Exotic superconductivity in graphene multilayers

Erez Berg

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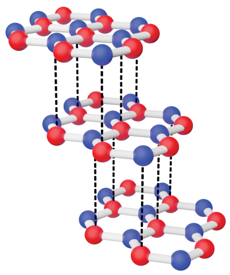
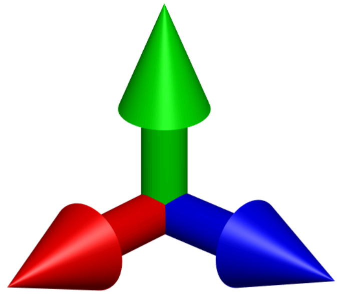
Outline

- Rhombohedral (ABC stacked) trilayer graphene

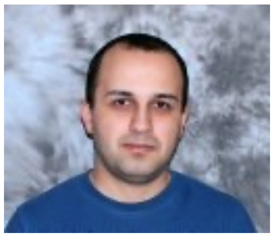


- Puzzles
- Electronic mechanism for SC in 2D system with annular Fermi surfaces

- Spin-polarized triplet superconductors: Order parameter topology and current dissipation



Superconductivity in Rhombohedral Trilayer Graphene



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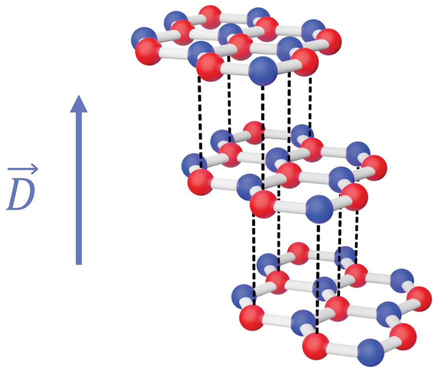
A. Ghazaryan, T. Holder, M. Serbyn, EB, arXiv:2109.00011

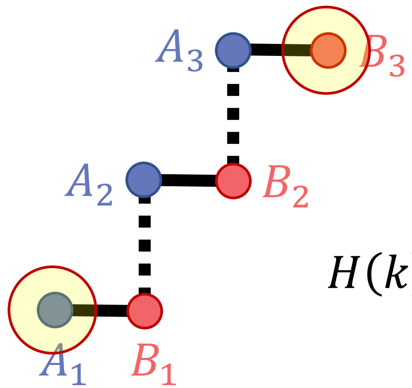
S. Chatterjee, T. Wang, EB, M. Zaletel, arXiv: 2109.00002

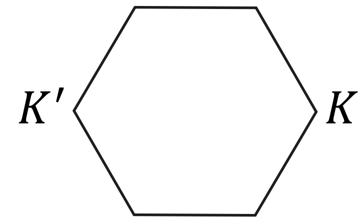
Thanks to: Andrea Young, Haoxin Zhou

See also: Dong, Levitov; Cea, Pantaleon, Phong, Guinea; Szabo, Roy; You, Vishwanath (2021)

Rhombohedral (ABC) trilayer graphene

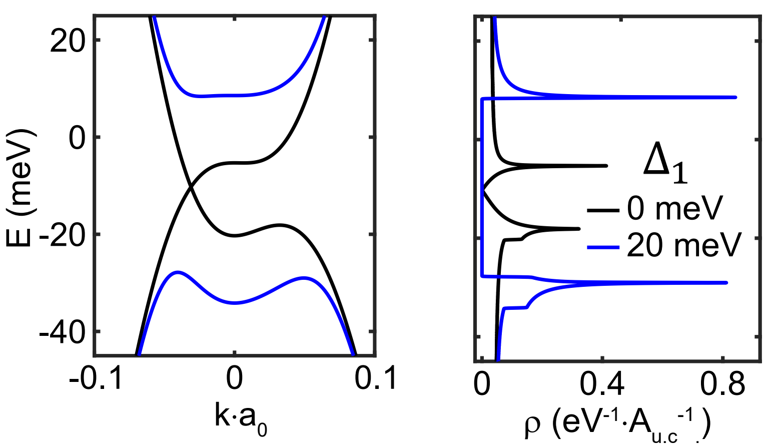


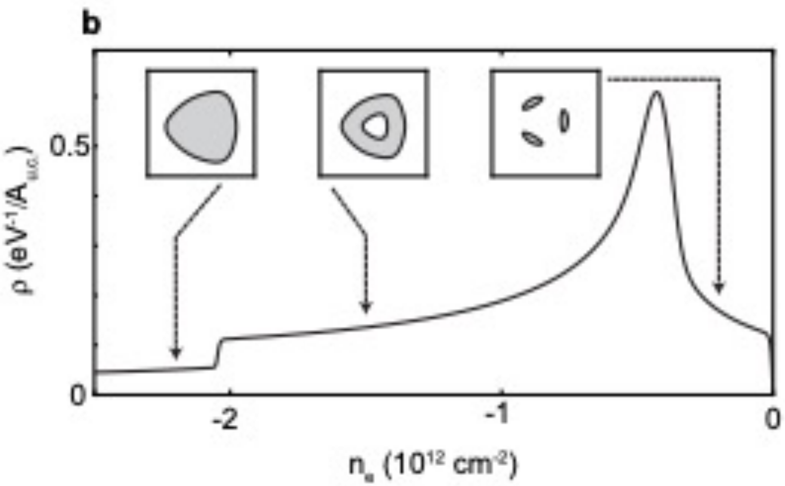




$$H(k) = \begin{pmatrix} \Delta_1 & \beta\left(k_x + \tau i k_y\right)^3 + \gamma \\ \beta\left(k_x - \tau i k_y\right)^3 + \gamma & -\Delta_1 \end{pmatrix}$$

$\Delta_1 = eDd_0$, $\tau = \pm 1$: valley index

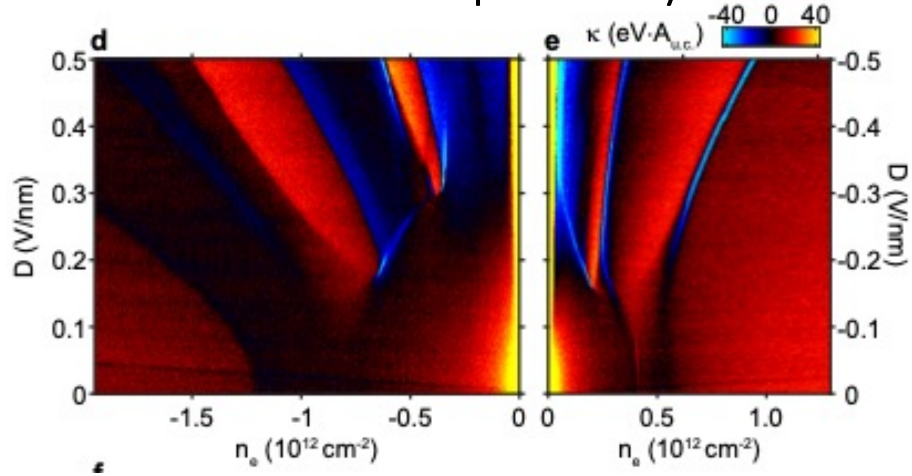




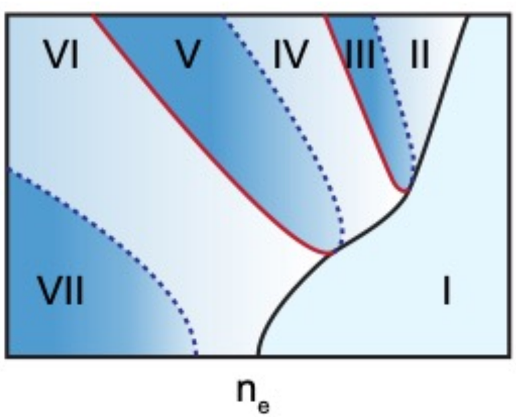
Phase diagram

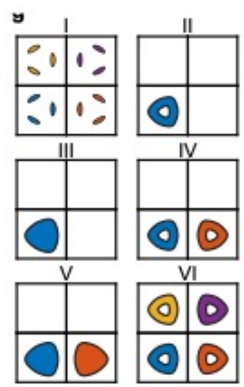
H. Zhou, …,A. Ghazaryan T. Holder, EB, M. Serbyn, A. Young (2021)

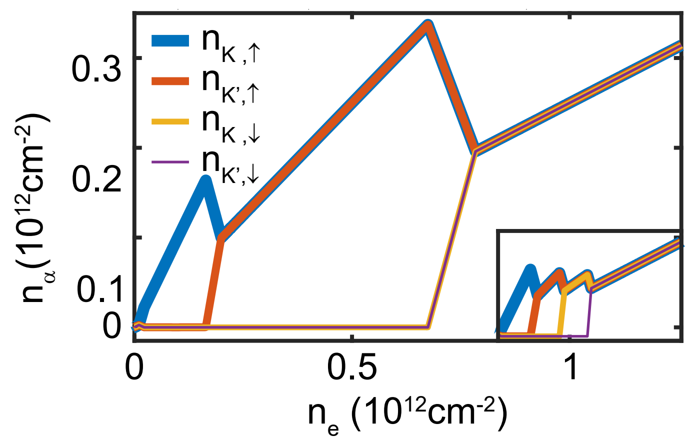
Inverse compressibility



$$E = \sum_{\alpha=1}^{4} \int_{-\infty}^{\mu_\alpha} d\varepsilon\, \varepsilon\, \rho(\varepsilon) + \frac{1}{2} \sum_{\alpha \neq \beta} n_\alpha n_\beta - J \vec{S}_K \cdot \vec{S}_{K'}$$



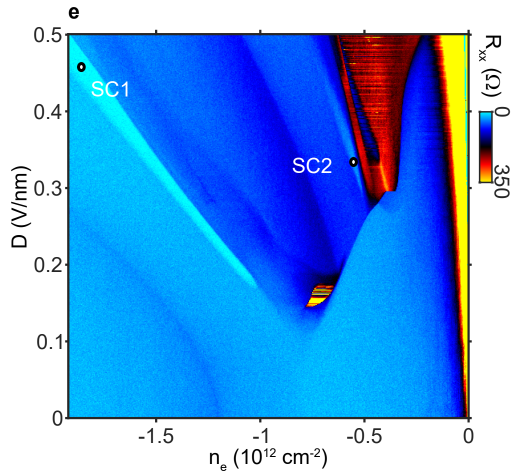


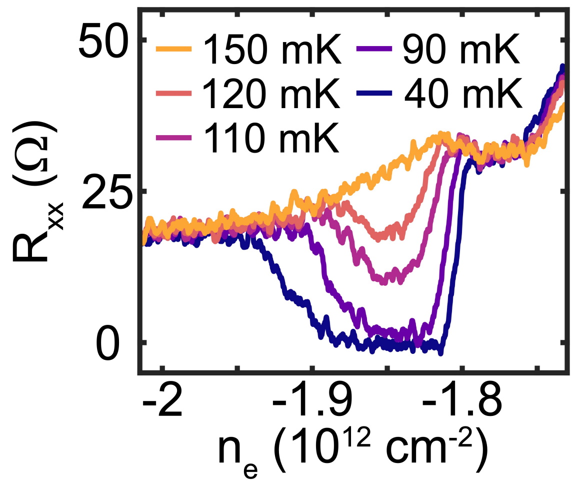


Similar phenomena in MATBG: Zondiner et al., Wong et al. (2020)

Superconductivity!

Zhou, Xie, Taniguchi, Watanabe, Young (2021)





	SC1	SC2
T_c	$\sim 100\text{mK}$	$\lesssim 50\text{mK}$
$\frac{\xi}{\ell}$ (from $B_{c,\perp}$)	~ 0.05	~ 0.1
Singlet/triplet? **(from $B_{c,\parallel}$)**	**Singlet (probably)** (Pauli limited)	**Triplet!** $B_{c,\parallel} > 10 B_{\text{Pauli}}$ Normal state: spin polarized FM

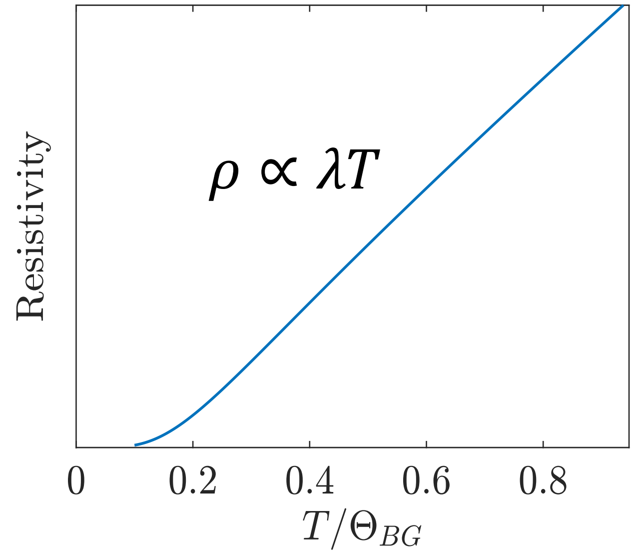
Puzzles

Conventional (acoustic phonon-mediated) s-wave?

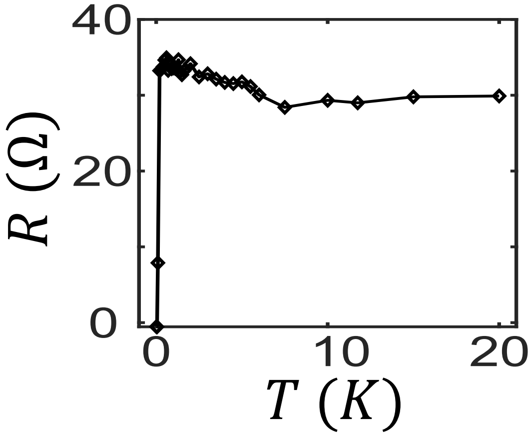
Chou, Wu, Sau, Das Sarma (2021)

$\Theta_{BG} = 2v_s k_F \approx 40\text{K}$

Resistivity should be linear in T for $T \gtrsim \Theta_{BG}/4$



H. Zhou et al. (2021)

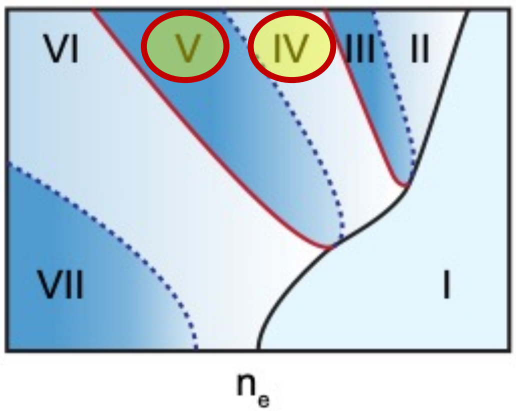


Puzzles (2)

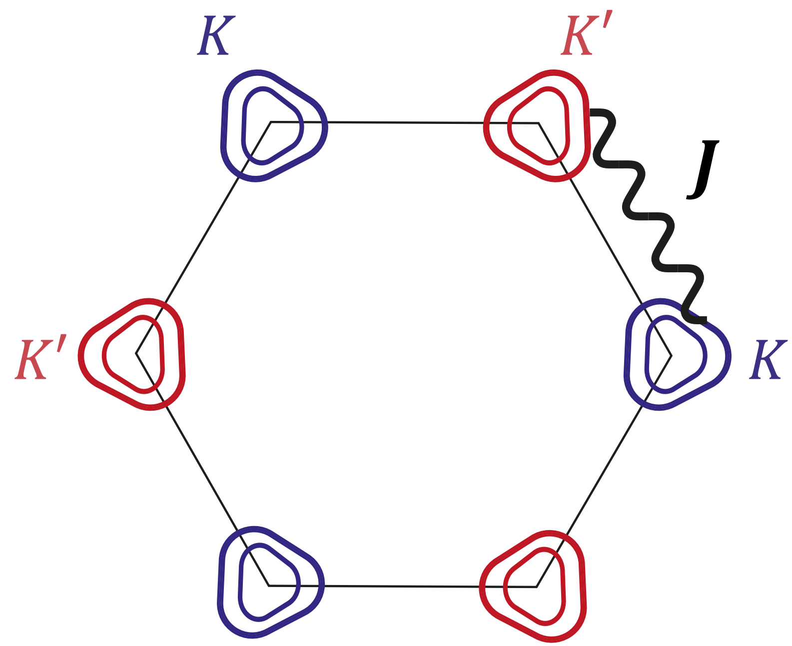
SC1: singlet or triplet?

$$H_J = -J \int d^2r\, \vec{S}_K \cdot \vec{S}_{K'}$$

Spin-polarized, valley-unpolarized phases: $J > 0$



SC1 is spin singlet: $J < 0$??



Electronic mechanism

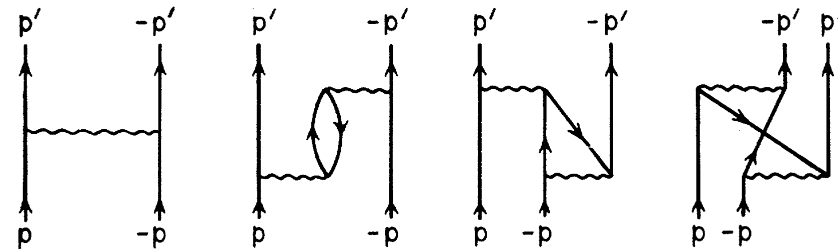
NEW MECHANISM FOR SUPERCONDUCTIVITY*

W. Kohn

University of California, San Diego, La Jolla, California

and

J. M. Luttinger (1965)

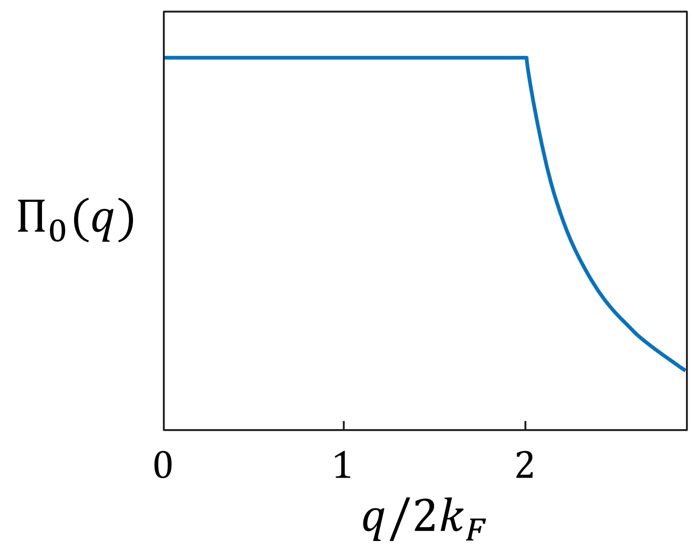


2D, parabolic dispersion:

$$\Pi_0(q < 2k_F) = const$$

No superconductivity to second order

A. Chubukov (1992)



Non-parabolic dispersion/multiple sub-bands:

Unconventional superconductivity!

E.g.: Raghu, Kivelson, Scalapino (2011);

Raghu, Kivelson (2015); Chubukov, Kivelson (2017)

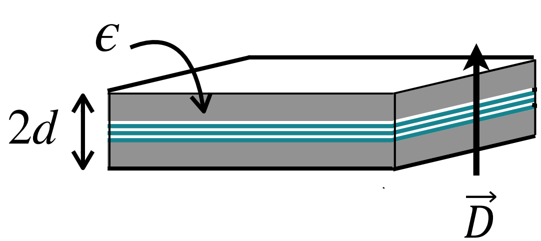
Electronic mechanism

In both SC1,2: annular FS

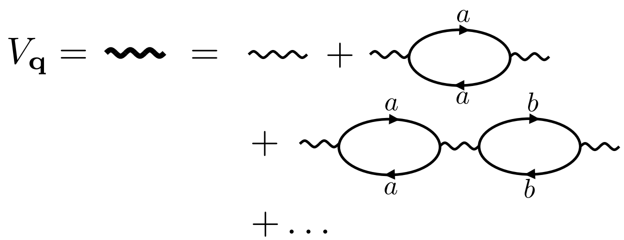
$$H = H_0 + H_C$$

$$H_0 = \sum_{\boldsymbol{k},\alpha=1,\dots,4} \varepsilon_{\boldsymbol{k}}\, \psi^{\dagger}_{\alpha\boldsymbol{k}} \psi_{\alpha\boldsymbol{k}} \qquad \varepsilon_{\boldsymbol{k}} = -\varepsilon_0 \left(\frac{k^2}{k_0^2} - 1 \right)^2 - \mu$$

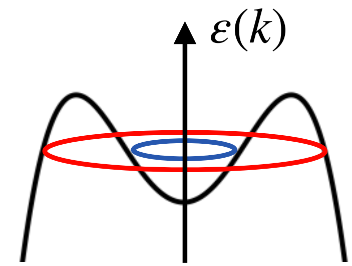
$$H_C = \frac{1}{2L^2} \sum_{\boldsymbol{q}} V_{0,\boldsymbol{q}}\, \rho_{\boldsymbol{q}} \rho_{-\boldsymbol{q}} \qquad V_{0,\boldsymbol{q}} = \frac{2\pi e^2}{\epsilon q} \tanh(qd)$$



Circularly symmetric model

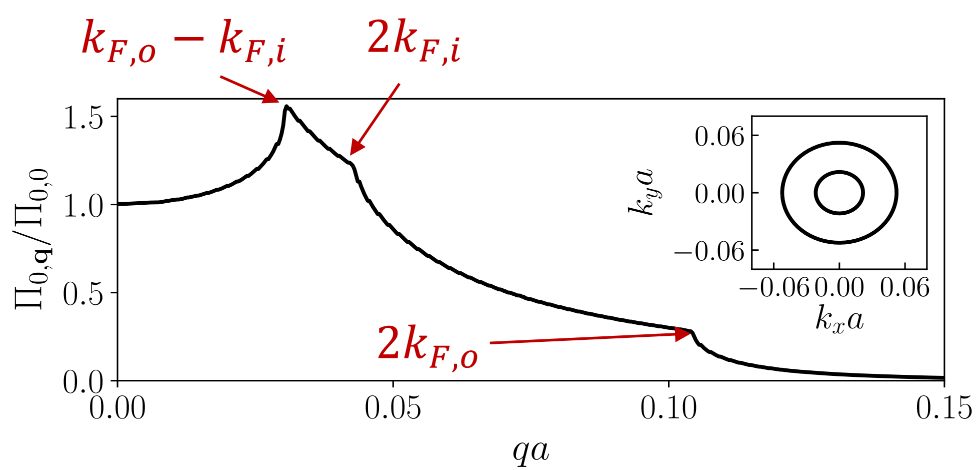


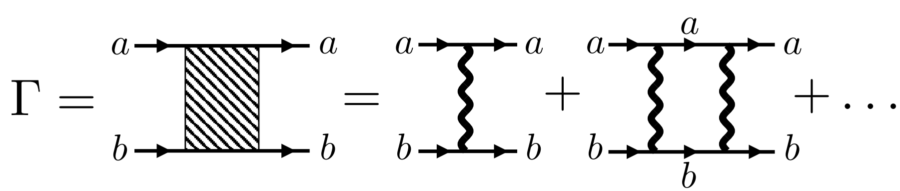
$$= \frac{V_{0,\boldsymbol{q}}}{1 + N\,\Pi_{0,\boldsymbol{q}}\,V_{0,\boldsymbol{q}}}$$



SC1: $N = 4$

SC2: $N = 2$ (spin polarized)

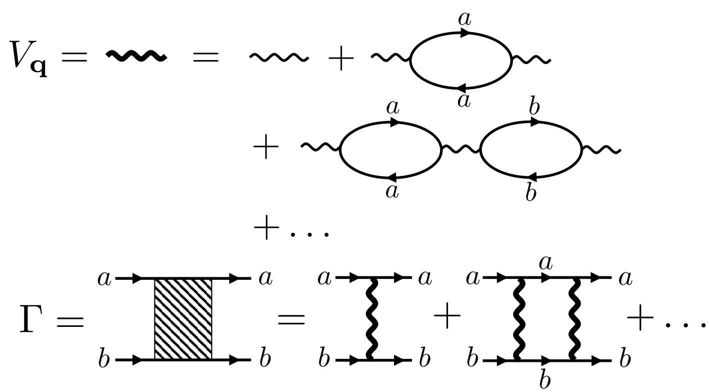


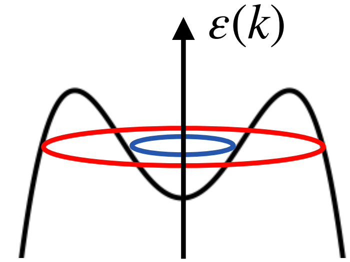


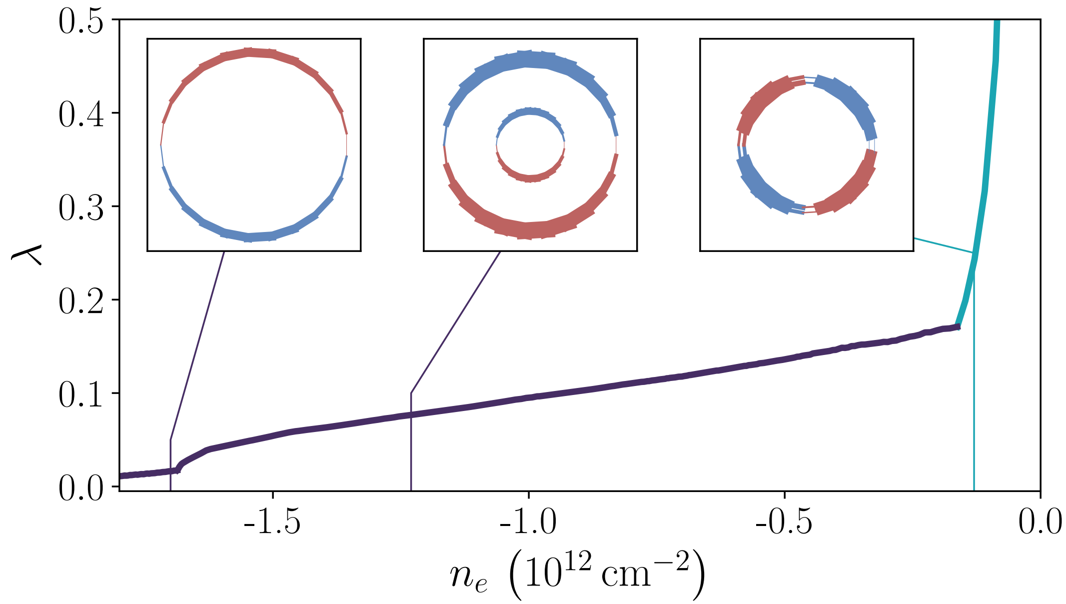
A. Ghazaryan, T. Holder, M. Serbyn, EB, arXiv:2109.00011

SC from isospin fluctuations: S. Chatterjee, T. Wang, EB, M. Zaletel, arXiv: 2109.00002

Circularly symmetric model





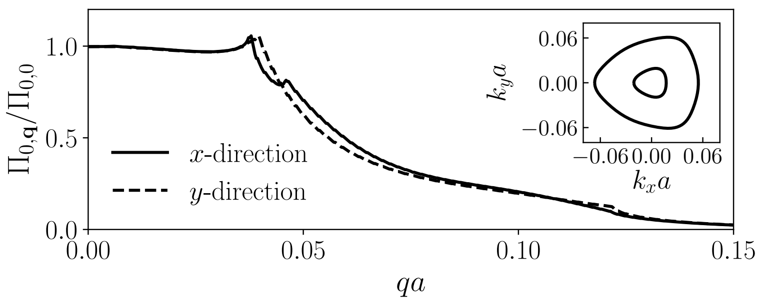


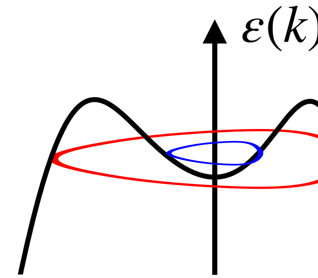
$$T_c = We^{-1/\lambda}$$

$$W \sim E_F$$

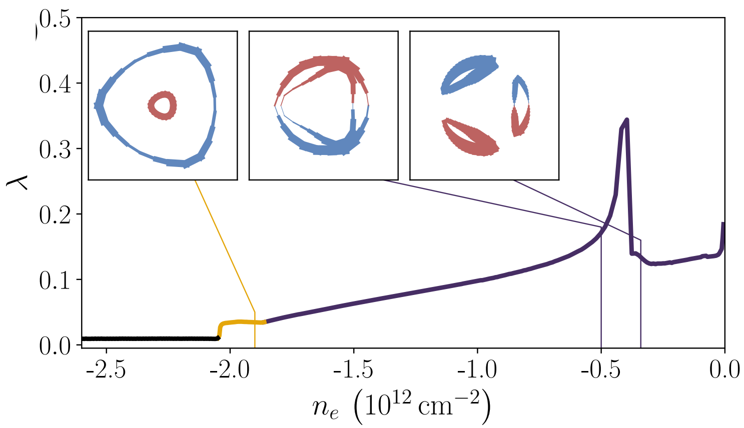
A. Ghazaryan, T. Holder, M. Serbyn, EB, arXiv:2109.00011

Realistic dispersion





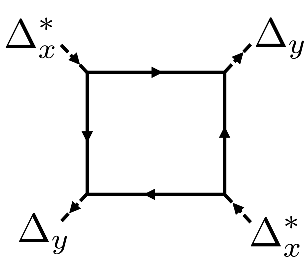
$s_\pm$ wave p-wave



$$H_C = \frac{1}{2L^2}\sum_{\mathbf{q}} V_{0,\mathbf{q}}\,\rho_{\mathbf{q}}\rho_{-\mathbf{q}}$$

$$\rho_{\mathbf{q}} = \sum_{\mathbf{q},a} \Lambda_{a,\mathbf{k},\mathbf{q}}\,\psi^\dagger_{a,\mathbf{k}}\psi_{a,\mathbf{k}+\mathbf{q}}$$

$$\Lambda_{a,\mathbf{k},\mathbf{q}} = \langle u_{n,a,\mathbf{k}} | u_{n,a,\mathbf{k}+\mathbf{q}}\rangle$$



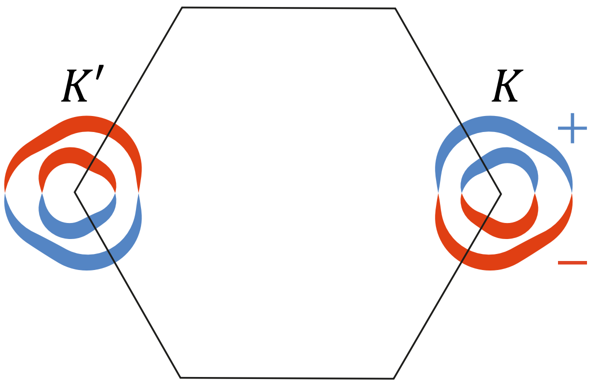
Beyond the linearized BCS equation:
Chiral $\Delta_x + i\Delta_y$

A. Ghazaryan, T. Holder, M. Serbyn, EB, arXiv:2109.00011

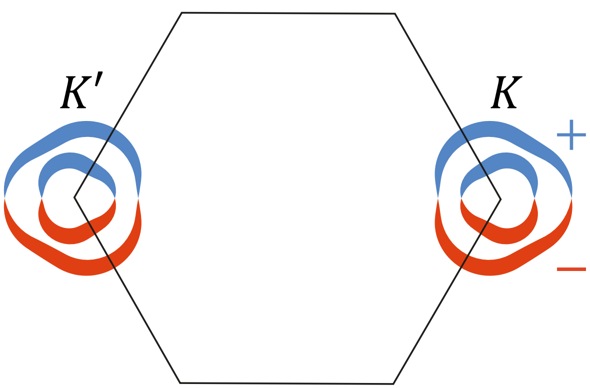
Singlet or Triplet: Hund's Term

p-wave can be either singlet or triplet

$$\left\langle \psi^{\dagger}_{K,\boldsymbol{k},\uparrow}\psi^{\dagger}_{K',-\boldsymbol{k},\downarrow}\right\rangle = \left\langle \psi^{\dagger}_{K',-\boldsymbol{k},\uparrow}\psi^{\dagger}_{K,\boldsymbol{k},\downarrow}\right\rangle = \phi_{\boldsymbol{k}} \neq 0$$



$$\left\langle \psi^{\dagger}_{K,\boldsymbol{k},s}(i\sigma_2\vec{\sigma})_{s,s'}\psi^{\dagger}_{K',-\boldsymbol{k},s'}\right\rangle = \vec{d}_{\boldsymbol{k}} \neq 0$$



Long-range Coulomb interactions: $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry

Singlet and triplet are degenerate!

Singlet or Triplet: Hund's Term

$$H_J = -J\int d^2r\, \vec{S}_K \cdot \vec{S}_{K'} \qquad J \sim 10^{-2} \cdot \frac{e^2}{\epsilon k_F}$$

Chiral $\Delta_x + i\Delta_y$: $\langle \psi_s^\dagger(\boldsymbol{r})\psi_{s'}^\dagger(\boldsymbol{r})\rangle = 0$

H_J drops out of gap equation!

$$\widetilde{H_J} = -\int_{\boldsymbol{r},\boldsymbol{r}'} J(\boldsymbol{r}-\boldsymbol{r}')\vec{S}_K(\boldsymbol{r}) \cdot \vec{S}_{K'}(\boldsymbol{r}')$$

$$\tilde{J}(q) = J_0 + J_2(a_0 q)^2 + \cdots$$

E.g.:

$J_0 > 0, J_2 > 0$ favors: spin polarized, valley unpolarized state ✓

spin singlet $\Delta_x + i\Delta_y$ SC ✓

Order Parameter Topology and Current Dissipation and in Spin-Polarized Triplet Superconductors

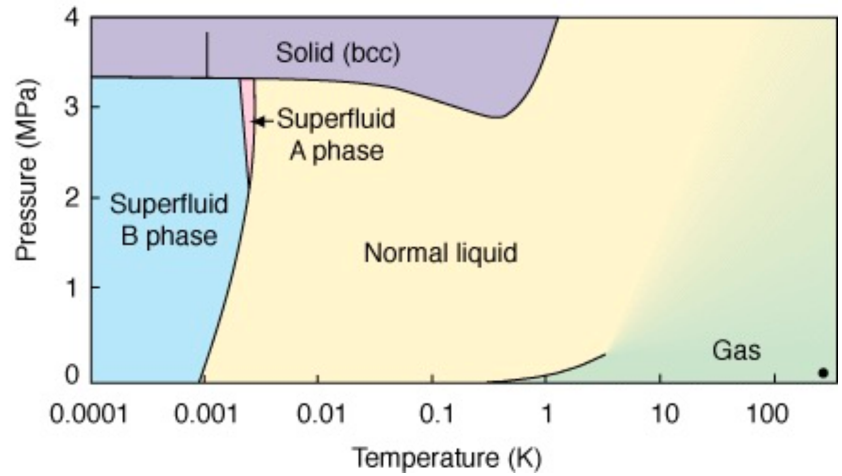
Eyal Cornfeld
(WIS→Classiq Technologies)

Mark Rudner
(Copenhagen→U. Washington)

E. Cornfeld, M. Rudner, EB, Phys. Rev. Research 3, 013051 (2021)

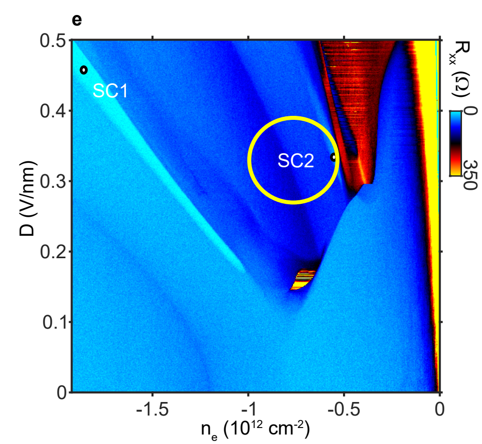
Triplet superconductivity in RTG

Solid state analogue of superfluid ^3He?



Triplet superconductivity:

- Strong electronic correlations ✅
- Nearby/coexisting ferromagnetism ✅
- Extremely clean ✅



Very small spin-orbit: SC and magnetism intertwined in interesting way?

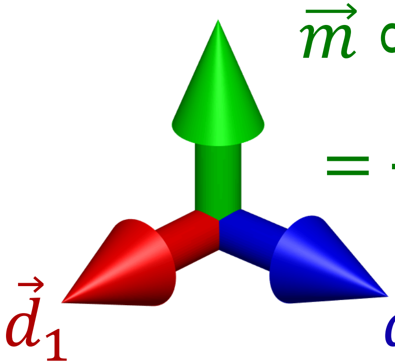
Order parameter of a spin-polarized superconductor

Order parameter of a spin-triplet SC:

$$\vec{d}_k = \langle c_k^\dagger i\sigma_2 \vec{\sigma} c_{-k}^\dagger \rangle \equiv \vec{d}_{1,k} + i\vec{d}_{2,k}$$

Fully spin polarized SC: $|\vec{d}_{1,k}| = |\vec{d}_{2,k}|$, $\vec{d}_{1,k} \perp \vec{d}_{2,k}$

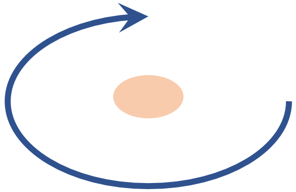
Order parameter space: $SO(3)$
(*Neglecting spin-orbit coupling)*



No finite T transition in $d = 2$
Mukerjee, Xu, Moore (2006)

Topological defects

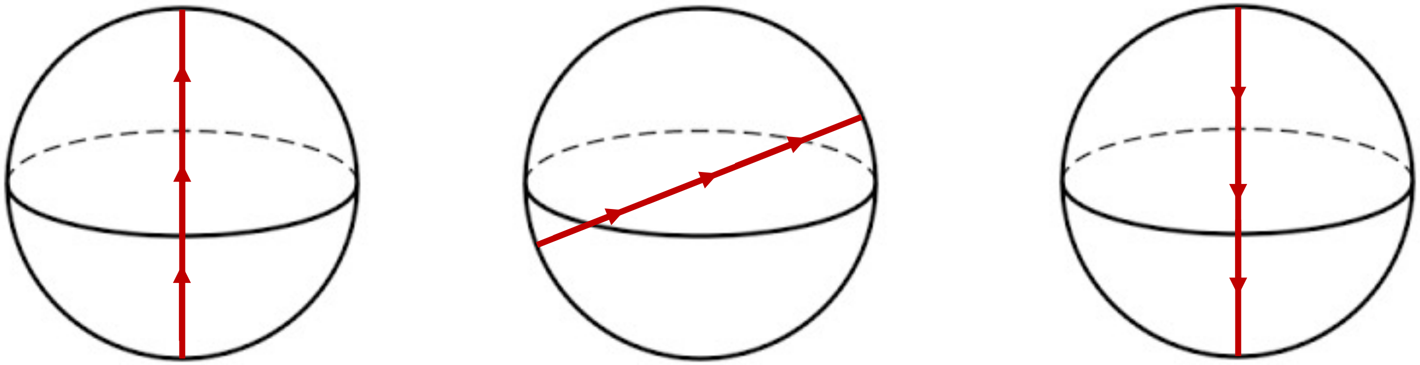
Polarized triplet superconductor:



$$\pi_1\big(SO(3)\big) = Z_2$$

Z_2 superconducting vortex

Direction: axis of rotation
Radius: rotation angle
(antipodal points of radius π identified)



Consequences for current relaxation

Free energy density (assuming spin rotation invariance):

$$f = \frac{\kappa_d}{2}\left|\nabla\vec{d}\right|^2 + \frac{\kappa_m}{8}\left|\nabla\left(\vec{d}^*\times\vec{d}\right)\right|^2$$

Represent order parameter by 2×2 unitary matrix u:

$$\vec{d} = \mathrm{Tr}[u(\sigma_1 + i\sigma_2)u^\dagger\vec{\sigma}]$$

Spin rotation: $u \rightarrow e^{\frac{i}{2}\vec{\theta}\cdot\vec{\sigma}}u$, *Gauge transformation:* $u \rightarrow ue^{\frac{i}{2}\varphi\sigma_3}$

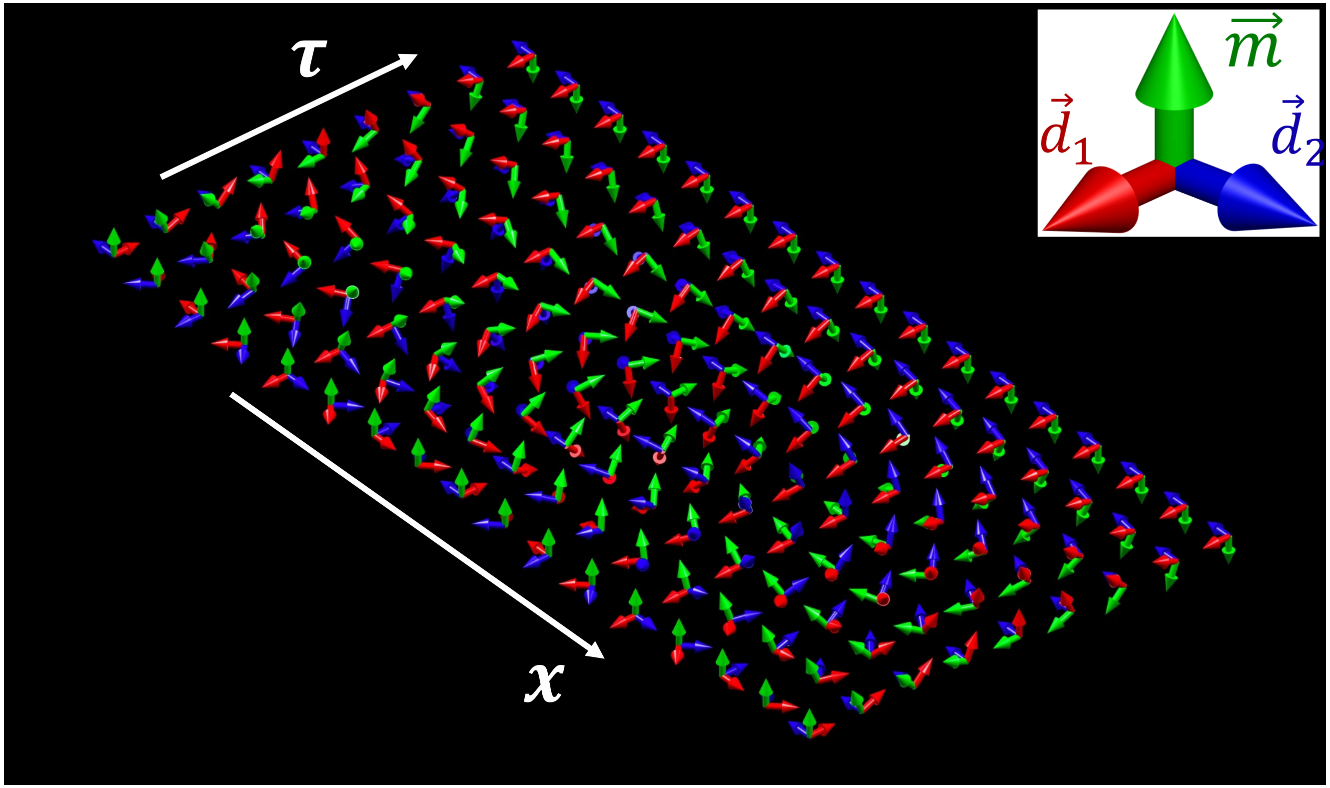
Supercurrent carrying state: $u(\vec{r}) = e^{i\pi n\sigma_3\frac{x}{L_x}}$

E. Cornfeld, M. Rudner, EB, Phys. Rev. Research 3, 013051 (2021)

Consequences for current relaxation

Unwinding a phase twist of 4π:

$$u(\vec{r}, 0 \leq \tau \leq 1) = e^{i\pi\sigma_3 \frac{x}{L_x}} e^{\frac{i\pi}{2}\sigma_1\tau} e^{i\pi(n-1)\sigma_3 \frac{x}{L_x}}$$



E. Cornfeld, M. Rudner, EB, Phys. Rev. Research 3, 013051 (2021)

Consequences for current relaxation

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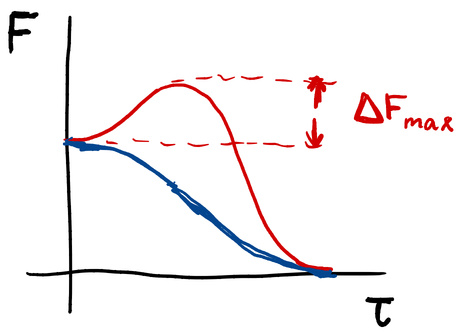
$$u(\vec{r}, 0 \leq \tau \leq 1) = e^{i\pi\sigma_3 \frac{x}{L_x}} e^{\frac{i\pi}{2}\sigma_1 \tau} e^{i\pi(n-1)\sigma_3 \frac{x}{L_x}}$$

Path requires mechanism to dissipate magnetization (spin bath/coupling to leads)

Energy landscape:

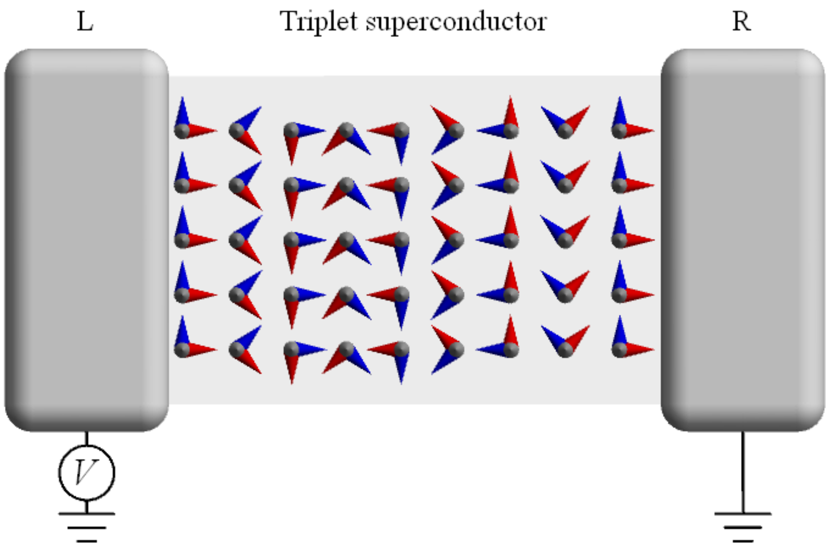
$$\Delta F_{\max} = \begin{cases} 0 & \frac{\kappa_m}{\kappa_d} \leq 2n-1, \\ \frac{2\pi^2 L_y (\kappa_m - (2n-1)\kappa_d)^2}{L_x(\kappa_m - \kappa_d)} & \frac{\kappa_m}{\kappa_d} > 2n-1. \end{cases}$$

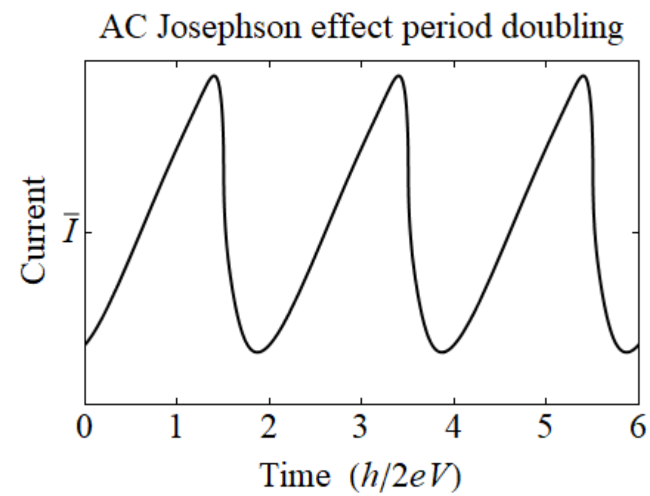


"Critical current density" depends on the system size!

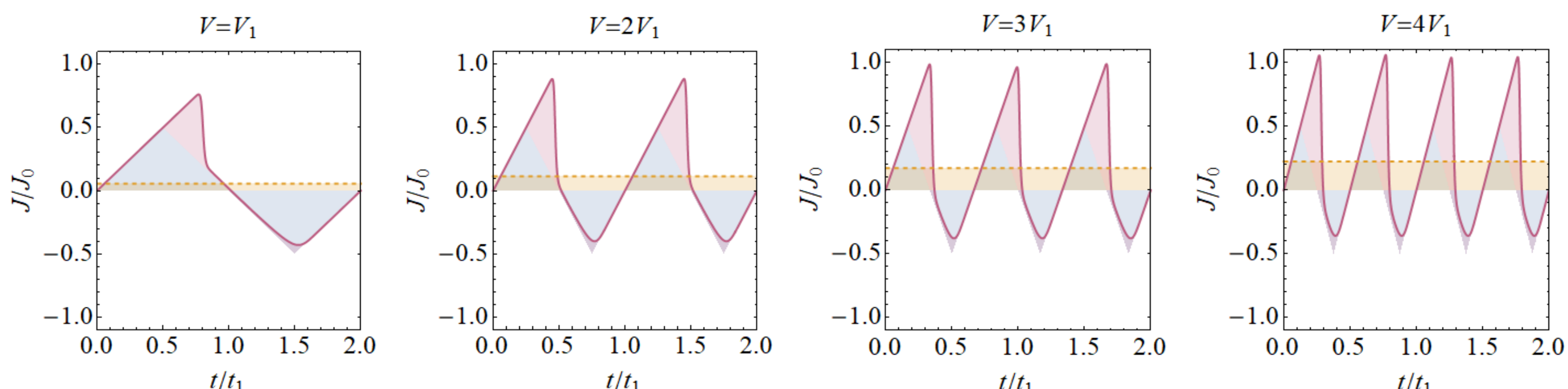
$$J_c \sim \frac{\kappa_d}{L_x}\left(\frac{\kappa_m}{2\kappa_d} + 1\right)$$

Double-period Josephson effect





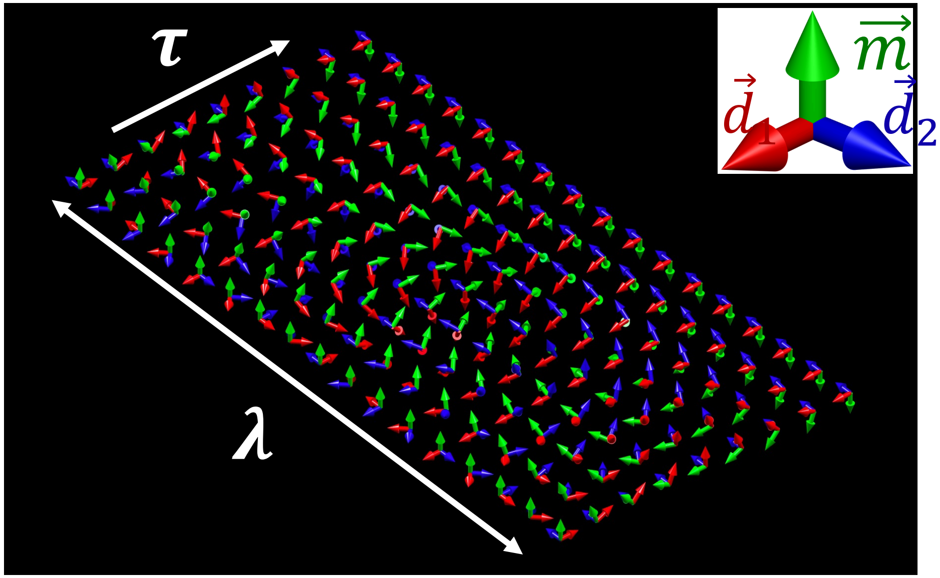
Langevin simulation (assuming coupling to a spin bath)



Half the usual Josephson frequency: $\omega = \dfrac{eV}{\hbar}$. $\qquad \bar{J} - J_c \sim \sqrt{V}$

**T is low enough such that vortex-antivortex dissociation is suppressed.*

In-plane magnetic field



Optimize λ:

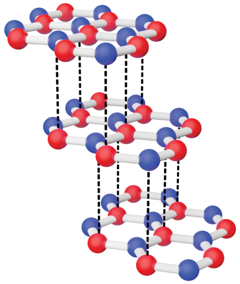
$$\lambda \sim \frac{1}{\sqrt{B}}$$

Critical current: $J_c \sim 1/\lambda \sim \sqrt{B}$

Summary

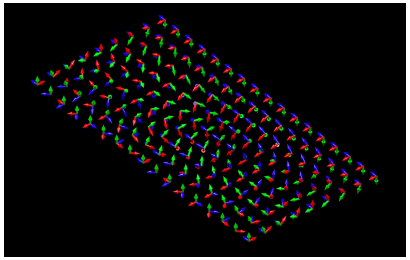
2D Annular Fermi surfaces are favorable for unconventional superconductivity driven by Coulomb interactions.

- Unconventional SC in ABC trilayer graphene?
  Most likely state: chiral p-wave



 $+e^{2\pi i/3}$  $+e^{4\pi i/3}$ 

- Fully spin polarized SC: fragility of supercurrent due to topology, double-period Josephson effect



Thank you!