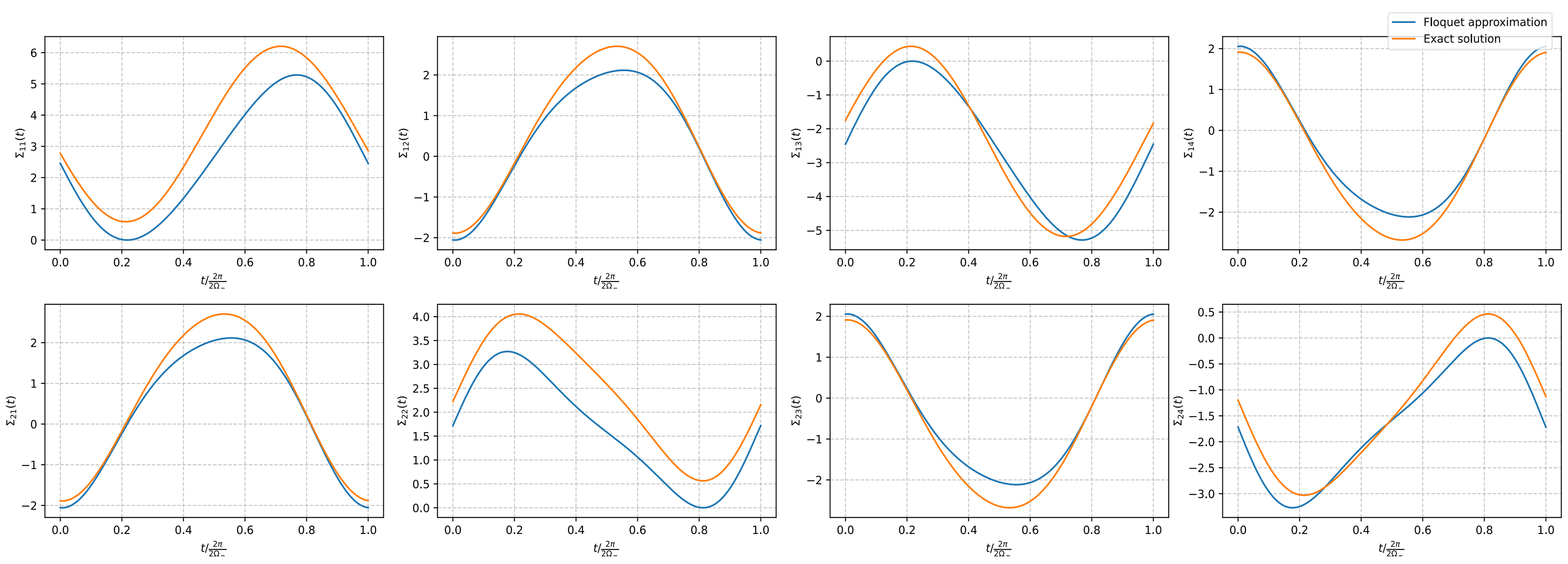
$$\frac{d\Sigma^F}{dt} = \boldsymbol{A}(t)\Sigma^F + \Sigma^F\boldsymbol{A}(t)^T \qquad \Sigma_F(t) = e^{\lambda t}\,\widetilde{\Sigma_F}(t),$$

- The system tends to follow the strongest Floquet mode, while the nonlinearity cancels the divergence.

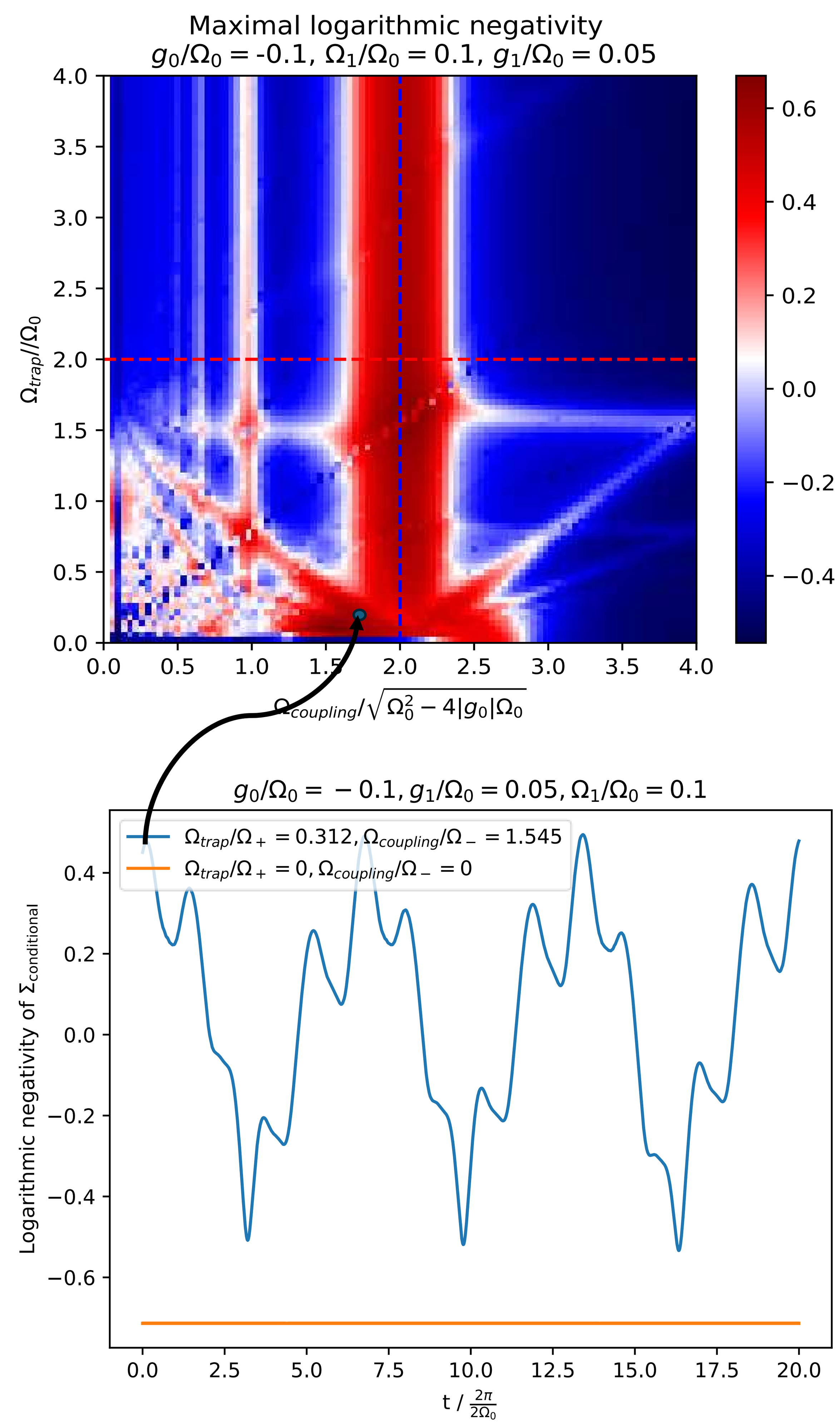
$$\Sigma^c(t) \propto \widetilde{\Sigma_F}(t)$$

- The proportionality constant can be approximately calculated by analytics.

Analytical vs numerical results



Numerical results



References

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