14/05/19

4. de Haas – van Alphen effect.

→ allows us to measure fermi surface which is crucial for various transport and electronic properties of the material.

→ In 1930 de Haas & van Alphen measured the magnetization of bismuth as a function of magnetic field at 14.2K and found oscillatory behaviour.

Refs : Ashcroft and Mermin.

Bloch electrons in uniform magnetic field

semiclassical equations of motion (EOM) are given as

$$\dot{r} = v(k) = \frac{1}{\hbar}\frac{\partial \epsilon(k)}{\partial k}$$

$$\hbar\dot{k} = \frac{-e}{c} v(k) \times H$$

→ $(\dot{k})_{\parallel} = 0$ ⇒ $(\bar{k})_{\parallel} = $ const.

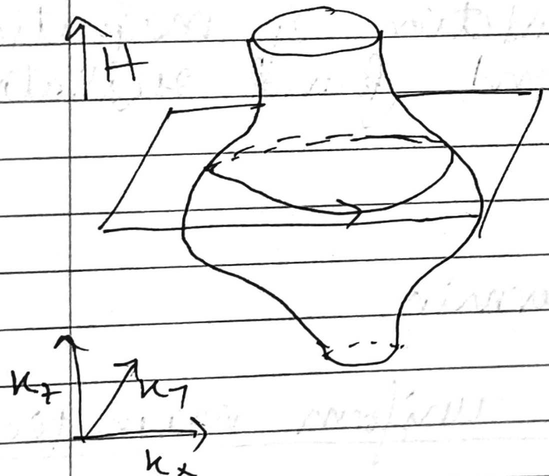
component of k in the direction of the magnetic field.

and

$$\frac{d\,\epsilon(k)}{dt} = \frac{\partial \epsilon(k)}{\partial k} \cdot \dot{k} = \frac{-e}{c} v(k) \cdot [v(k) \times H]$$

$$= 0$$

This shows both $(\bar{k})_{\parallel}$ and $\epsilon(k)$ are constants of motion.



Electrons will move along curves in the plane perpendicular to the magnetic field at constant energy.

→ The orbital period (T) of the electrons can be calculated as

$$t_2 - t_1 = \int_{t_1}^{t_2} dt = \int_{k_1}^{k_2} \frac{dk}{|\dot{k}|}$$

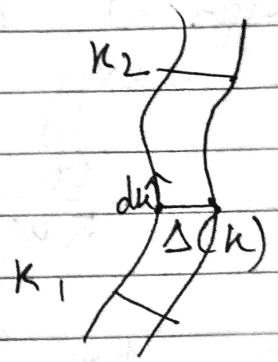
(plug in EOM)

$$= \frac{\hbar^2 c}{eH} \int_{k_1}^{k_2} \frac{dk}{|(\partial\epsilon/\partial\bar{k})_{\perp}|}$$

↑ component ⊥ to the magnetic field.

→ geometrical interpretation of $\left|(\partial \epsilon/\partial k)_\perp\right|$

$$\Delta \epsilon = \left(\frac{\partial \epsilon}{\partial k}\right)_\perp \cdot \Delta(k)$$



Since $\left(\frac{\partial \epsilon}{\partial k}\right)_\perp$ is $\perp$ to SCE

and $\perp$ to the orbit, it is $\parallel$ to $\Delta(k)$

so,

$$\Delta \epsilon = \left|\left(\frac{\partial \epsilon}{\partial k}\right)_\perp\right| \Delta(k)$$

$$t_2 - t_1 = \frac{\hbar^2 c}{eH} \frac{1}{\Delta \epsilon} \int_{k_1}^{k_2} \Delta(k)\, dk$$

in the limit $\Delta \epsilon \to 0$ $\quad = \dfrac{\hbar^2 c}{eH} \dfrac{\partial A_{1,2}}{\partial \epsilon}$

for a closed orbit we get

$$T(\epsilon, k_z) = \frac{\hbar^2 c}{eH} \frac{\partial}{\partial \epsilon} A(\epsilon, k_z)$$

Onsager's formula

→ typical quantum number in metals is very high

$$\nu = \frac{\epsilon_F}{\hbar\omega_c} = \frac{\epsilon_F}{\frac{e\hbar}{mc}\cdot H}, \quad \frac{e\hbar}{mc} \sim 10^{-8}\ \text{eV/G}$$

so for typical metals with $\epsilon_F \sim 1\,\text{eV}$, even for 1 T or 10^4 G

$$\nu \sim 10^4.$$

Therefore reasonable to use Bohr's correspondence principle

$$\Delta\epsilon_n = \frac{h}{T}, \quad T \text{ is the period of semi-classical motion.}$$

$$\rightarrow \epsilon_{\nu+1}(k_z) - \epsilon_\nu(k_z) = \frac{h}{T(\epsilon_\nu, k_z)}$$

$$\Rightarrow (\epsilon_{\nu+1} - \epsilon_\nu)\frac{\partial A(\epsilon_\nu)}{\partial\epsilon} = \frac{2\pi e H}{\hbar c}$$

$$A(\epsilon_{\nu+1}) - A(\epsilon_\nu) = \boxed{\frac{2\pi e H}{\hbar c} = \Delta A}$$

using Bohr-Sommerfeld quantization condition

$$A(\epsilon_\nu, k_z) = (\nu + \gamma)\Delta A$$

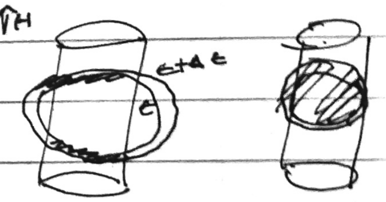
Origin of Oscillation

$$A = (\nu + \gamma)\,\frac{2\pi e H}{\hbar c}$$

→ oscillatory structure in the electronic density of levels imposed by quantization condition.

→ For an orbit with extremal area A_e, a small variation of H will have large change in area.

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c}\,\frac{1}{A_e(\epsilon_F)}$$



Conditions to observe dHvA

1. low T. $k_B T < \hbar \omega_c = \frac{\hbar e H}{m^* c}$

large magnetic field.

2. Finite scattering time mixes different LLs, hence

$$\omega_c \tau > 1.$$