2. Landauer-Büttiker Formalism

Ref: Datta, Electronic Transport in Microscopic Systems

- widely used for the interpretation of mesoscopic transport experiments.
- Current-voltage relation of devices

- Landauer: related the linear response of conductance to the transmission probability.

- Büttiker extended this approach to multi-terminal measurements (and magnetic field)

Most of our discussion will be limited to ballistic coherent transport at 0 temperature.

- From Ohm's law $G = \frac{eW}{L}$, but it was experimentally seen that as $L$ is decreased, conductance approaches a limiting value $G_c$. 
Q how does a ballistic conductor have finite resistance?

\[ M_1 \quad M_2 \quad M_3 \]

Turns out this resistance is due to contacts (pioneered by Joe Imry)

Current is carried by many modes in the contact while a few in the conductor.

Current distribution at the interface gives rise to the contact resistance.
Landau's formula

What is the current carried by the transverse mode of the conductor?

Current carried by the +k state:

\[ I^+ = nev = \frac{e}{c} \sum \nu f^+(E) \]

\[ = \frac{e}{c} \sum \frac{2E}{h} \frac{\partial f^+(E)}{\partial k} \]

Using \( \frac{2L}{\hbar} \rightarrow 2L \int \frac{dk}{\hbar} \) (for spin)

we get

\[ I^+ = \frac{2e}{\hbar} \int f^+(E) dE \]

\( \epsilon = \text{bottom of the band} \).

Extending it to multiple modes

\( \Rightarrow I^+ = \frac{2e}{\hbar} \int f^+(E) M(E) dE \]

\[ M(E) = \sum_{\text{# of modes}} \theta (E - E_0) \]

Assume \( M(E) = \text{constant in the energy window } \mu_1 < E < \mu_2 \).
Note: The above result is independent of the actual energy dispersion \( E(k) \).

\[
I = \frac{2e^2}{h} \frac{M}{\hbar} \left( \frac{M_1 - M_2}{e} \right)
\]

\[
G_C = \frac{2e^2}{h} M
\]

contact resistance

for \( M = 1 \), \( g_C = R_C = 12.9 \ \text{K}\Omega \)

What happens when there is a transmission probability?

\[
I_1^+ = \frac{2e^2}{h} M \left( \frac{M_1 - M_2}{e} \right)
\]

\[
I_2^- = \frac{2e^2}{h} M T \left( \frac{M_1 - M_2}{e} \right) = \text{net current in the system}
\]
hence, \[ G = \frac{2e^2}{\hbar} MT \] Landauer formula

where \( M = \# \) of modes
\( T = \) Transmission probability
Buttiker formula

- Current voltage for multi-terminal measurement.

- Schematic of a simple four-terminal device device

- Voltage probes sense the local electrochemical potential.

- They are floating i.e. no external current.

- Buttiker noted: no qualitative difference between the current and voltage probes.

- Extension of the 2-terminal linear response

\[ I = \frac{2e^2}{h} T M \left( \frac{M_1 - M_2}{e} \right) \]

generalize to multi-terminal

\[ I_p = \frac{2e^2}{h} \sum_{q} \left[ T_{q+p} M_p - T_{p+q} M_q \right] \]
\[ T_{q-p} = T_{q-p} M_{q-p} \]

Define \( V = \frac{N}{e} \)

\[ J_p = \frac{3}{q} \left[ G_{qp} V_p - G_{pq} V_q \right] \]

Where \( G_{pq} = \frac{2e^2}{\hbar} T_{pq} \)

\[ \sum \text{ rule:} \]

If all potentials are equal then current should be zero.

\[ \sum \frac{3}{q} G_{qp} = \sum \frac{3}{q} G_{pq} \]

Thus \[ J_p = \frac{3}{q} G_{pq} [V_p - V_q] \] Butiker formula

\[ \sum (G_{qp})_{+b} = (G_{pq})_{-b} \]

TR symmetry \( \Rightarrow G_{qp} = G_{pq} \)

This can be shown using S matrix approach,