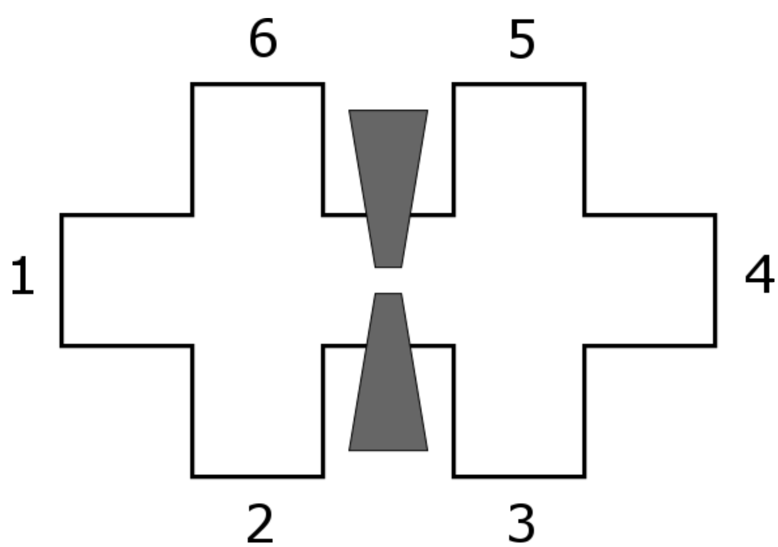
Spectroscopy and Microscopy in Condensed Matter Physics

Assignment #1 Due: 28/05/2019

1. Landauer-Buttiker in QHE

Consider the sketched Hall bar with filling factor ν where current is sourced from contact 1 to contact 4. The gate-defined constriction reflects N individual edge channels and transmits the other M channels (ν=N+M). Assume no spin degeneracy. Calculate the two terminal resistance R_{2T} ($=R_{14,14}$), the longitudinal resistance R_L, and the Hall resistance R_H. Calculate also the diagonal resistances R_{36} and R_{25}, how are they related to R_L and R_H?



2. **Shubnikov–de Haas effect in a topological insulator**

In this exercise we will analyze SdH oscillations measured on a TI, and understand how they can be used to identify and characterize the surface states dispersion.

The attached zip file contains measurement of the sample's resistance as a function of the magnetic field (B = 0 - 30 T), applied at different angles θ ($\theta = 0°$ is perpendicular to the surface).

a. Extract and plot the data, with each angle plotted at a different curve.

In class you learned the Lifshitz-Kosevich formula, which states that the oscillatory part follows $\cos\left(2\pi\left(\frac{F}{B}+\frac{1}{2}+\beta\right)\right)$ behavior, where $2\pi\beta$ is the Berry phase. The SdH oscillations assume their maximum whenever $2\pi\left(\frac{F}{B}+\frac{1}{2}+\beta\right) = 2\pi n$, which corresponds to a Landau-level crossing the Fermi energy. You may find a good reference for that in the first chapter of *Physics of Graphene* (Springer 2013).

b. For the $\theta = 0°$ data, plot n as a function of $1/B_n$. Fit this data and extract the Berry phase. What can you conclude about the topological nature of these states?

Ideal TI is insulating in the bulk and conducting on the surface. In reality, however, it's not always possible to neglect the bulk contribution to the magnetoresistance. In the following, we would like to show that the measured SdH oscillations reflect a 2D Fermi surface, and therefore come from the Dirac-like topological surface states.

According to the Onsager relation, the frequency of the SdH oscillations F $\left(F = \frac{1}{\Delta B}\right)$ is given by:

$$F = \frac{\hbar}{2\pi e} A_{ext}(\epsilon_F)$$

where $A_{ext}(\epsilon_F)$ is the extremal cross-section of the Fermi surface in the plane perpendicular to the applied magnetic field.

c. What is the functional form of $F(\theta)$ at the limit of a 2D Fermi surface? Use FFT to determine F, and plot it is a function of θ. Show that it follows the expected behavior by fitting it to the function you found on b.
   Note: in the experiment $R(B)$ was sampled linearly, but if you consider $R(1/B)$ you will get a non-linear sampling. You will have to fix that before FFT, for example by interpolation (hint: Matlab spline function can be useful).
d. Calculate the Fermi wave vector k_F of the surface Dirac electrons, and estimate how high is the Fermi energy above the Dirac point.
e. Assuming you can fabricate a gate to control the chemical potential of the TI, propose an experiment to measure the Dirac dispersion of the topological surface states. Given a fermi velocity $v_F = 5 \times 10^5$ m/s plot the expected results of your experiment over a ±0.5 eV energy range around the Dirac point.

The Dingle scattering time τ_D is a measure of the dephasing of the Landau states. As you heard in class, when the electron mean-free-path l_e is smaller than the cyclotron radius r_c the oscillations will be damped. This will lead to suppression of the SdH oscillations with decreasing magnetic field, since r_c becomes larger. This suppression is characterized by the Dingle factor:

$$R_D = exp\left(-\frac{\pi}{\omega_c \tau_D}\right)$$

f. Extract τ_D from $R(1/B)$ at $\theta = 0°$. Note the oscillations ride on top a smooth background that you have to account for.