4. de Haas - van Alphen effect.

This allows us to measure Fermi surface which is crucial for various transport and electronic properties of the material.

In 1930 de Haas and van Alphen measured the magnetization of bismuth as a function of magnetic field at 14.2K and found oscillatory behaviour.

Refs: Ashcroft and Mermin.

Bloch electrons in uniform magnetic field

Semi-classical equations of motion (EOM) are given as

\[ \dot{v} = \nabla E(k) = \frac{1}{m} \frac{\partial E(k)}{\partial k} \]

\[ \dot{k} \times H = -\frac{e}{c} v(k) \times H \]

\[ \dot{k}_x = 0 \Rightarrow (k)_x = \text{const.} \]

Component of \( k \) in the direction of the magnetic field.
and

\[ \frac{dE(k)}{dt} = \Theta E(k) \cdot \frac{i}{\theta k} = -\frac{e}{c} v(k) \cdot [v(k) \times H] \]

This shows both \( i(k) \) and \( v(k) \) are constants of motion.

Electrons will move along curves in the plane perpendicular to the magnetic field at constant energy.

The orbiting period \( T \) of the electron can be calculated as

\[ t_2 - t_1 = \int_{t_1}^{t_2} dt = \int \frac{dp}{dk} \left| \frac{k}{\theta k} \right| \]

plugging in

\[ \left( \Theta v / \theta k \right) \]

component \( \pm \) to the magnetic field.
The geometrical interpretation of \( \frac{\partial t}{\partial k} \)

\[ \Delta t = \frac{\Delta E}{(\Delta k)_1}, \Delta(c(k)) \]

Since \( \frac{\partial E}{\partial k} \) is \( \perp \) to \( k \), it is \( \perp \) to \( \Delta(k) \)

So,

\[ \Delta t = \left| \frac{\partial E}{\partial k} \right| \Delta c(k) \]

\[ t_2 - t_1 = \frac{\hbar^2}{eH} \frac{1}{\Delta t} \int_{k_1}^{k_2} \Delta(c(k)) \, dk \]

In the limit \( \Delta t \to 0 \),

\[ T = \frac{\hbar^2}{eH} \frac{\partial A_{1,2}}{\partial E} \]

for a closed orbit we get

\[ T(E, k_2) = \frac{\hbar^2}{eH} \frac{\partial}{\partial E} A(E, k_2) \]
Onsager's formula.

Typical quantum number in metals is very high

\[ n = \frac{E_F}{\hbar \omega_c} = \frac{E_F}{\frac{e^2}{mc} \cdot H}, \quad \frac{e^2}{mc} \sim 10^{-8} \text{ eV/g} \]

So for typical metal with \( E_F \sim 1 \text{ eV} \), even for \( H \sim 10^4 \text{ eV} \)

\[ n \sim 10^4. \]

Therefore reasonable to use

Bohr's correspondence principle

\[ \Delta E_n = \frac{\hbar}{T}, \quad T \text{ is the period of semi-classical motion.} \]

\[ E_{n+1}(k_F) = E_n(k_F) = \frac{\hbar}{T(E_F, k_F)} \]

\[ \Rightarrow \left( E_{n+1} - E_n \right) \frac{\partial A(E)}{\partial t} = \frac{8 \pi e \hbar}{\hbar c} \]

\[ A(E_{n+1}) - A(E_n) = \left| \frac{8 \pi e \hbar}{\hbar c} \right| = \Delta A \]

Using Bohr–Sommerfeld quantization condition

\[ A(E_n, k_F) = (n + \gamma) \Delta A \]
Origin of Oscillation

\[ A = \left( 2 + \gamma \right) \frac{2\pi e H}{\hbar c} \]

- Oscillatory structure in the electronic density of levels imposed by quantization condition.

- For an orbit with extremal area \( A_e \), a small variation of \( H \) will have large change in area.

\[ \Delta A(H) = \frac{2\pi e l}{\hbar c} \frac{1}{A_e(t_0)} \]

Conditions to observe dHvA

1. Low T. \( k_B T < \hbar \omega_c = \frac{\hbar e H}{m^* c} \)
   
   Large magnetic field.

2. Finite scattering time mixes different l.h.s. Hence

\[ \omega_c t > 1 \]