Subradiant states in waveguide quantum electrodynamics

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Jan Brehm, Alexey Ustinov

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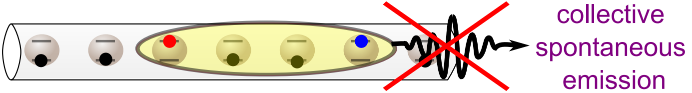
Institute of Science and Technology, Austria
Elena Redchenko, Johannes Fink

Outline

All happy families are alike,
but every unhappy family is unhappy in its own way
(Leo Tolstoy, Anna Karenina, 1878)

All superradiant states are alike,
but each subradiant state is subradiant in its own way



1. Intro. Waveguide Quantum Electrodynamics
2. Single-excited subradiant states
3. Double-excited subradiant states
4. Many-body subradiant states

Atom in free space

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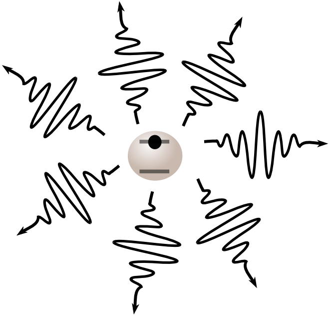

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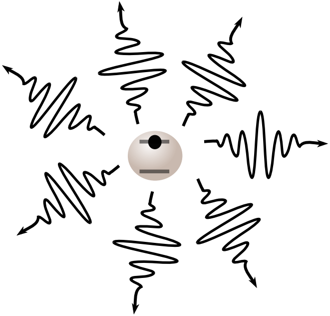
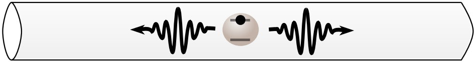
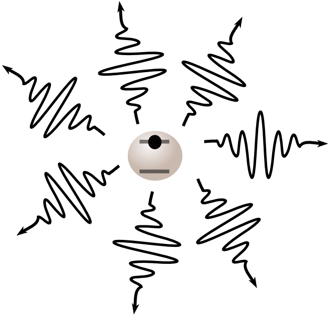


Atom coupled to a waveguide



Waveguide quantum electrodynamics

Atom in free space



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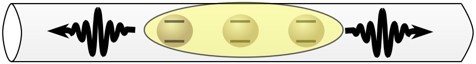


Reviews on Waveguide Quantum Electrodynamics

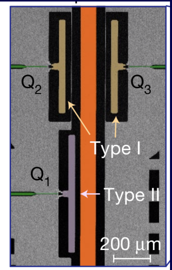
D. Roy, C. Wilson, and O. Firstenberg,
"Colloquium: Strongly interacting photons in one-dimensional continuum", Rev. Mod. Phys. **89**, 021001 (2017)

D.E. Chang, J.S. Douglas, A. González-Tudela, C.-L. Hung, and H.J. Kimble,
"Colloquium: Quantum matter built from nanoscopic lattices of atoms and photons", Rev. Mod. Phys. **90**,031002(2018)

A.S. Sheremet, M.I. Petrov, I.V. Iorsh, A.V. Poshakinskiy and ANP,
"Waveguide quantum electrodynamics:
collective radiance and photon-photon correlations",
Rev. Mod. Phys. **95**, 015002 (2023)

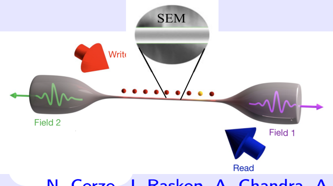
Platforms for Waveguide Quantum Electrodynamics

Superconducting qubits



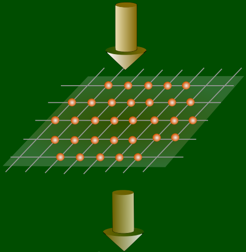
M. Mirhosseini et al., Nature **569**, 692 (2019)

Cold atoms near nanofiber



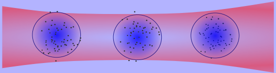
N. Corzo, J. Raskop, A. Chandra, A. Sheremet, B. Gouraud, and J. Laurat, Nature 566, 359 (2019)

Quantum metasurfaces



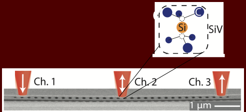
J. Rui et al., Nature **583**, 369 (2020)

Clouds of Rydberg superatoms



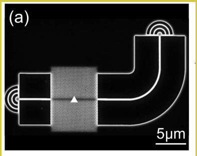
N. Stiesdal et al.,Nat. Comm. 12, 4328 (2021).

Quantum defects



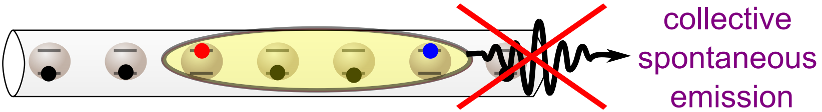
Sipahigil et al., Science **354**, 847 (2016)

Quantum dots

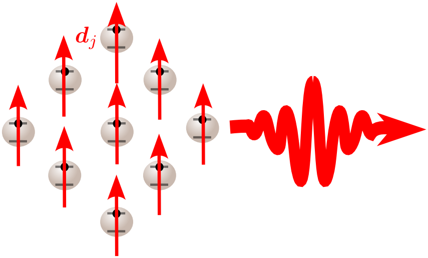


Foster et al., PRL **122**, 173603 (2019)

Outline



Bright (superradiant) state:



- Constructive interference,
  $$\sum_{j=1}^{N} \boldsymbol{d}_j = N\boldsymbol{d}_1$$
- Spontaneous decay is enhanced $\times N$

Bright and dark states

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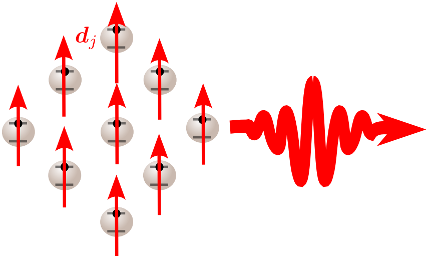

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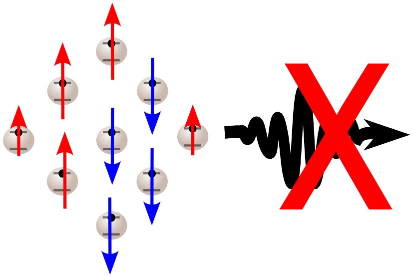
Dark state examples:



- Destructive interference, $\sum_{j=1}^{N} \boldsymbol{d}_j = 0$
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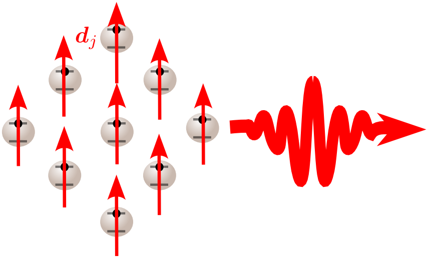

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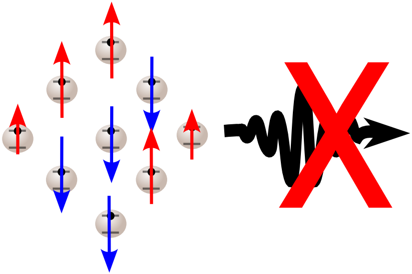

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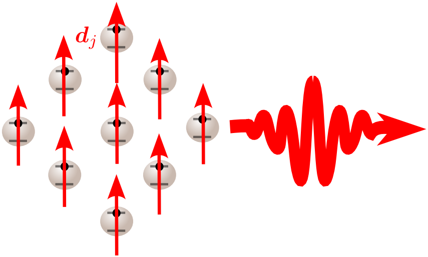

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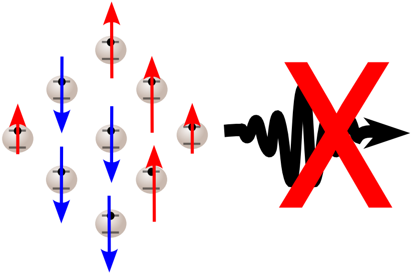

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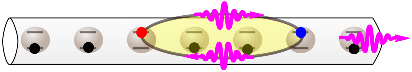
Subradiant states in array coupled to the waveguide



$$H = -\mathrm{i}\gamma_{1\mathrm{D}} \underbrace{\sum_{j,j'=1}^{N} \sigma_j^\dagger \sigma_{j'} \mathrm{e}^{\mathrm{i}\omega_0 |z_j - z_{j'}|/c}}_{\text{long-range coupling of atoms via photons}}$$

Single-excited states: $\psi = \sum_{j=1}^{N} \psi_j \sigma_j^\dagger |0\rangle$

Eigenfrequencies of single-excited states

$$-\mathrm{i}\gamma_{1\mathrm{D}} \sum_{j'=1}^{N} \mathrm{e}^{\mathrm{i}\omega_0 |z_j - z_{j'}|/c} \psi_{j'} = \omega \psi_j$$

Example: $N = 2$

$$-\mathrm{i}\gamma_{\mathrm{1D}} \begin{pmatrix} 1 & \mathrm{e}^{\mathrm{i}\omega_0 d/c} \\ \mathrm{e}^{\mathrm{i}\omega_0 d/c} & 1 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \omega \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

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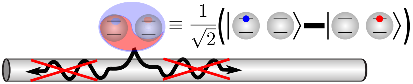
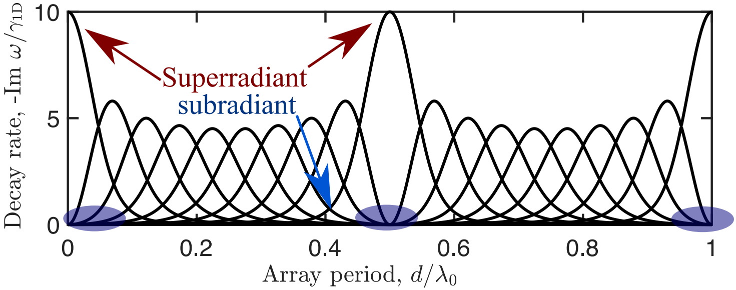
Antisymmetric (subradiant)
dimer excitation $\frac{1}{\sqrt{2}}(b_1^\dagger - b_2^\dagger)|0\rangle$



$$\omega_\pm = -\mathrm{i}\gamma_{1\mathrm{D}}(1 \pm \mathrm{e}^{\mathrm{i}\omega_0 d/c})$$

Decay rate $-\operatorname{Im}\omega_- = \gamma_{1\mathrm{D}}(1 - \cos\frac{\omega_0 d}{c})$

Decay rate vs period



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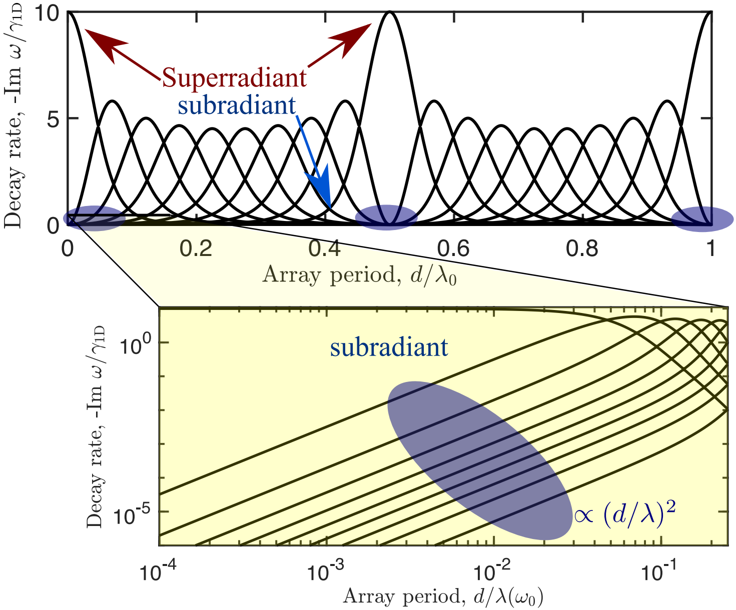

Fermi golden rule: decay rate $\gamma \propto |M|^2, \; \underbrace{M = \sum_{m=1}^{N} \mathrm{e}^{\mathrm{i}(\omega_0/c)dm}\psi_m}_{\text{light-matter interaction}}$

fast spontaneous decay needs $k_{\mathrm{polariton}} \approx \omega_0/c$

Universal scaling of the decay rate

Fermi golden rule: decay rate $\gamma \propto |M|^2$, $\underbrace{M = \sum_{m=1}^{N} \mathrm{e}^{\mathrm{i}(\omega_0/c)dm}\psi_m}_{\text{light-matter interaction}}$

Superradiant state

$$\psi_m = \frac{1}{\sqrt{N}}, \quad \omega_0 d/c \to 0$$

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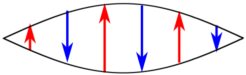

$$\psi_m = \sqrt{\frac{2}{N}}(-1)^m \sin\left(\pi\frac{m-\frac{1}{2}}{N}\right),$$

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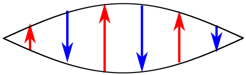

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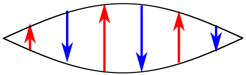

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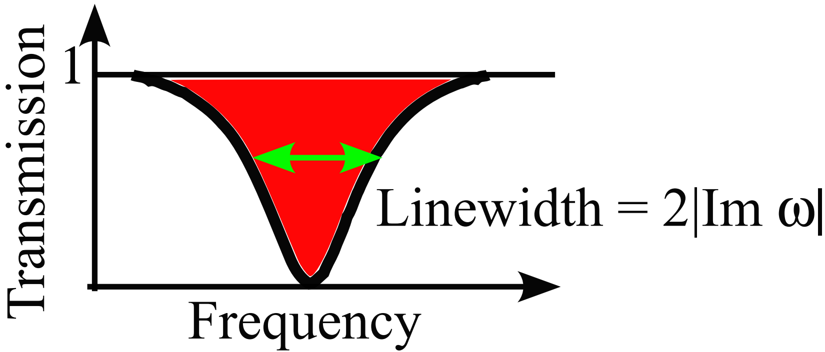
General consideration:

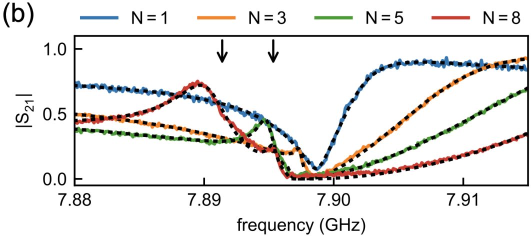
Y.-X. Zhang and K. Mølmer, PRL **125**, 253601 (2020)

Subradiant states manifest as narrow dips in transmission

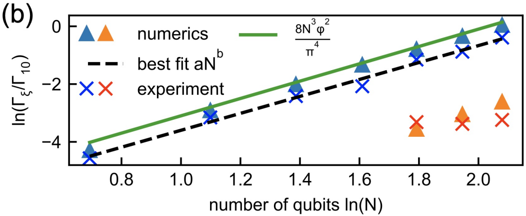


Subradiant states with transmon qubits: **Experiment**

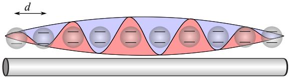
Transmission vs number of qubits N



Linewidth vs number of qubits N



Experiment: U of Karlsruhe, theory: ANP



Spontaneous decay rate for the brightest subradiant mode

$$\Gamma(N) = \Gamma_{1D}\frac{8N^3\varphi^2}{\pi^4}$$

$\varphi = \omega_0 d/c$

npg "Quantum Materials" **6**, 10 (2021)

Outline

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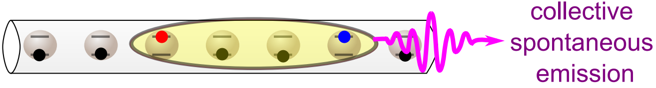
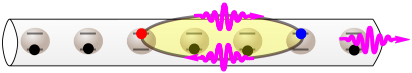


Double-excited states



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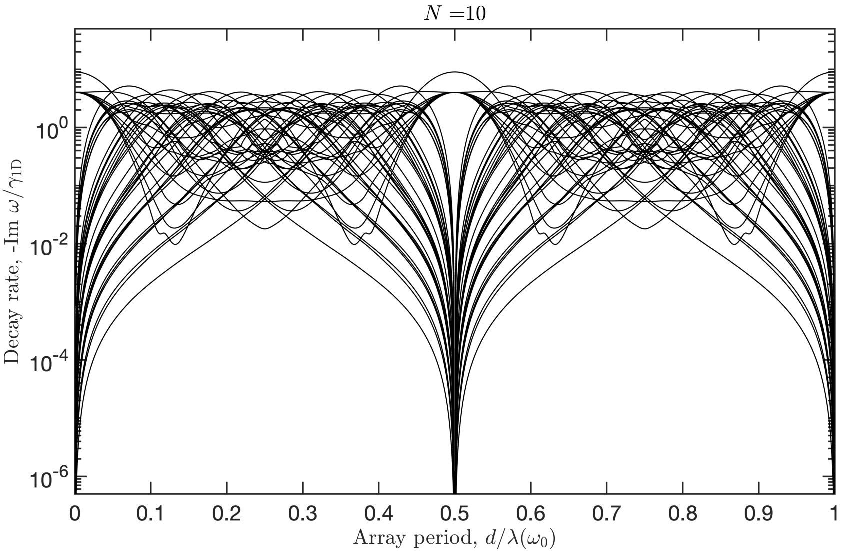
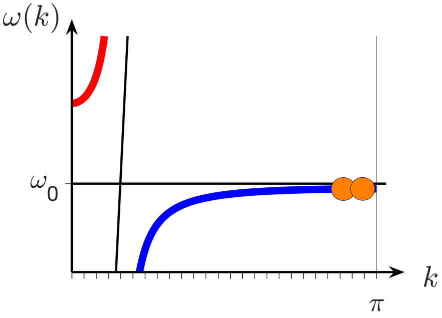
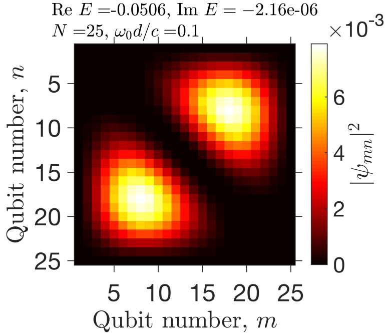
Dependence of decay rate on the array period



Fermionized state of two polaritons





Antisymmetric combination of single-particle subradiant states

$$\psi_{nm} \approx [\cos k_1 n \cos k_2 m - (n \leftrightarrow m)] \times \begin{cases} 1, & n > m \\ -1, & n < m \end{cases}, \quad k_{1,2} \approx \pi$$

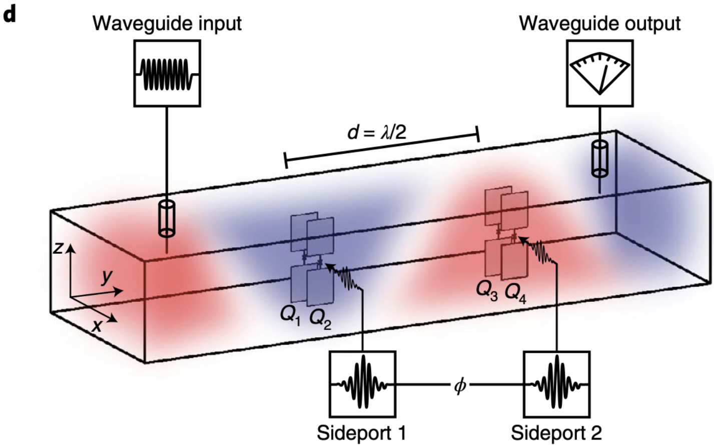
A. Albrecht, L. Henriet, A.Asenjo-Garcia, P.B. Dieterle, O.Painter and D.E Chang, NJP **21**, 025003 (2019)

Y.-X. Zhang and K. Mølmer, PRL **122**, 203605 (2019)

Photon pair scattering enhanced by subradiant states:

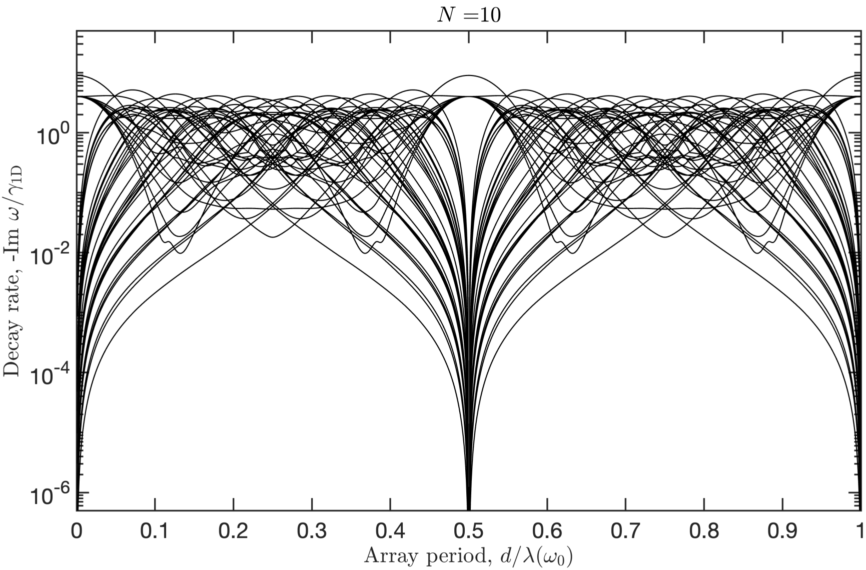
Y. Ke, A.V. Poshakinskiy, C. Lee, Yu.S. Kivshar & ANP, PRL **123**, 253601 (2019)

Double-excited subradiant states: two pairs of qubits



M.Zanner, T. Orell, C.M. F. Schneider, R. Albert, S. Oleschko, M.L. Juan, M. Silveri & G. Kirchmair, Nature Physics **18**, 538 (2022)

Dependence of decay rate on the array period



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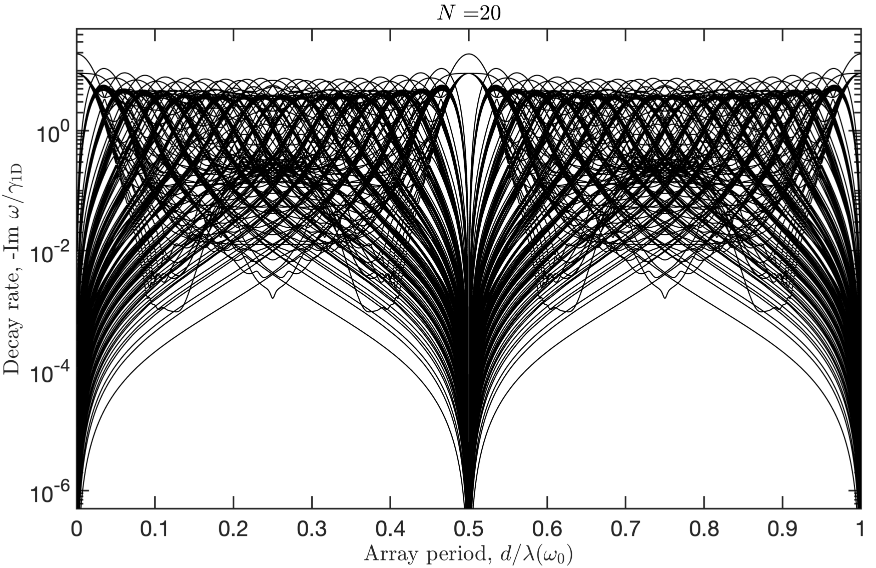

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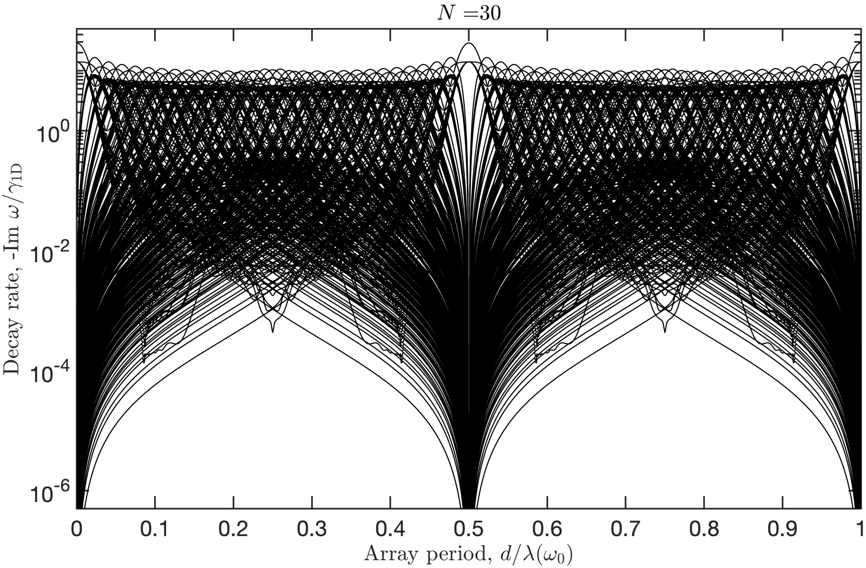

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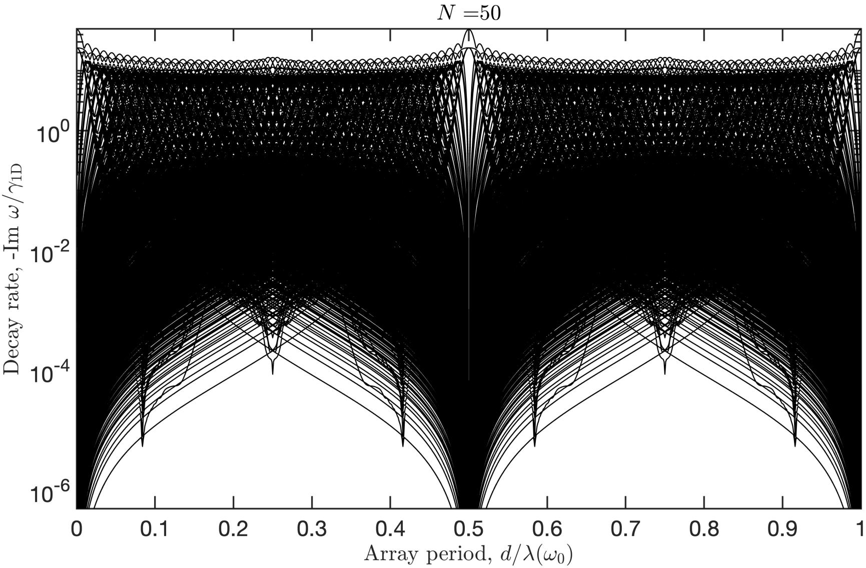

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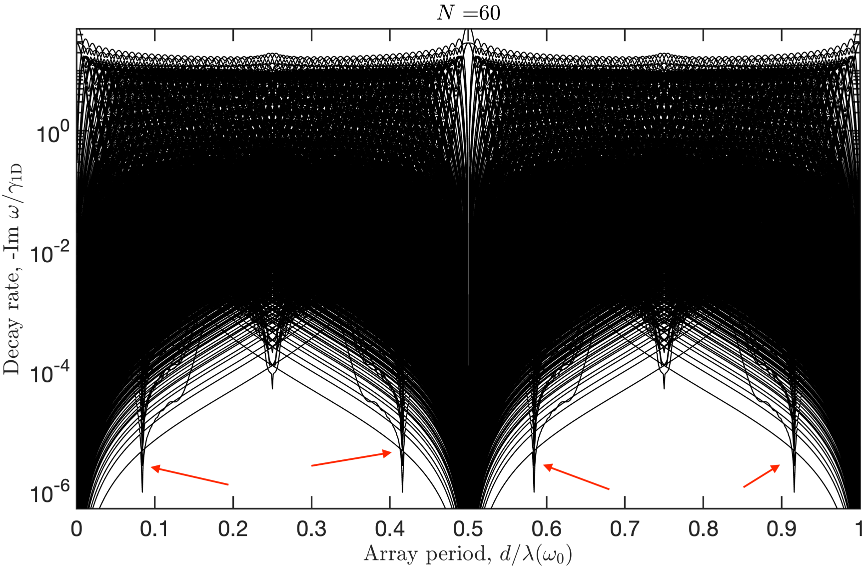
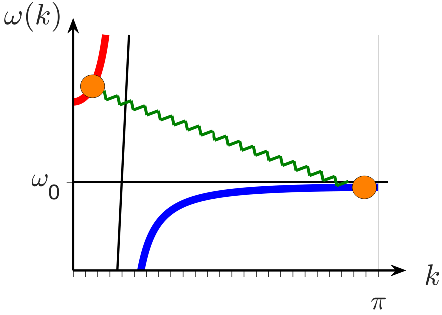
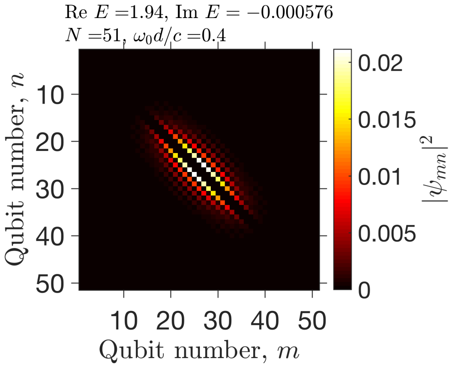


Subradiant states for $d = \lambda(\omega_0)/12$?

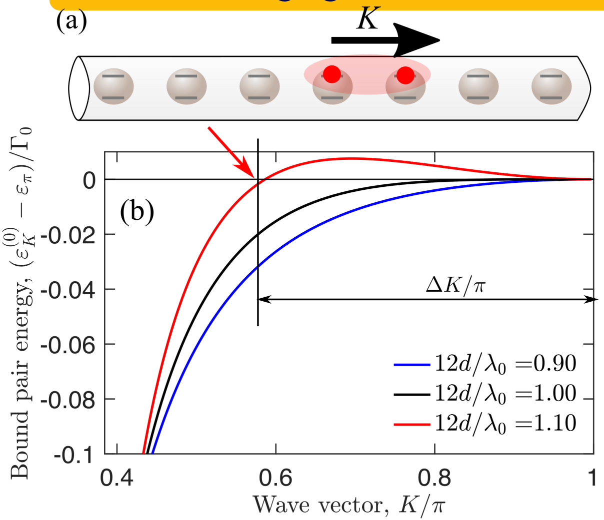




$$\psi_{nm} \propto \cos \frac{K(n+m)}{2} \mathrm{e}^{-\kappa|n-m|}$$

Y.X. Zhang, C. Yu, and K. Mølmer, PRR **2**, 013173 (2020)

Diverging effective mass of bound pairs



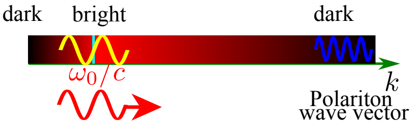
$$\frac{1}{m} \equiv \left.\frac{d^2\varepsilon}{dK^2}\right|_{K=\pi} = -\frac{\Gamma_0 \sin\varphi \cos 3\varphi}{8\cos^6\varphi}$$

For $\varphi = \pi/6$
$(d = \lambda/12)$
one has $m \to \infty$

$$\Psi_{j,j'} \propto e^{iK(j+j)')/2}\Phi_{j-j'}$$

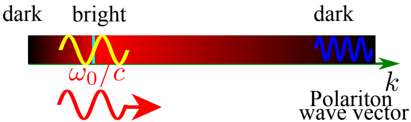
Bound state has long lifetime for $d = \lambda(\omega_0)/12$
due to quasi-flat center-of-mass dispersion.
ANP, PRA **101**, 043845 (2020);
general theorem: Y.-X. Zhang and K. Mølmer, PRL **125**, 253601 (2020)

Single-excited: $\psi(x) \propto \mathrm{e}^{\mathrm{i}kx}$

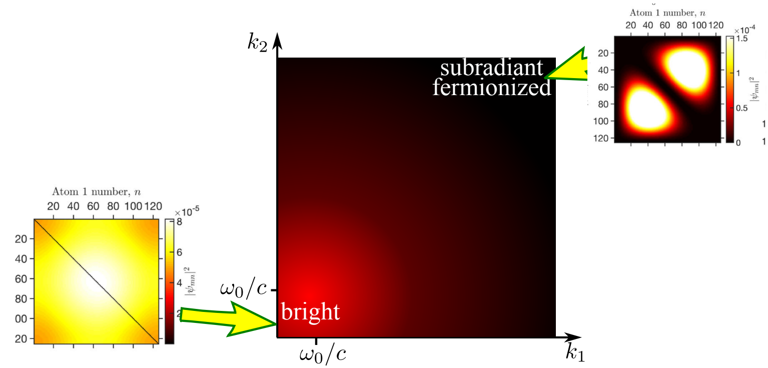


Beyond most subradiant states

Single-excited: $\psi(x) \propto \mathrm{e}^{\mathrm{i}kx}$



Double-excited: $\psi(n,m) \propto \mathrm{e}^{\mathrm{i}k_1 n + \mathrm{i}k_2 m} + (n \leftrightarrow m)\ldots$



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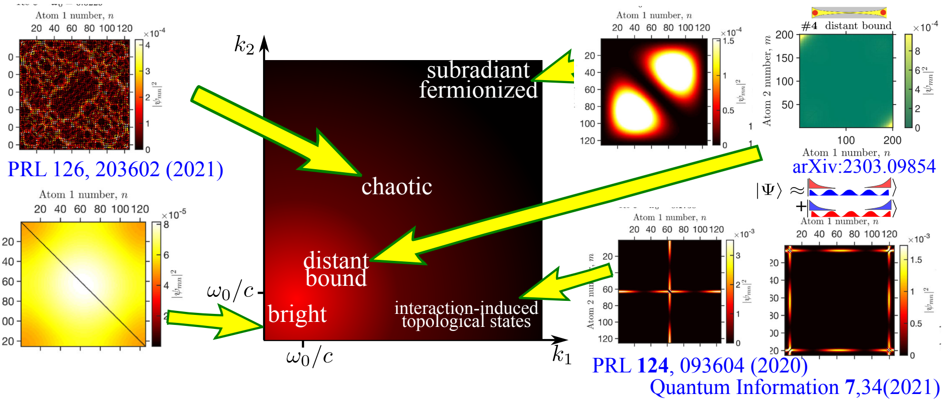
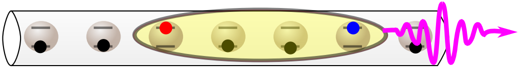


All happy families are alike,
but every unhappy family is unhappy in its own way
(Leo Tolstoy, Anna Karenina, 1878)

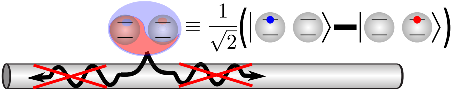
All superradiant states are alike,
but each subradiant state is subradiant in its own way



collective
spontaneous
emission

Many-body subradiant states in waveguide

Antisymmetric (subradiant) dimer excitation
$\frac{1}{\sqrt{2}}(b_1^\dagger - b_2^\dagger)|0\rangle$



Many-body subradiant states in waveguide

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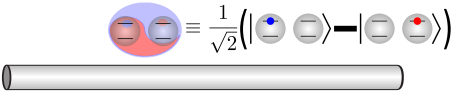

Antisymmetric (subradiant) dimer excitation
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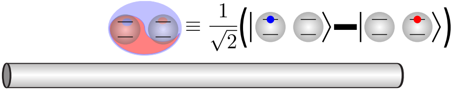
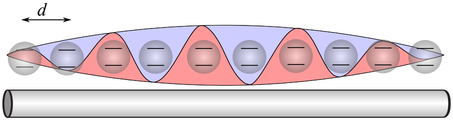


Weakly excited subradiant state for N atoms.
(Fill factor $f = n_{\rm exc}/N \ll 1$).

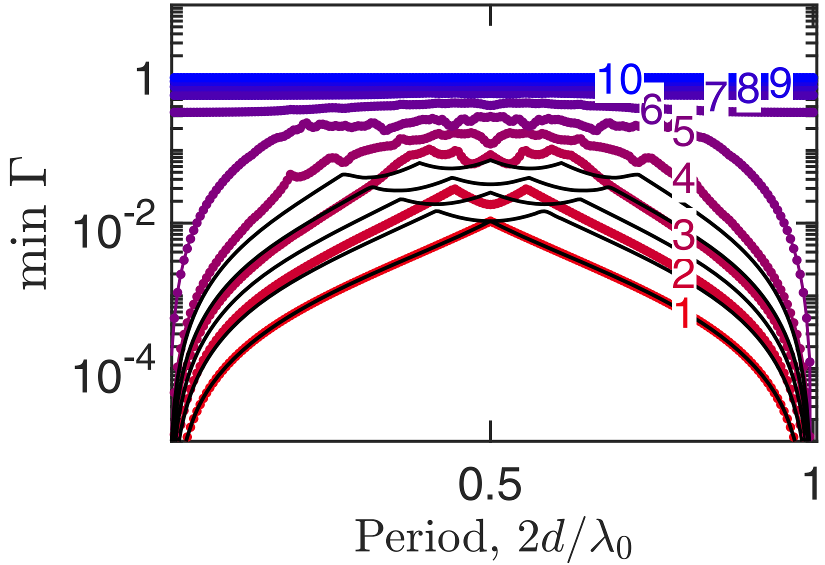
- decay rate:
  $\Gamma_{\rm darkest}(N) \sim \Gamma_0 \frac{(\omega_0 d/c)^2}{N^3}$
- Fermionization.



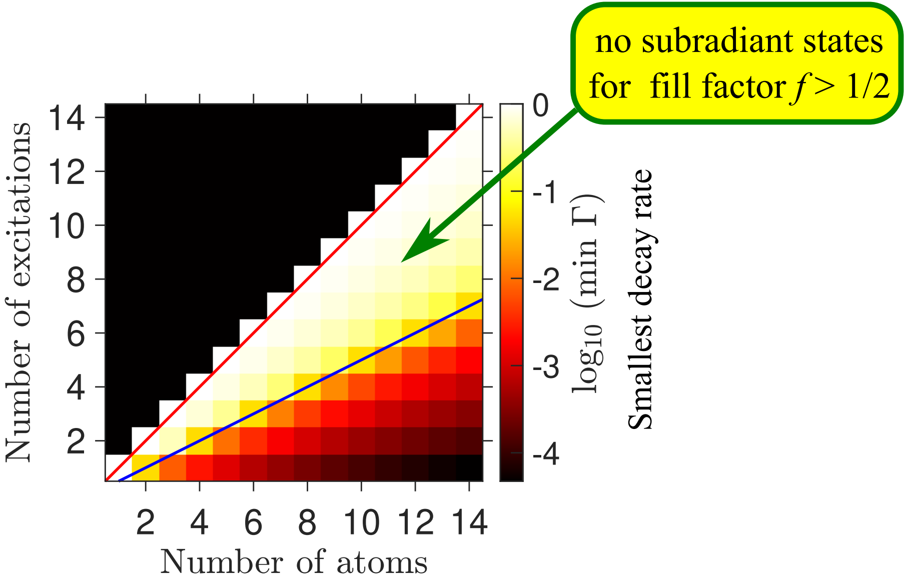
Zhang and Molmer, PRL 122, 203605 (2019)

What happens for large number of excitations m?

Decay rate for different numbers of excitations



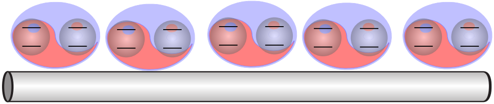
Smallest decay rate vs number of atoms and excitations



Poshakinskiy and ANP, PRL **127**, 173601 (2021)

Most-excited subradiant states

Multiply excited subradiant state ($f = 1/2$)

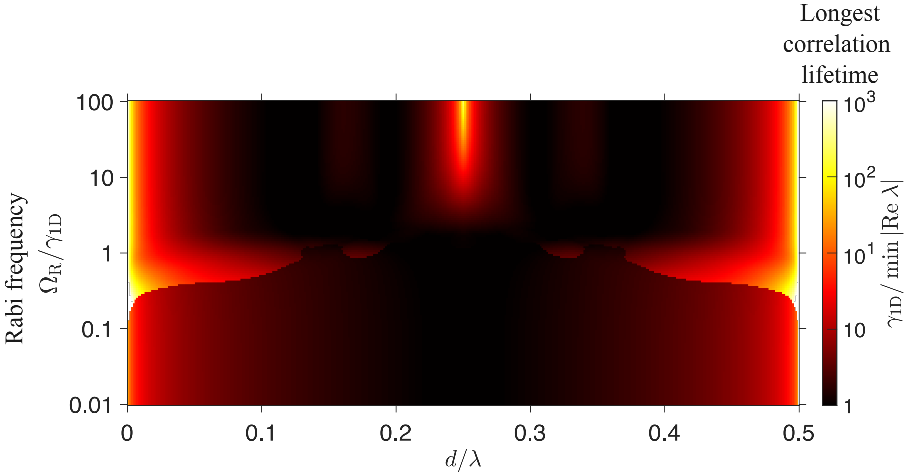


- subradiant states exist for fill-factor $f \leq 1/2$
- most subradiant state for $f = 1/2$ is the dimer product

$$|\psi\rangle = \frac{1}{2^{N/4}}(b_1^\dagger - b_2^\dagger)(b_3^\dagger - b_4^\dagger)\dots(b_{N-1}^\dagger - b_N^\dagger)|0\rangle$$

Poshakinskiy and ANP, PRL **127**, 173601 (2021)

Correlations in driven-dissipative dynamics

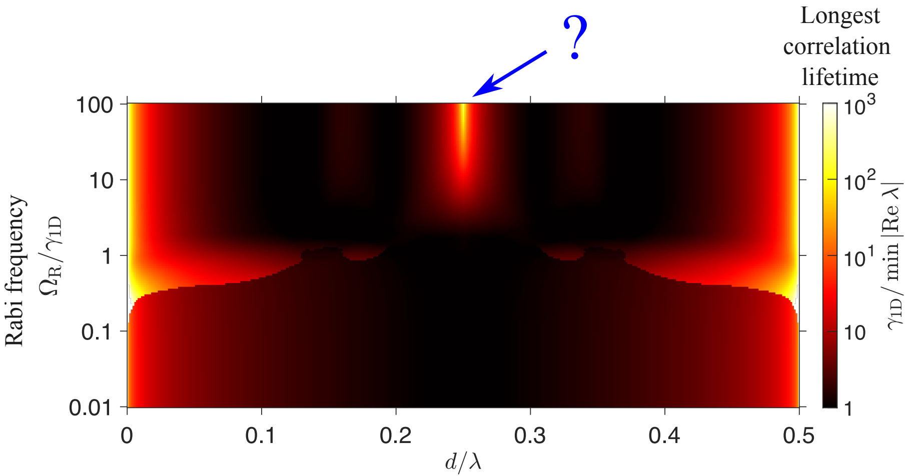


$$\mathcal{L}\rho = \Lambda\rho\ ,$$

$$\mathcal{L}\rho = \mathcal{L}_0\rho + \underbrace{\mathrm{i}[\rho, V]}_{\text{driving}}, \quad V = \Omega_\mathrm{R}\sum_{n=1}^{N}(\sigma_n^\dagger \mathrm{e}^{-\mathrm{i}\varphi n} + \mathrm{H.c.})$$

ANP, PRA **106**, L031702 (2022)

Correlations in driven-dissipative dynamics



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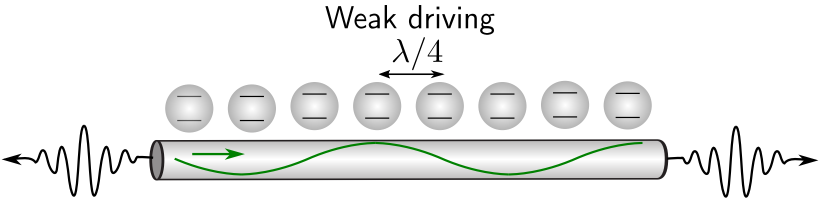
ANP, PRA **106**, L031702 (2022)

Explanation: array splits into two



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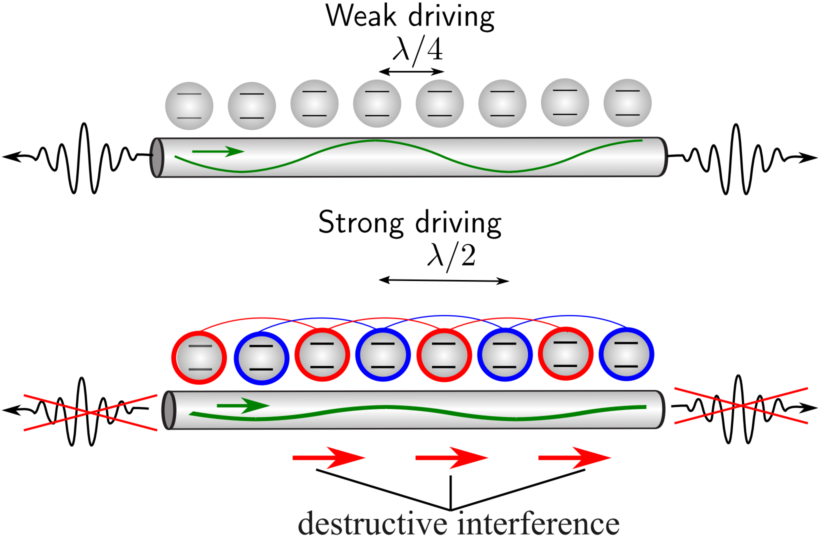


ANP, PRA **106**, L031702 (2022)

Summary

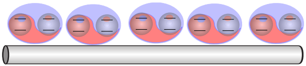
- Weakly-excited subradiant states
  - universal scaling $\propto (d/\lambda)^2/N^3$
  - polariton wave vectors $\sim \pi$

PRL **123**, 253601 (2019); Quantum Materials **6**, 10 (2021);

- Bound and single-excited states have long radiative lifetime if polariton mass is large

PRA **101**, 043845 (2020);

- Multiple-excited subradiant states:
  - fermionization →dimerization
  - fill-factor$\leq 1/2$



PRL **127**, 173601 (2021)

- Driven system can have special subradiant correlations

PRA **106**, L031702 (2022)

Review: Rev. Mod. Phys. 95, 0105002 (2023)