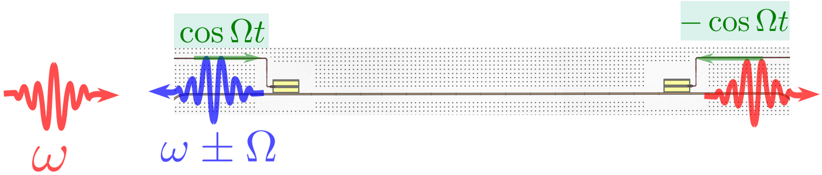
Routing classical and quantum light by time modulation

Alexander (Sasha) Poddubny

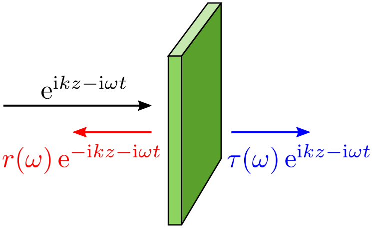
Weizmann Institute of Science

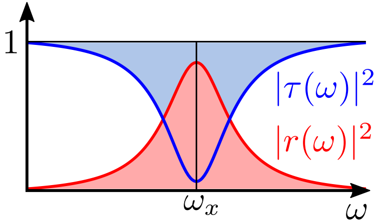


Tel-Aviv University,

Jan 10, 2024

Transmission and reflection of light by a resonant layer





reflection coefficient

$$r(\omega) = -\frac{i\Gamma_0}{\omega - \omega_x + i(\Gamma_0 + \Gamma)}$$

transmission coefficient

$$\tau(\omega) = 1 + r(\omega) = \frac{\omega - \omega_x + i\Gamma}{\omega - \omega_x + i(\Gamma + \Gamma_0)}$$

E. L. Ivchenko, *Optical spectroscopy of semiconductor nanostructures* (2005)

ω_x — resonant frequency ,
Γ_0 — radiative decay rate (light-matter interaction strength)

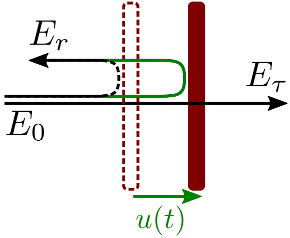
Γ — nonradiative decay rate

TMDC: $\Gamma_0 \sim 1$ meV, QW: $\Gamma_0 \sim 0.05$ meV

Light reflection by a resonant layer

Plane wave $E(t) = E_0 e^{-i\omega t}$ incident on a layer trembling as $u(t) = u e^{-i\Omega t} + u^* e^{i\Omega t}$

Layer displacement modifies the reflected wave phase by $\phi(t) = 2(\omega/c)u(t)$



Reflected wave up to u-linear terms

$$E_r = rE_0 e^{-i\omega t} e^{i\phi(t)} \approx rE_0 e^{-i\omega t} + 2i\frac{\omega}{c} rE_0 \Big[\underbrace{e^{-i(\omega+\Omega)t} u}_{\text{anti}-\text{Stokes}} + \underbrace{e^{-i(\omega-\Omega)t} u^*}_{\text{Stokes}}\Big]$$

Transmitted wave phase remains **unaffected**: $E_\tau = \tau E_0 e^{-i\omega t} + 0 + 0$

No forward-scattered Stokes and anti-Stokes waves

LIMITATION

In case of frequency-dependent r, shall we take $r(\omega)$ or $r(\omega \pm \Omega)$ or ... ?

Plane wave $E(t) = E_0 \mathrm{e}^{-\mathrm{i}\omega t}$ incident on a layer trembling as $u(t) = u\mathrm{e}^{-\mathrm{i}\Omega t} + u^* \mathrm{e}^{\mathrm{i}\Omega t}$

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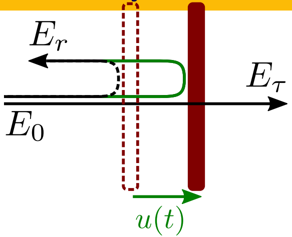

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$$E_r = rE_0\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}\phi(t)} \approx rE_0\mathrm{e}^{-\mathrm{i}\omega t} + 2\mathrm{i}\frac{\omega}{c}rE_0\Big[\underbrace{\mathrm{e}^{-\mathrm{i}(\omega+\Omega)t}u}_{\text{anti}-\text{Stokes}} + \underbrace{\mathrm{e}^{-\mathrm{i}(\omega-\Omega)t}u^*}_{\text{Stokes}}\Big]$$

Transmitted wave phase remains **unaffected**: $E_\tau = \tau E_0 \mathrm{e}^{-\mathrm{i}\omega t} + 0 + 0$

Resonant scatterer:

Forward scattering: $S^{\rightarrow} = \mathrm{i}\frac{\omega'}{c}u\left[r(\omega \pm \Omega) - r(\omega)\right]$

Backscattering: $S^{\leftarrow} = \mathrm{i}\frac{\omega'}{c}u\left[r(\omega \pm \Omega) + r(\omega)\right]$

Need $\Omega \gtrsim \Gamma$

(sideband-resolved regime)

Poshakinskiy, ANP, PRX **9**, 011008 (2019)
M.-T. Jaekel and S. Reynaud, Quantum Opt. 4 39 (1992)

Directional scattering by a small trembling particle

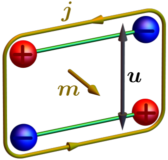
Scattered field is a sum of electric dipole, magnetic dipole and electric quadrupole

$$\boldsymbol{p} = \frac{\mathrm{i}\omega}{c}\,\alpha(\omega')(\boldsymbol{n}_0\cdot\boldsymbol{u})\boldsymbol{E}_0$$

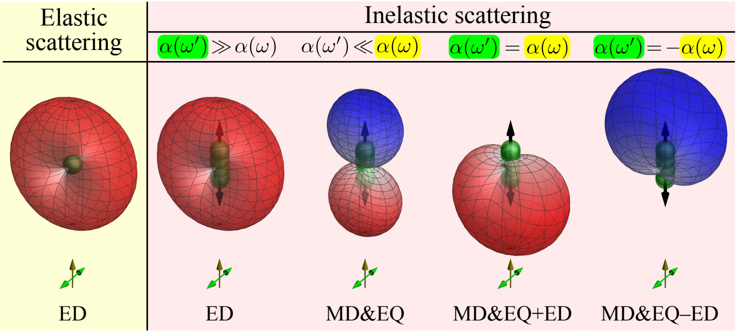
$$\boldsymbol{m} = \frac{\mathrm{i}\omega}{2c}\,\alpha(\omega)(\boldsymbol{E}_0\times\boldsymbol{u})$$

$$\boldsymbol{Q} = 3\alpha(\omega)(\boldsymbol{E}_0\otimes\boldsymbol{u}+\boldsymbol{u}\otimes\boldsymbol{E}_0)$$

Moving & oscillating ED yields MD and EQ

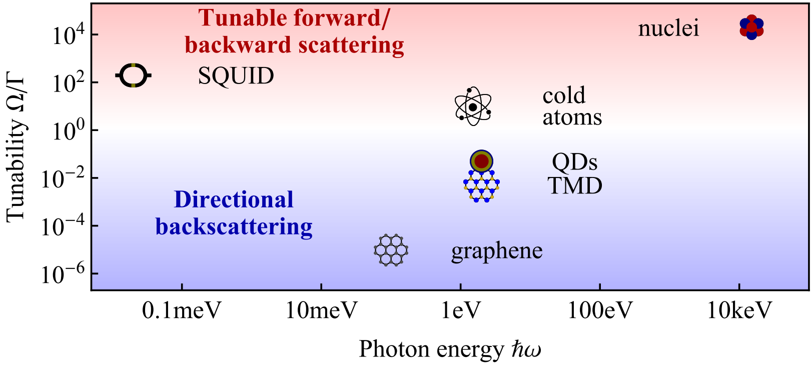


Frequency dependence of α can be used to achieve Kerker condition

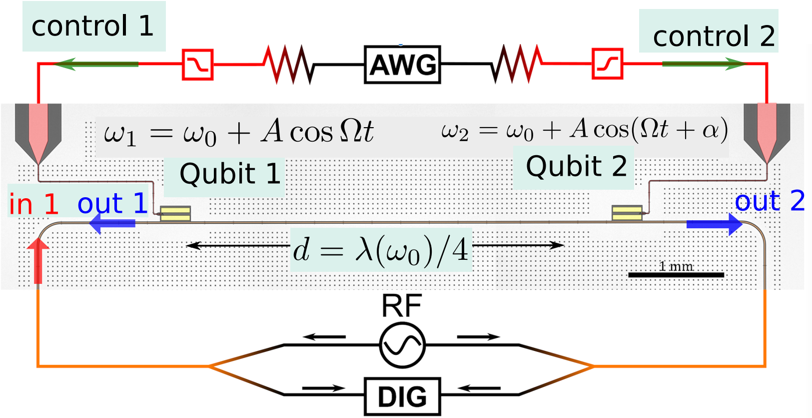


Poshakinskiy, Poddubny, Phys. Rev. X 9, 011008 (2019)

Possible platforms for observation

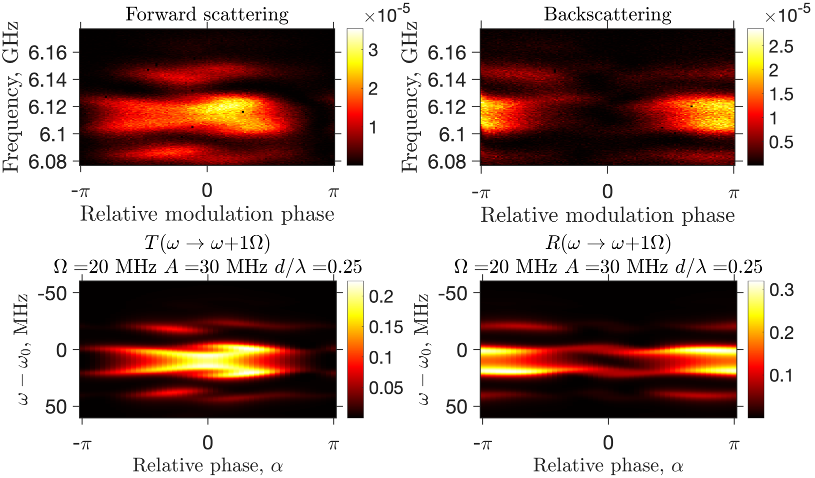


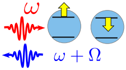
Parameter / System	Photon energy $\hbar\omega_x$	Resonance width Γ	Trembling frequency Ω	Trembling amplitude u
Plasmon in graphene	0.1 – 1 eV	10 THz	0.1 GHz	10 nm
TMD monolayers	2 eV	20 GHz	0.1 GHz	10 nm
Colloidal QDs	2 eV	400 MHz	20 MHz	200 nm
Cold atoms	1.5 eV	10 kHz	100 kHz	200 nm
Superconducting qubits	20 μeV	0.1 MHz	20 MHz	200 nm
Resonance in nuclei	15 keV	0.5 MHz	10 GHz	0.001 nm



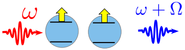
Idea of experiment:
control of photon scattering direction at $\omega \pm \Omega$
using **time modulation** of qubit resonant frequency $\omega_{1,2}(t)$
with phase delay α
Collaborators: Johannes Fink group, IST Austria

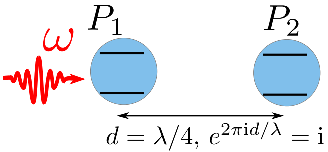
Measured Stokes reflection and transmission





- modulation in phase: forward scattering
- modulation in anti-phase: backscattering





$$P_{1,2}(t)=\sum_{n=-\infty}^{\infty} P_{1,2}^{(n)}\mathrm{e}^{-\mathrm{i}(\omega+n\Omega)t}$$

driving: $P_2^{(0)}=\mathrm{i}P_1^{(0)}$

Modulation in phase: $P_2^{(1)}=\mathrm{i}P_1^{(1)}$

Back-scattering:
$S_{\hookleftarrow}\propto \mathrm{i}P_2^{(1)}+P_1^{(1)}=0$

Forward scattering:
$S_{\rightarrow}\propto P_2^{(1)}+\mathrm{i}P_1^{(1)}=2\mathrm{i}P_1^{(1)}\neq 0$

Modulation in anti-phase: $P_2^{(1)}=-\mathrm{i}P_1^{(1)}$

Back-scattering:
$S_{\hookleftarrow}\propto \mathrm{i}P_2^{(1)}+P_1^{(1)}=2P_1^{(1)}\neq 0$

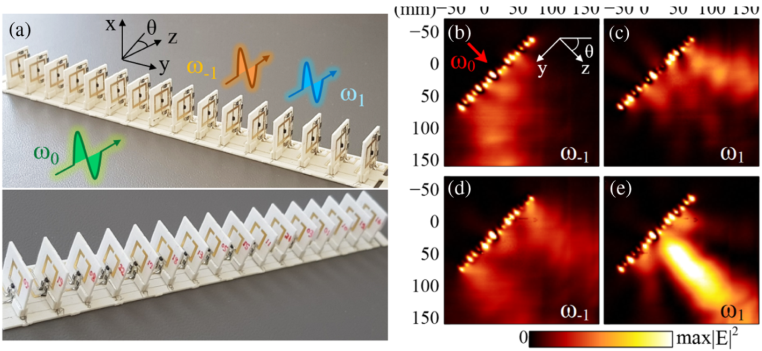
Forward scattering:
$S_{\rightarrow}\propto P_2^{(1)}+\mathrm{i}P_1^{(1)}=0$

Classical electromagnetic setup

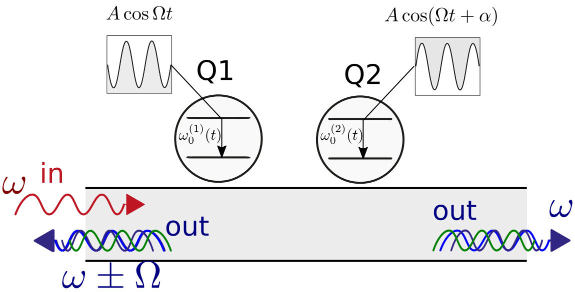
PHYSICAL REVIEW X **8**, 031077 (2018)

Huygens' Metadevices for Parametric Waves

Mingkai Liu,[1,*] David A. Powell,[2,1] Yair Zarate,[1] and Ilya V. Shadrivov[1]
[1]*Nonlinear Physics Centre, Research School of Physics and Engineering, Australian National University, Canberra, Australian Capital Territory 2601, Australia*
[2]*School of Engineering and Information Technology, University of New South Wales, Canberra, Australian Capital Territory 2610, Australia*



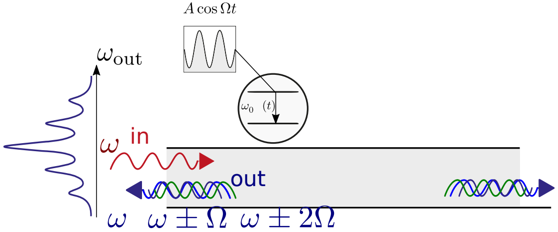
Intermediate summary:



Different harmonics can be scattered in different directions

E.Redchenko, A.Poshakinskiy, R.Sett, M.Zemlicka, ANP and J.M.Fink, Nat. Commun. **14**, 2998 (2023)

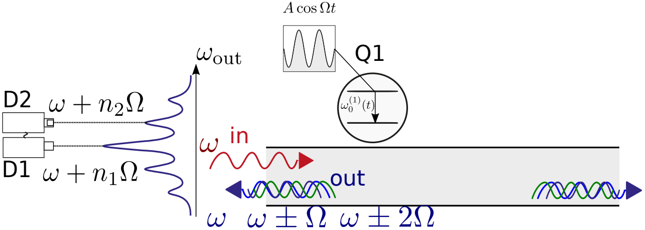
Next: Let us go quantum



$$\omega_0(t) = \omega_0 + A\cos\Omega t$$

$$\psi_{\rm out} \propto \mathrm{e}^{-\mathrm{i}\omega t}\sum_{n=-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}n\Omega t} J_n(A/\Omega)$$

Two-photon correlations: 1 qubit



Single-photon subspace:

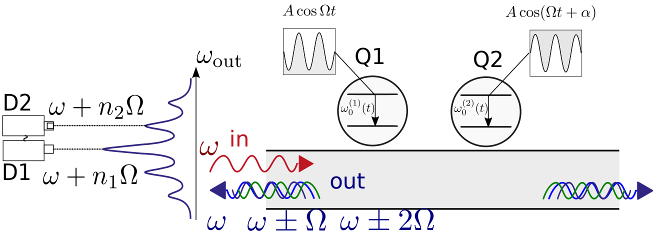
$$\psi_{\text{out}}(t)\propto \mathrm{e}^{-\mathrm{i}\omega t}\sum_{n=-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}m\Omega t}J_n(A/\Omega)$$

Two-photon subspace: product state

$$\psi_{\text{out}}^{(2)}(t,t+\tau)\propto \mathrm{e}^{-2\mathrm{i}\omega t}\sum_{n_1,n_2=-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}(n_1+n_2)\Omega t}J_{n_1}(A/\Omega)J_{n_2}(A/\Omega)f_{n_1,n_2}(\tau)$$

$f_{n_1,n_2}(\tau=0)=0$ (antibunching).

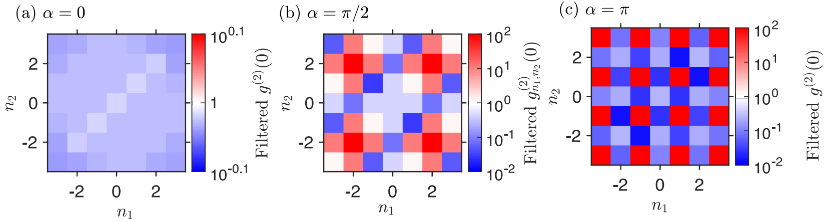
G. Nienhuis, "Spectral correlations in resonance fluorescence," PRA 47, 510 (1993)



$$\psi^{(2)}_{\rm out}(t,t)\propto \mathrm{e}^{-2\mathrm{i}\omega t}\sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}(n_1+n_2)\Omega t}$$
$$\times J_{n_1}(A/\Omega)J_{n_2}(A/\Omega)(\mathrm{e}^{-\mathrm{i}n_1\alpha}+\mathrm{e}^{-\mathrm{i}n_2\alpha})$$

Two-photon interference

$$\psi^{(2)}_{\rm out}(t,t) \propto \mathrm{e}^{-2\mathrm{i}\omega t} \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}(n_1+n_2)\Omega t} \times J_{n_1}(A/\Omega) J_{n_2}(A/\Omega)(\mathrm{e}^{-\mathrm{i}n_1\alpha}+\mathrm{e}^{-\mathrm{i}n_2\alpha})$$



Entanglement entropy

Schmidt decomposition: $\psi_{n_1,n_2} = \sum_\nu \lambda_\nu u^\nu_{n_1} v^\nu_{n_2}$

$$S = -\sum_\nu |\lambda_\nu|^2 \ln |\lambda_\nu|^2$$

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- One qubit. $\psi_{n_1,n_2} = J_{n_1}(A/\Omega) J_{n_2}(A/\Omega)$

$$\lambda_1 = 1, S = 0$$

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$$|\lambda_{1,2}| = \frac{1 \pm x}{\sqrt{2(1+x^2)}}, \quad x = J_0\left(\frac{2A}{\Omega} \sin\frac{\alpha}{2}\right)$$

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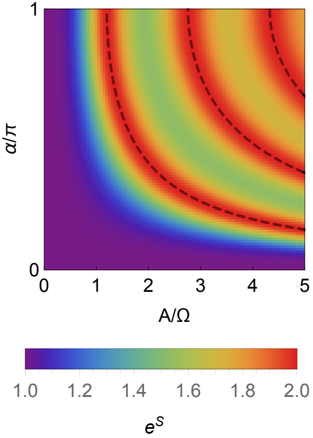


Generalized multipartite entanglement entropy

Generalized singular value decomposition:

$\psi_{n_1,n_2,n_3} =$

Generalized multipartite entanglement entropy

Generalized singular value decomposition:

$$\psi_{n_1,n_2,n_3} = \sum \Lambda_{\nu_1\nu_2\nu_3} U_{n_1}^{\nu_1} U_{n_2}^{\nu_2} U_{n_3}^{\nu_3}\,, \quad \sum_{\nu_2,\nu_3} \Lambda_{\nu_1\nu_2\nu_3} \Lambda_{\nu_1'\nu_2\nu_3} = \delta_{\nu_1,\nu_1'}$$

L. D. Lathauwer, B. D. Moor, and J. Vandewalle, A multilinear singular value decomposition, SIAM J. Matrix Anal. Appl. **21**, 1253 (2000).

Generalized multipartite entanglement entropy

Generalized singular value decomposition:

$$\psi_{n_1,n_2,n_3} = \sum \Lambda_{\nu_1\nu_2\nu_3} U^{\nu_1}_{n_1} U^{\nu_2}_{n_2} U^{\nu_3}_{n_3}\,, \quad \sum_{\nu_2,\nu_3} \Lambda_{\nu_1\nu_2\nu_3} \Lambda_{\nu_1'\nu_2\nu_3} = \delta_{\nu_1,\nu_1'}$$

L. D. Lathauwer, B. D. Moor, and J. Vandewalle, A multilinear singular value decomposition, SIAM J. Matrix Anal. Appl. **21**, 1253 (2000).

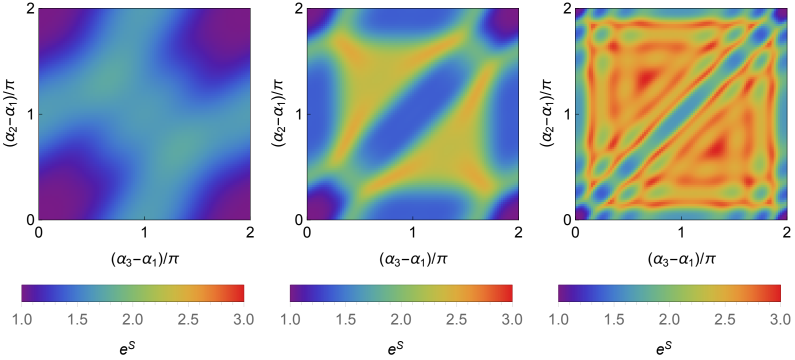
Generalized entanglement entropy:

$$|\lambda_\nu|^2 = \sum_{\nu_1\nu_2} |\Lambda_{\nu_1\nu_2\nu}|^2\,,$$

$$S = -\sum_{\nu} |\lambda_\nu|^2 \ln |\lambda_\nu|^2$$

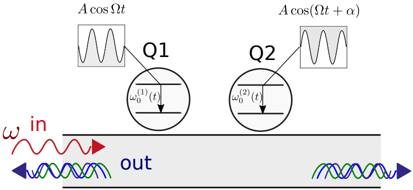
A.V. Poshakinskiy and ANP, PRL **127**, 173601 (2021)

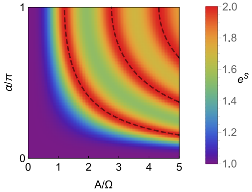
Entropy depending on phase



Color plots of e^S, where S is the entanglement entropy, for the three photons reflected from three modulated qubits as a function of the relative phases of the modulation α calculated for different modulation amplitudes (from left to right): $A/\Omega = 1$, 2, and 5.
Cluster state reached at $\alpha_1 - \alpha_2 = \alpha_2 - \alpha_3 = 2\pi/3$, $\mathrm{e}^S = 3$

Summary





Modulation of resonance frequencies in time enables

- directional scattering

Phys. Rev. X 9, 011008 (2019); Nat. Commun. **14**, 2998 (2023)

- selective bunching and antibunching, cluster states, Bell states, on-demand entangled states

Phys. Rev. Lett. **130**, 023601 (2023)

More on time-modulated qubit arrays:

I. Iorsh, A.V. Poshakinskiy & ANP,PRL **125**, 183601 (2020); PRA **104**, 063719 (2021)

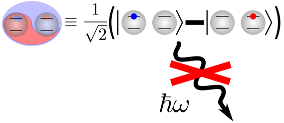
More on waveguide QED:

A.S. Sheremet, M.I. Petrov, I.V. Iorsh, A.V. Poshakinskiy and ANP,

Rev. Mod. Phys. **95**, 015002 (2023)

Some other projects

Subradiant states



Y. Ke, A.V. Poshakinskiy, C. Lee, Yu.S. Kivshar & ANP, PRL **123**, 253601 (2019)

ANP, PRA **101**, 043845 (2020)

A.V. Poshakinskiy & ANP, PRA **103**, 043718 (2021); PRL **127**, 173601 (2021)

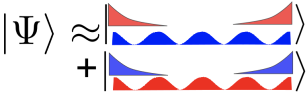
J. Brehm, ANP, A. Stehli, T. Wolz, H. Rotzinger, and A.V. Ustinov, npj Quantum Materials **6**, 10 (2021)

ANP, PRA **106**, L031702 (2022)

D.Kornovan, ANP and A. Poshakinskiy,PRA **108**, 033707 (2023)

E.Vlasiuk, A.V. Poshakinskiy, and ANP,PRA **108**, 033705 (2023)

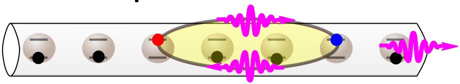
Topological edge states:



Y. Ke, J. Zhong, A.V. Poshakinskiy, Yu.S. Kivshar, ANP, and Ch. Lee, PRR **2**, 033190 (2020)

A.V. Poshakinskiy, J. Zhong, Y. Ke, N.A. Olekhno, C. Lee, Yu.S. Kivshar, ANP, npj Quantum Information **7**, 34 (2021)

Polariton-polariton interactions



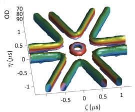
J. Zhong, N.A. Olekhno, Y. Ke, A.V. Poshakinskiy, C. Lee, Yu.S. Kivshar, and ANP, PRL **124**, 093604 (2020)

J. Zhong and ANP, PRA **103**, 023720 (2021)

A.V. Poshakinskiy, J. Zhong, ANP, PRL **126**, 203602 (2021)

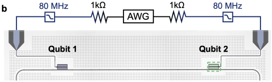
A.V. Poshakinskiy and ANP, PRA **108**, 023707 (2023)

Polariton-polariton interactions with Rydberg atoms:



L. Drori, B.C. Das, T.D. Zohar, G. Winer, E. Poem, ANP, and O. Firstenberg, Science **381**, 193 (2023)

Time-modulated WQED



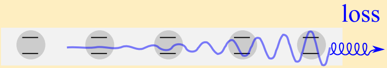
I. Iorsh, A.V. Poshakinskiy & ANP,PRL **125**, 183601 (2020); PRA **104**, 063719 (2021)

D. Ilin, A.V. Poshakinskiy, ANP, I.Iorsh, PRL **130** 023601 (2023)

E.Redchenko, A.Poshakinskiy, R.Sett, M.Zemlicka, ANP and J.M.Fink, Nat. Commun. **14**, 2998(2023)

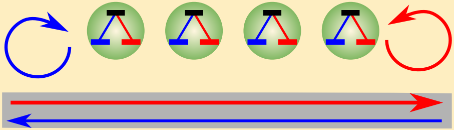
Main activities in 2023

Non-Hermitian skin effect in chiral waveguide-QED



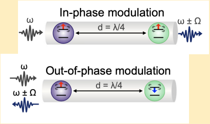
Collaborations: S. Fan @ Stanford
arXiv:2310.04025

waveguide-QED with Λ-atoms



Collaborations: B. Dayan, S. Rosenblum @ Weizmann

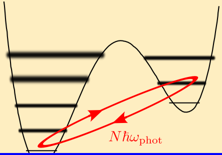
Directional scattering from modulated qubits



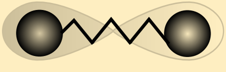
E.Redchenko, A.Poshakinskiy, R.Sett, M.Zemlicka, ANP and J.M.Fink, Nat. Commun. 14, 2998(2023)

D. Ilin, A.V. Poshakinskiy, ANP, I.Iorsh
PRL 130 023601 (2023)

Floquet dynamics of complex qubits



entanglement in optomechanics



Collaborations: M. Aspelmeyer @ UWien

Master students, PhD students, postdocs and beyond are welcome!
poddubny@weizmann.ac.il

WEIZMANN INSTITUTE OF SCIENCE

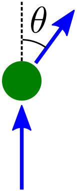
Light scattering by a particle

Rayleigh scattering

scattering by a small particle
with **dipole polarizability only**

$$I(\theta) \sim \frac{\alpha^2}{\lambda^4}(1+\cos^2\theta)$$

Explains the blue color of sky



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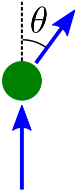
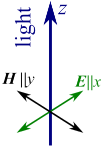
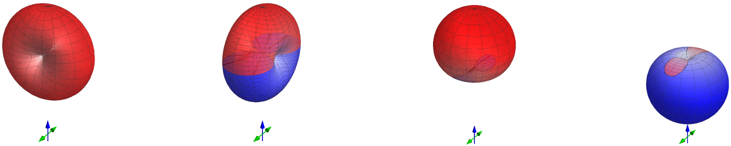


Kerker effect: Light scattering by a particle with **both electric dipole** (ED) and **magnetic dipole** (MD) polarizabilities



ED | MD | ED+MD | ED-MD



$E_x(z) = E_x(-z)$ | $E_x(z) = -E_x(-z)$ | forward scattering | backward scattering

LIMITATION: in optics, MD is relativistically weaker than ED (for sub-wavelength particles)

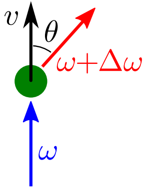
Light scattering by a moving particle

C.V. Raman,
Nature **103**, 65(1919)

Doppler shift
is strongly asymmetric

$$\Delta\omega = \omega \frac{v}{c} (1 - \cos\theta)$$

LIMITATION:
Asymmetry of **scattering intensity**
is relativistically small $\sim v/c$



Idea: Scattering by a trembling particle

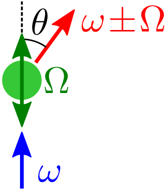
Particle oscillation with the frequency Ω

$\Downarrow$

Periodically varying (asymmetric!) Doppler shift

$\Downarrow$

Shifted harmonics at $\omega \pm \Omega$ in scattered light

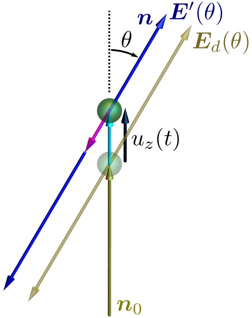


Effect of polarizability dispersion on inelastic scattering

Two contributions to the phase come from the light path variation before and after scattering:

$$\phi(\theta, t) = (\omega/c)(1 - \cos\theta)u_z(t)$$

Corresponding scattering amplitudes feature polarizabilities $\alpha(\omega \pm \Omega)$ and $\alpha(\omega)$



Inelastic scattering cross-section for unpolarized light

$$\frac{d\sigma}{do} = |u_z|^2 \frac{\omega^6}{c^6} \left| \alpha(\omega \pm \Omega) - \alpha(\omega)\cos\theta \right|^2 \underbrace{\frac{1+\cos^2\theta}{2}}_{\text{polarization averaging}}$$

Dispersion effects arise when the trembling frequency Ω is comparable to the width of polarizability resonance Γ

Poshakinskiy, Poddubny, Phys. Rev. X **9**, 011008 (2019)

Multipole decomposition of the scattered light

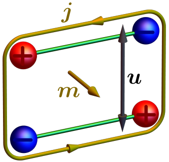
Scattered field is a sum of electric dipole, magnetic dipole and electric quadrupole

$$\boldsymbol{p} = \frac{\mathrm{i}\omega}{c}\alpha(\omega')(\boldsymbol{n}_0 \cdot \boldsymbol{u})\boldsymbol{E}_0$$

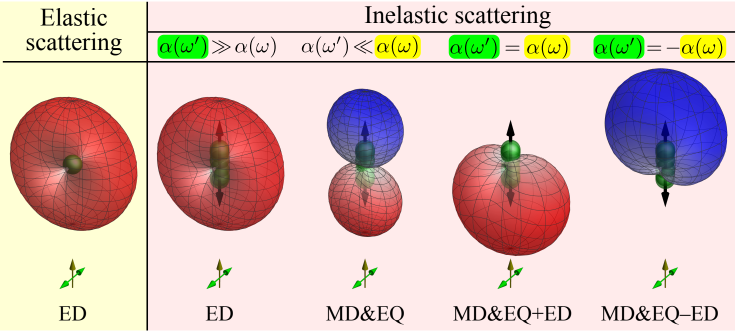
$$\boldsymbol{m} = \frac{\mathrm{i}\omega}{2c}\alpha(\omega)(\boldsymbol{E}_0 \times \boldsymbol{u})$$

$$\boldsymbol{Q} = 3\alpha(\omega)(\boldsymbol{E}_0 \otimes \boldsymbol{u} + \boldsymbol{u} \otimes \boldsymbol{E}_0)$$

Moving & oscillating ED yields MD and EQ



Frequency dependence of α can be used to achieve Kerker condition



Poshakinskiy, Poddubny, Phys. Rev. X 9, 011008 (2019)

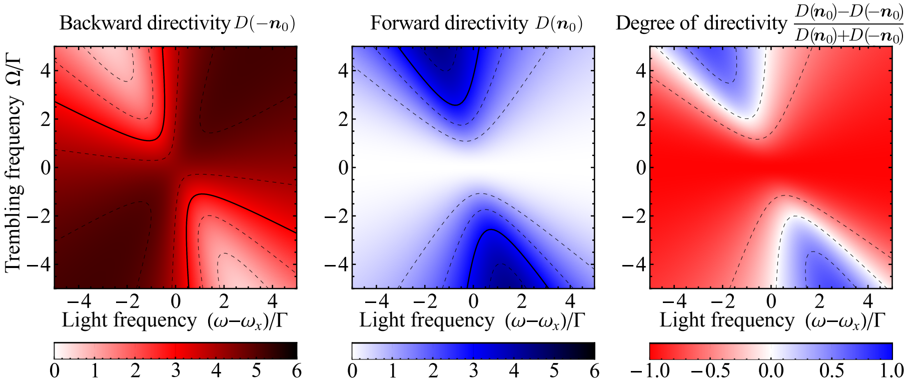
Directivity of inelastic resonant scattering

Resonant polarizability

$$\alpha(\omega) = \frac{A}{\omega - \omega_x + \mathrm{i}\Gamma}$$

Scattering in direction $\boldsymbol{n}$ is quantified by directivity

$$D(\boldsymbol{n}) = 4\pi \frac{dI(\boldsymbol{n})/do}{\int dI}$$



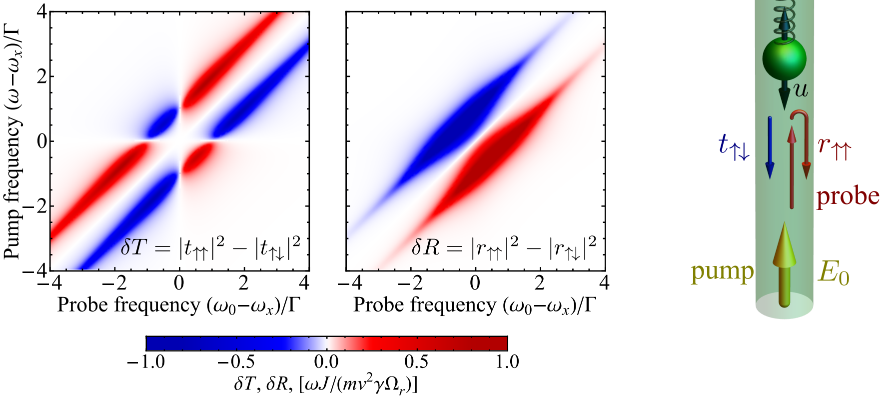
Maximal directivity is 5.25, which surpasses the conventional Kerker effect limiting value 3, because EQ is involved in addition to ED and MD

Optomechanically induced non-reciprocity

In the presence of optical pump at ω_0,
co- $(s=+)$ and counter- $(s=-)$ propagating probe light couples to **vibrations** differently

Optomechanical coupling is chiral at nanoscale

Spectra of
non-reciprocal **transmission** & asymmetric **reflection**



Optomechanical Spin-Hall effect

Polarization conversion at inelastic scattering is due to relativistic effects

Unpolarized light acquires **circular polarization** at inelastic scattering

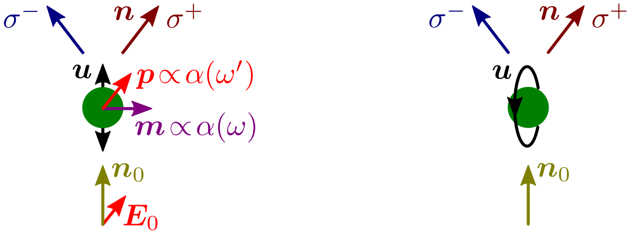
$$P_c \sim \boxed{\frac{\Omega}{\omega}} [\boldsymbol{n} \times \boldsymbol{n}_0] \cdot \mathrm{Im}\, \frac{[\alpha(\omega') + (\boldsymbol{n}_0 \cdot \boldsymbol{n})\alpha(\omega)]\boldsymbol{u}}{[\alpha(\omega')\boldsymbol{n}_0 - \alpha(\omega)\boldsymbol{n}] \cdot \boldsymbol{u}}$$

cf. spin-Hall effect for light
Bliokh *et al.*,
Nat. Phot. 9, 796 (2015)

Two mechanisms of conversion

Phase difference between motion-induced ED and MD $\propto \alpha(\omega')$ and $\alpha(\omega)$

Circular motion of the scatterer, i.e., phase difference between components u_x, u_y, u_z



Optomechanical Huygens' surface

An **array** of particles exhibiting Kerker effect can act as a Huygens' surface

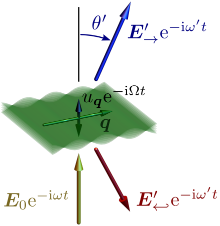
Pfeiffer, Grbic, PRL **110**, 197401 (2013)
Staude *et al*, ACS Nano **7**, 7824 (2013)

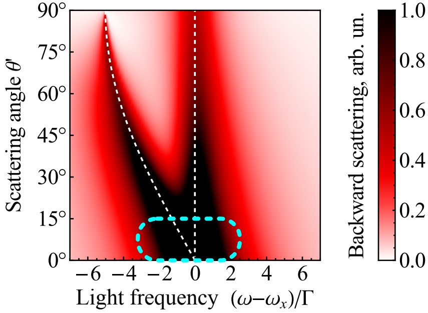
Scattering amplitude

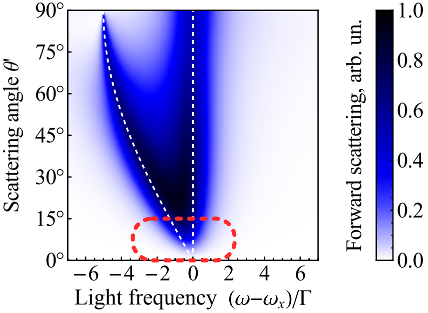
$$S_{ss}^{\rightarrow(\leftarrow)} = \mathrm{i}\frac{\omega'}{c} u_{\boldsymbol{q}} \left[r_s(\theta', \omega') \mp r_s(\theta, \omega) \right]$$

Trembling resonant membrane scatters light **preferably backward**

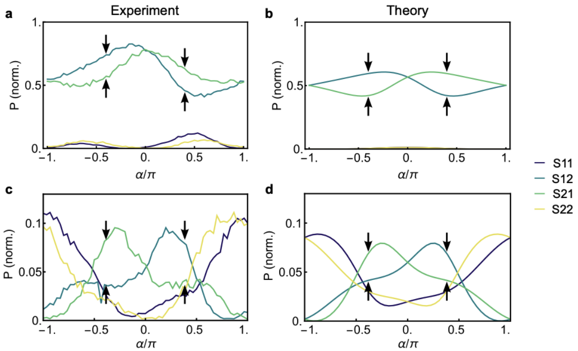
Light scattering on trembling membrane





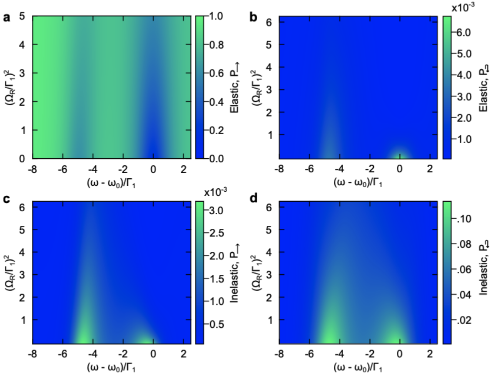


Power dependence



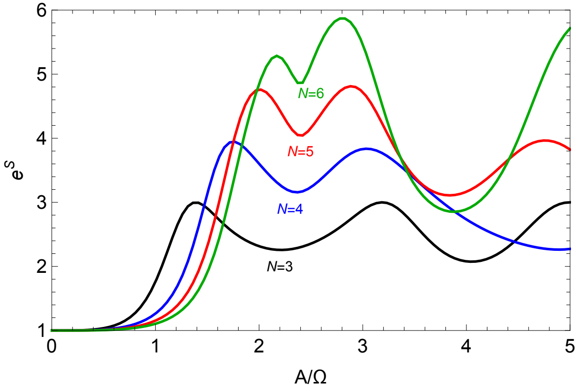
Supplementary Figure S5. **Isolator scattering matrix. a.** Measured coherent elastic scattering power normalized to the background transmission coefficient shown as a function of the relative phase between modulation tones α where $\Omega/(2\pi) = 20\,\mathrm{MHz}$ and $A_m/(2\pi) = 30\,\mathrm{MHz}$. **b.** Theoretically predicted elastic scattering. **c.** Measured coherent inelastic scattering power of the Stokes component normalized to theory shown as a function of the relative phase between modulation tones α where $\Omega/(2\pi) = 20\,\mathrm{MHz}$ and $A_m/(2\pi) = 30\,\mathrm{MHz}$. **d.** Theoretically predicted inelastic scattering.

Scattering matrix



Supplementary Figure S4. **Dynamic range of directional scattering.** Calculated coherent elastic (**a, b**) and inelastic (**c, d**) scattering spectra as a function of normalized drive frequency $(\omega - \omega_0)/\Gamma_1$ and drive power $(\Omega_R/\Gamma_1)^2$ theoretically predicted for reflection (**a, c**) and transmission (**b, d**). The calculation has been performed for the modulation phase difference $\alpha_2 - \alpha_1 = \pi$ and $\Omega = A_m = 5\Gamma_1$.

Entropy depending on amplitude



Dependence of e^S, where S is 3-photon the entanglement entropy, for the three photons reflected from N modulated qubits as a function of the modulation amplitude A for different N. The qubits are modulated with the phases $\alpha_i = 2\pi i/N$ and equal amplitudes.

Cluster state for 3 photons

$$\psi_{n_1,n_2,n_3} = \sum_{a\neq b, b\neq c, c\neq a} a_{n_1}^{(a)} a_{n_2}^{(b)} a_{n_3}^{(c)}$$

Maximum entropy $S = \ln 3$ is reached for the cluster state

$$\begin{aligned}\psi^{(\mathrm{cl})}_{n_1,n_2,n_3} &= a_{n_1}^{(1)} a_{n_2}^{(2)} a_{n_3}^{(3)} + a_{n_1}^{(1)} a_{n_2}^{(3)} a_{n_3}^{(2)} + a_{n_1}^{(2)} a_{n_2}^{(1)} a_{n_3}^{(3)} \\ &+ a_{n_1}^{(2)} a_{n_2}^{(3)} a_{n_3}^{(1)} + a_{n_1}^{(3)} a_{n_2}^{(1)} a_{n_3}^{(2)} + a_{n_1}^{(3)} a_{n_2}^{(2)} a_{n_3}^{(1)} \, .\end{aligned}$$

$$a_n = J_n(A/\Omega)$$

$\alpha_1 - \alpha_2 = \alpha_2 - \alpha_3 = 2\pi/3$ and $A/\Omega = j_{0,1}/\sqrt{3} \approx 1.39$,
$J_0(j_{0,1}) = 0$

Designing correlations

1. Do an SVD factorization of the target wave function:
$$\psi_{n_1,n_2} \approx \lambda^{(1)}\alpha^{(1)}_{n_1}\alpha^{(1)}_{n_2} + \lambda^{(2)}\alpha^{(2)}_{n_1}\alpha^{(2)}_{n_2} \tag{1}$$
If the SVD decomposition has more than 2 terms,
more than 2 qubits are required

2. Define $a^{(1,2)}_n = C^{(1,2)}\left[\sqrt{\lambda^{(1)}}\alpha^{(1)}_n \pm i\sqrt{\lambda^{(2)}}\alpha^{(2)}_n\right]$,
Then $\psi_{n_1,n_2} \approx 2C^{(1)}C^{(2)}\left(a^{(1)}_{n_1}a^{(2)}_{n_2} + a^{(2)}_{n_1}a^{(1)}_{n_2}\right)$.

3. Calculate required modulation of the qubit resonance frequencies
$$\omega^{(1,2)}(t) = \mathrm{i}\frac{d}{dt}\ln\left[\sum_n a^{(1,2)}_n e^{-in\Omega t}\right] . \tag{2}$$

Designing correlations

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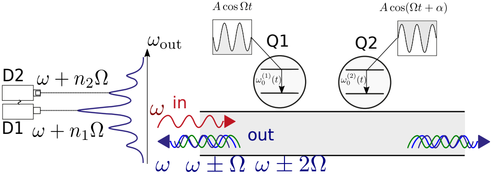
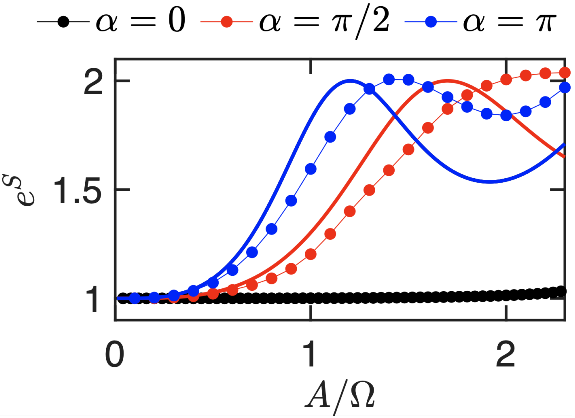
$$\omega^{(1,2)}(t) = \mathrm{i}\frac{d}{dt}\ln\left[\sum_n a^{(1,2)}_n e^{-in\Omega t}\right] . \tag{2}$$

Example target: $\Psi_{n_1,n_2} = \frac{1}{\sqrt{2}}(f_{n_1}f_{n_2}+g_{n_1}g_{n_2})\,,\, f_n = \delta_{n,0}, g_n = \frac{\delta_{n,1}+\delta_{n,-1}}{\sqrt{2}}$

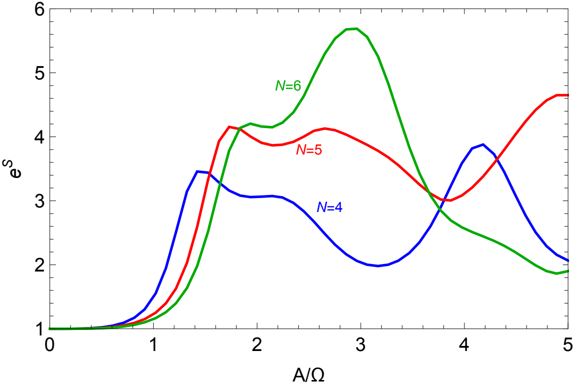
$\omega^{(1,2)}(t) = \mathrm{Im}\,\frac{\mathrm{d}}{\mathrm{d}t}\ln(1\pm\mathrm{i}\sqrt{2}\cos\Omega t)$, Fidelity: $F = \left|\sum_{n_1,n_2}\Psi^*_{n_1,n_2}\psi_{n_1,n_2}\right|^2 \approx 0.9\,,$

Comparison with a master equation result





Entropy depending on amplitude for 4 photons



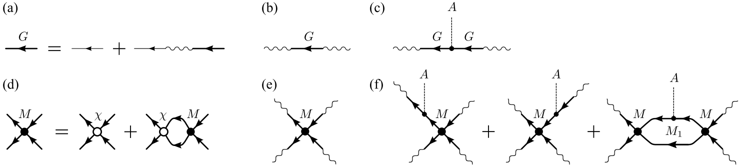
Dependence of e^S, where S is 4-photon the entanglement entropy, for the three photons reflected from N modulated qubits as a function of the modulation amplitude A for different N. The qubits are modulated with the phases $\alpha_i = 2\pi i/N$ and equal amplitudes.

$$\psi(t,t+\tau) = \sum_{n_1,n_2} J_{n_1}(A/\Omega) J_{n_2}(A/\Omega) \times \frac{\mathrm{e}^{\mathrm{i}(n_1+n_2)\Omega t}(\mathrm{e}^{\mathrm{i}n_1\Omega\tau} + \mathrm{e}^{\mathrm{i}n_2\Omega\tau} - 2\mathrm{e}^{-\gamma\tau - \mathrm{i}\Delta_0\tau})}{(\Delta_0 + n_1\Omega - \mathrm{i}\gamma)(\Delta_0 + n_2\Omega - \mathrm{i}\gamma)}$$

G. Nienhuis, "Spectral correlations in resonance fluorescence," PRA **47**, 510 (1993)

$$g_1^{(2)}(\tau) = \frac{1}{2}\Big|\mathrm{e}^{-\mathrm{i}\Delta\tau} + \mathrm{e}^{-\mathrm{i}(\Delta+\Omega)\tau} - \frac{(\Omega+\Delta+\Omega+\mathrm{i}\gamma_{1\mathrm{D}})\mathrm{e}^{-\gamma_{1\mathrm{D}}\tau - \mathrm{i}\Omega\tau} + (\Omega-\Delta-\mathrm{i}\gamma_{1\mathrm{D}})\,\mathrm{e}^{-\gamma_{1\mathrm{D}}\tau}}{\Omega}\Big|^2,$$

where $\Delta = \varepsilon - \omega_0$.



Diagrammatic representation of photon scattering on modulated qubits. (a) Dyson equation for the Green's function of qubit excitation. Thin solid line is the bare qubit excitation, wavy line is the photon in the waveguide, thick solid line is the qubit excitation dressed by interaction with photons. (b) Diagram describing elastic scattering of a single photon. (c) Diagram describing inelastic scattering of a single photon with emission/absorption of a modulation quantum (dashed line). (d) Dyson equation for a pair of qubit excitations. Open dot denotes bare vertex of excitation interaction. Solid dot is the dressed interaction vertex. (b) Diagram describing scattering of a photon pair without change of the total energy. (c) Diagrams describing inelastic scattering of a photon pair with emission/absorption of a modulation quantum.

Elastic and inelastic reflection

Elastic scattering:

$$r(\omega_k) = \langle a_k | S | a_k^\dagger \rangle = -\mathrm{i}\gamma_{1D} \sum_{ij} G_{ij} \mathrm{e}^{iq(z_i+z_j)} \,. \tag{3}$$

Stokes scattering:

$$\langle a_{k'} | S | a_k^\dagger \rangle = r_1(\omega_k) \, \frac{2\pi c}{L} \delta(\omega_{k'} + \Omega - \omega_k) \,, \tag{4}$$

$$r_1(\omega) = \gamma_{1D} \sum_{ijk} G_{ik}(\omega + \Omega) u_k G_{kj}(\omega) \mathrm{e}^{iq(z_i+z_j)} \tag{5}$$

Green function:

$$(\omega - \omega_0) G_{ij}(\omega) + \mathrm{i}\gamma_{1\mathrm{D}} \sum_m \mathrm{e}^{\mathrm{i}(\omega/c)|z_i - z_m|} G_{mj}(\omega) = \delta_{ij} \,,$$

Two-photon inelastic S-matrix

$$\langle a_{k_1'} a_{k_2'} | S | a_{k_1}^\dagger a_{k_2}^\dagger \rangle = 2\pi\delta(\omega_{k_1'} + \omega_{k_2'} - \omega_{k_1} - \omega_{k_2} - \Omega)\, S_1(\omega_{k_1'}, \omega_{k_2'}; \omega_{k_1}, \omega_{k_2})\,.$$

$$\begin{aligned}
S_1(\omega_1', \omega_2'; \omega_1, \omega_2) = {} & 2\pi[\delta(\omega_1' - \omega_1) + \delta(\omega_2' - \omega_1)] r(\omega_1) r_1(\omega_2) \\
& + 2\pi[\delta(\omega_1' - \omega_2) + \delta(\omega_2' - \omega_2)] r(\omega_2) r_1(\omega_1) \\
& + 2\gamma_{1D}^2 \sum_{ijk} u_k \Big\{ M_{ij}(\epsilon) \big[s_k^+(\omega_1') G_{ki}(\omega_1' - \Omega) s_i^+(\omega_2') + s_i^+(\omega_1') G_{ki}(\omega_2' - \Omega) s_k^+(\omega_2') \big] s_j^+(\omega_1) s_j^+(\omega_2) \\
& \quad + M_{ij}(\epsilon + \frac{\Omega}{2}) s_i^+(\omega_1') s_i^+(\omega_2') \big[s_k^+(\omega_1) G_{kj}(\omega_1 + \Omega) s_j^+(\omega_2) + s_j^+(\omega_1) G_{kj}(\omega_2 + \Omega) s_k^+(\omega_2) \big] \Big\} \\
& + 4\gamma_{1D}^2 \sum_{ijkl} M_{ik}(\varepsilon + \tfrac{\Omega}{2}) M_{1,kl}(\epsilon) M_{lj}(\varepsilon) s_i^+(\omega_1') s_i^+(\omega_2') s_j^+(\omega_1) s_j^+(\omega_2)\,,
\end{aligned}$$

where

$$\begin{aligned}
M_{1,ij}(\epsilon) &= \mathrm{i} \int \sum_k u_k G_{ij}(\omega) G_{ik}(2\epsilon - \omega + \Omega) G_{kj}(2\epsilon - \omega) \frac{d\omega}{2\pi} \\
&= \left(\frac{1}{2\epsilon + \Omega - H \otimes I - I \otimes H} \,\mathsf{diag}(u) \otimes I\, \frac{1}{2\epsilon - H \otimes I - I \otimes H} \right)_{ii,jj}\,.
\end{aligned}$$

$$(\omega - \omega_0) G_{ij}(\omega) + \mathrm{i}\gamma_{1\mathrm{D}} \sum_m \mathrm{e}^{\mathrm{i}(\omega/c)|z_i - z_m|} G_{mj}(\omega) = \delta_{ij}\,,$$

S-matrix for homogeneous modulation

$$S(t_1', t_2'; t_1, t_2) = S_0(t_1', t_2'; t_1, t_2)\, \mathrm{e}^{-\mathrm{i}\,\frac{2u}{\Omega}\,[\sin\Omega t_1' + \sin\Omega t_2' - \sin\Omega t_1 - \sin\Omega t_2']}\,.$$

$$S(\omega_1', \omega_2'; \omega_1, \omega_2) = \sum_{k_1', k_2', k_1, k_2}^{\infty} J_{k_1'}\left(\frac{2u}{\Omega}\right) J_{k_2'}\left(\frac{2u}{\Omega}\right) J_{k_1}\left(\frac{2u}{\Omega}\right) J_{k_2}\left(\frac{2u}{\Omega}\right) S_0(\omega_1' - k_1'\Omega, \omega_2' - k_2'\Omega; \omega_1 - k_1\Omega, \omega_2 - k_2\Omega)\,. \tag{6}$$

Considering the limit of small u, we get

$$\begin{aligned} S_1(\omega_1', \omega_2'; \omega_1, \omega_2) &= \frac{u}{\Omega} \\ \times[S(\omega_1' - \Omega, \omega_2'; \omega_1, \omega_2) &+ S(\omega_1', \omega_2' - \Omega; \omega_1, \omega_2) \\ - S(\omega_1', \omega_2'; \omega_1 + \Omega, \omega_2) &- S(\omega_1', \omega_2'; \omega_1, \omega_2 + \Omega)]\,. \end{aligned} \tag{7}$$

Master equation with detectors

$$\omega_{\mathrm{D1}} = \varepsilon + n_1\Omega, \quad \omega_{\mathrm{D2}} = \varepsilon + n_2\Omega, \quad n_{1,2} = 0, \pm 1, \pm 2 \ldots$$

$$\dot{\rho} = -\mathrm{i}[\widetilde{H}_1, \rho] + \sum_{j,k=1}^{N} \gamma_{j,k} \left[2\sigma_j \rho \sigma_k^\dagger - \{\sigma_k^\dagger \sigma_j, \rho\} \right].$$

$$\gamma_{j,k} = \gamma_{\mathrm{1D}} \cos[q(z_j - z_k)] + \gamma_D(\delta_{j,2} + \delta_{j,3})$$

$$\widetilde{H}_1 = \sum_{j=1}^{2} [\omega_0 + A_n(t)] + \omega_{\mathrm{D1}} \sigma_3^\dagger \sigma_3 + \omega_{\mathrm{D2}} \sigma_4^\dagger \sigma_4$$

$$+ \gamma_{\mathrm{1D}} \sum_{j,k=1}^{N} \sigma_j^\dagger \sigma_k \sin(q|z_j - z_k|) - \frac{\mathrm{i}\Omega_R}{2} \sum_{j=1}^{2} (\mathrm{e}^{-\mathrm{i}qz_j - \mathrm{i}\varepsilon t} \sigma_j^\dagger - \mathrm{H.c.})$$

$$g^{(2)}_{n_1,n_2} = \frac{\langle \sigma_3^\dagger \sigma_4^\dagger \sigma_4 \sigma_3 \rangle}{\langle \sigma_3^\dagger \sigma_3 \rangle \langle \sigma_4^\dagger \sigma_4 \rangle}.$$