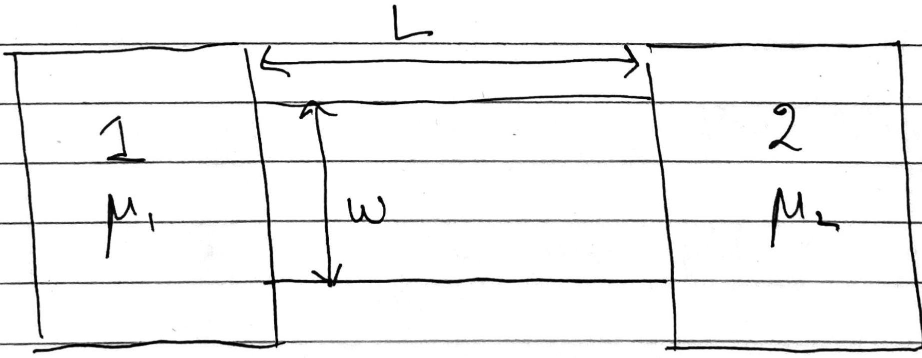
2. Landauer - Büttiker formalism

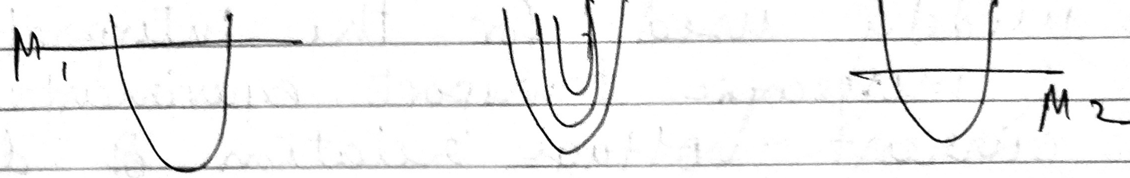
Ref: Datta, Electronic transport in mesoscopic systems.

- Widely used for the interpretation of mesoscopic transport experiments.
- current - voltage relation of devices
- Landauer: related the linear response of conductance to the transmission probability.
- Buttiker extended this approach to multi-terminal measurements (and magnetic field)



- Most of our discussion will be limited to ballistic coherent transport a 0 temperature.
- From ohm's law $G = \frac{\sigma W}{L}$, but it was experimentally seen that as L is decreased, conductance approaches a limiting value G_c.

→ Q how does a ballistic conductor have finite resistance?



→ Turns out this resistance is due to contacts (pioneered by Joe Imry)

→ Current is carried by many modes in the contact while a few in the conductor.

Current distribution at the interface gives rise to the contact resistance.

Landaur formula

→ What is the current carried by the transverse mode of the conductor?

current carried by the +k states

$$I^+ = nev = \frac{e}{L}\sum_k v f^+(E)$$

$$= \frac{e}{L}\frac{1}{\hbar}\sum_k \frac{\partial E}{\partial k} f^+(E)$$

Using $\sum_k \rightarrow \underset{(\text{for spin})}{2} \times \frac{L}{2\pi}\int dk$

We get $I^+ = \frac{2e}{h}\int_{\epsilon}^{\infty} f^+(E)\,dE$

ϵ = bottom of the band.

Extending it to multiple modes

$$\Rightarrow I^+ = \frac{2e}{h}\int_{-\infty}^{\infty} f^+(E)\,M(E)\,dE \qquad f = \frac{1}{e^{\frac{E-\mu}{K_BT}}+1}$$

$$M(E) = \sum_N \theta(E-E_n)$$

N = # of modes

Assume $M(E)$ = constant in the energy window $\mu_1 < E < \mu_2$

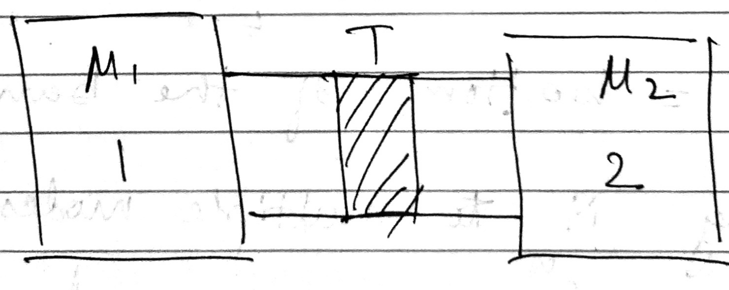
→ Note: The above result is independent of the actual energy dispersion $E(k)$.

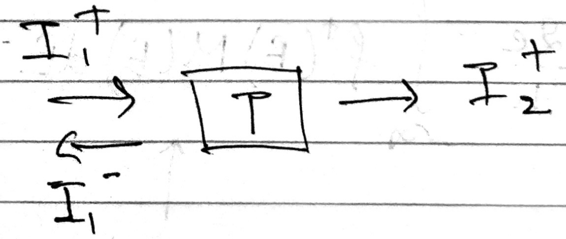
$$I = \frac{2e^2}{h} M \left(\frac{\mu_1 - \mu_2}{e}\right)$$

$$\boxed{G_c = \frac{2e^2}{h} M}$$ contact resistance

for $M = 1$, $G_c^{-1} = R_c = 12.9\ k\Omega$

→ What happens when there is a ~~for~~ Transmission probability?





$$I_1^+ = \frac{2e^2}{h} M \left(\frac{\mu_1 - \mu_2}{e}\right)$$

$$I_2^+ = \frac{2e^2}{h} M T \left(\frac{\mu_1 - \mu_2}{e}\right)$$ = net current in the system

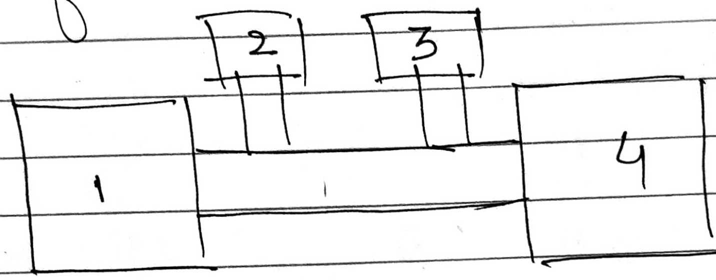
hence, $\boxed{G = \frac{2e^2}{h} M T}$ Landauer formula

Where M = # of modes
T = Transmission probability

Buttiker formula

→ current voltage for multi-terminal measurement.

→ Schematic of a simple four terminal ~~defice~~ device



→ Voltage probes sense the local electro chemical potential.

→ They are floating i.e no external current.

→ Buttiker noted: no qualitative difference between the current and voltage probes.

Extension of the 2 terminal linear response

$$I = \frac{2e^2}{h} TM \left(\frac{\mu_1 - \mu_2}{e}\right)$$

generalize to multi-terminal

$$I_p = \frac{2e^2}{h} \sum_q \left[\frac{\bar{T}_{q \leftarrow p} \mu_p - \bar{T}_{p \leftarrow q} \mu_q}{e}\right]$$

$$\bar{T}_{q\leftarrow p} = T_{q\leftarrow p} M_{q\leftarrow p}$$

~~we~~ define $V = \mu/e$

$$I_p = \sum_q \left[\bar{G}_{qp} V_p - G_{pq} V_q \right]$$

where $G_{pq} = \frac{2e^2}{h} \bar{T}_{pq}$

→ sum rule:

if all potentials are equal then current should be zero.

$$\sum_q G_{qp} = \sum_q G_{pq}$$

thus $\boxed{I_p = \sum_q G_{pq} [V_p - V_q]}$ Buttiker formula.

→ $[G_{qp}]_{+B} = [G_{pq}]_{-B}$

TR symmetry $\Rightarrow$ $G_{qp} = G_{pq}$.

This can be shown using S matrix approach.