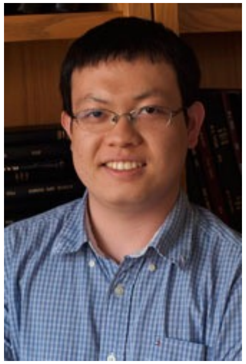
Quantum Critical Metals

Erez Berg

Weizmann Institute of Science

Sam Lederer (Cornell)

Yoni Schattner (WIS→Stanford)



Xiaoyu Wang (UChicago →UFL)

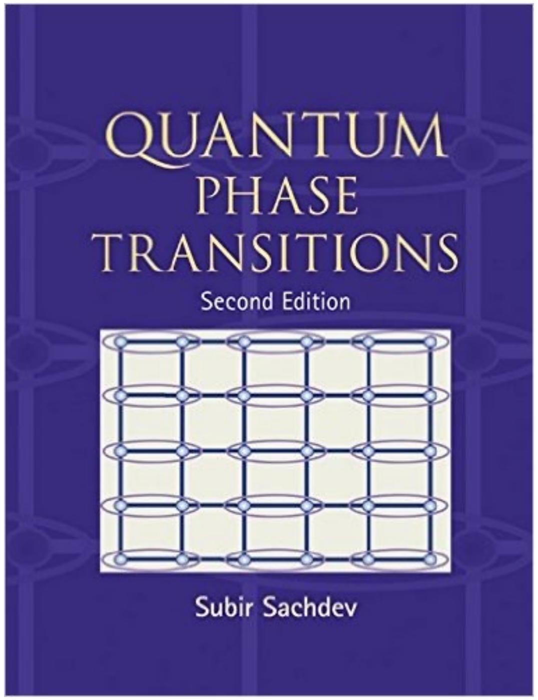
Ori Grossman (WIS)

Johannes Hofmann (WIS)

Tobias Holder (WIS)

Subir Sachdev, Max Metlitski, Steve Kivelson, Simon Trebst, Kai Sun, Rafael Fernandes, Andrey Chubukov, Yuxuan Wang, Avi Klein, Max Gerlach, Carsten Bauer, Zi-Yang Meng, Xiao Yan Xu

$T = 0$ **continuous transitions *in insulators* are fairly well understood.**



What happens when a system *with a Fermi surface* goes critical?

Outline

- Classical and Quantum Criticality
- Quantum critical *Fermi surfaces*
- Numerical Quantum Monte Carlo experiments: Results, intermediate conclusions, and outstanding mysteries

Kinetically Constrained

Scale invariance at the critical point

by Douglas Ashton

www.kineticallyconstrained.com

Why do we care about critical phenomena?

- *Emergent* scale invariance

$$\vec{r} \to b\vec{r}:$$

$$G(\vec{r}) = \langle \phi(\vec{r})\phi(0) \rangle \to b^{-2d_\phi} G(\vec{r}/b\ )$$

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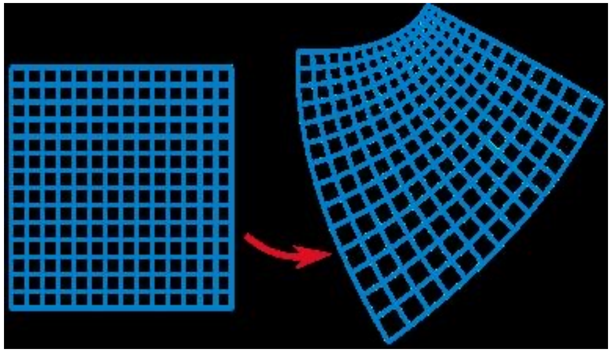
- *Emergent* conformal symmetry



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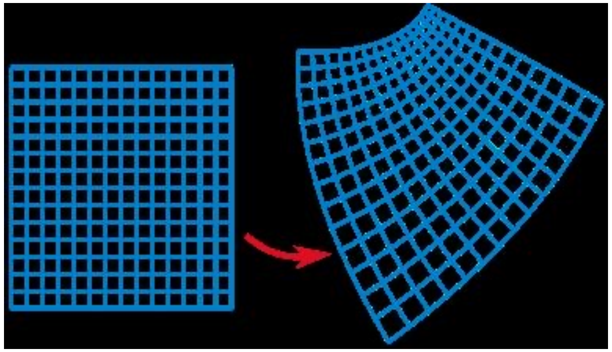

- Theoretical control (renormalization group, Monte Carlo simulations)

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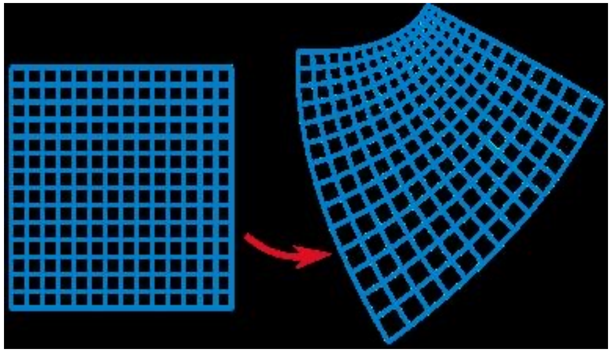


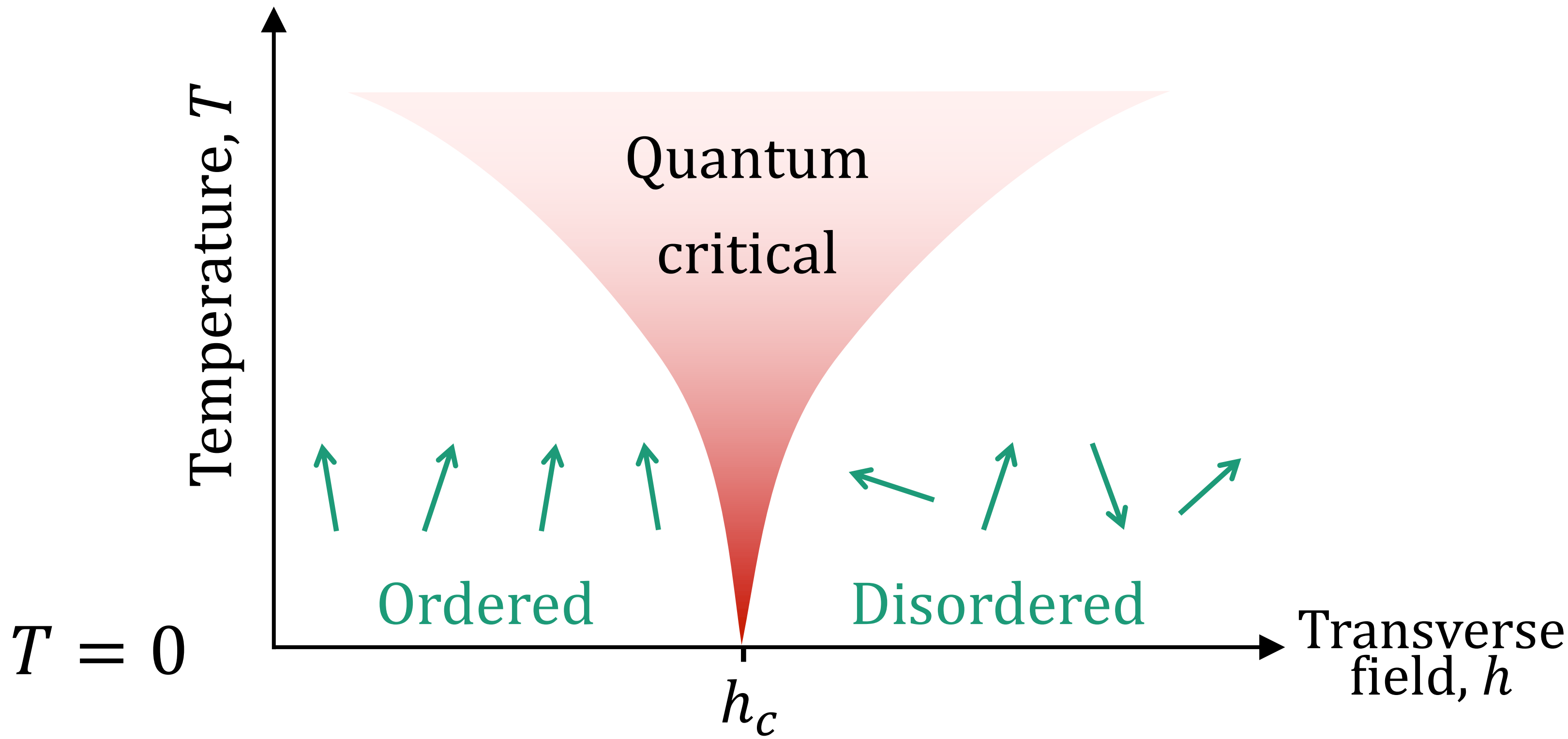
- Theoretical control (renormalization group, Monte Carlo simulations)

- Universality: divergent correlation length, “microscopic” details don’t matter!

Important concept for quantum field theory, too…

Quantum critical phenomena

- Continuous transition at $T = 0$ as a function of Hamiltonian parameter
- Example: the transverse field Ising model

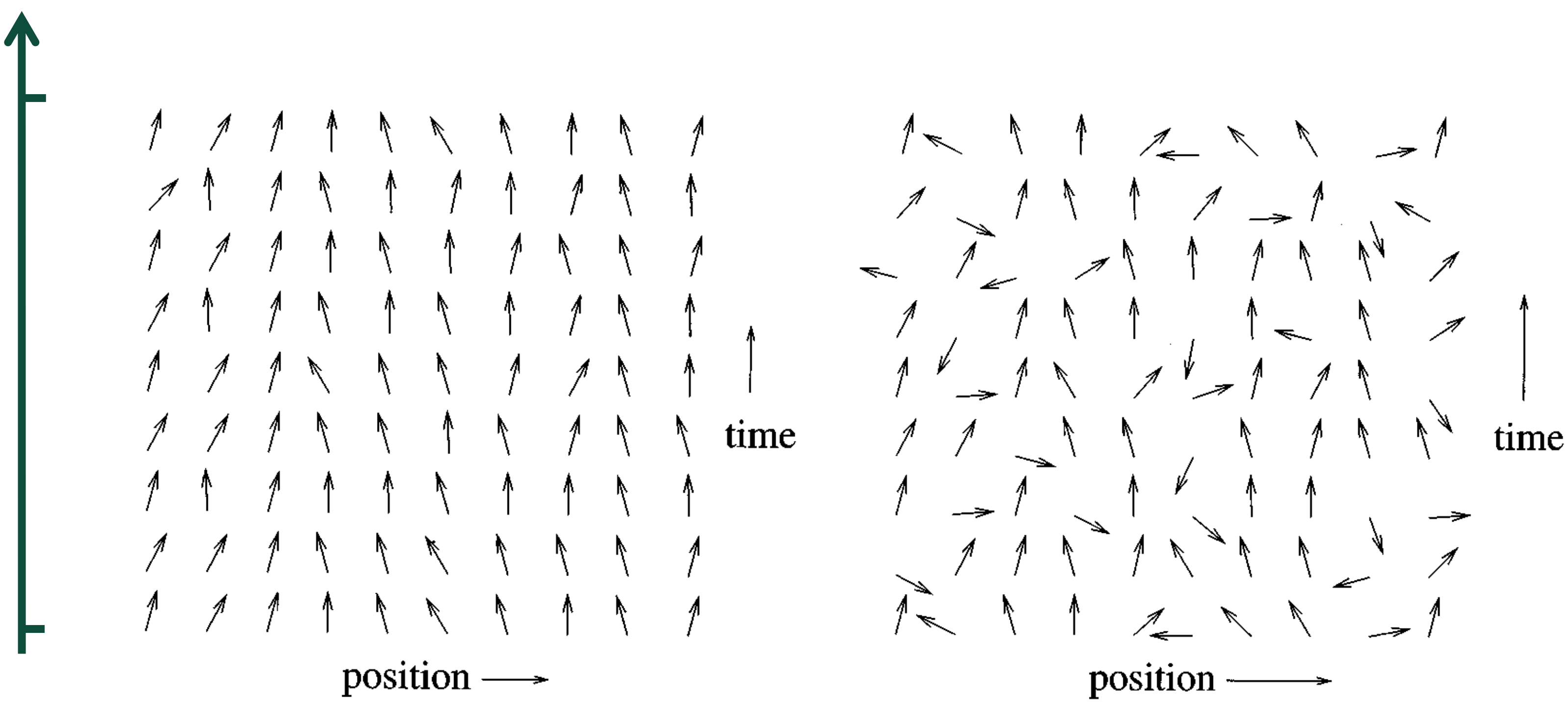


Quantum critical phenomena

$d-$dimensional quantum $\Leftrightarrow$
$d+1-$dimensional classical, size $\beta = \hbar/k_B T$ in the “imaginary time,” τ direction

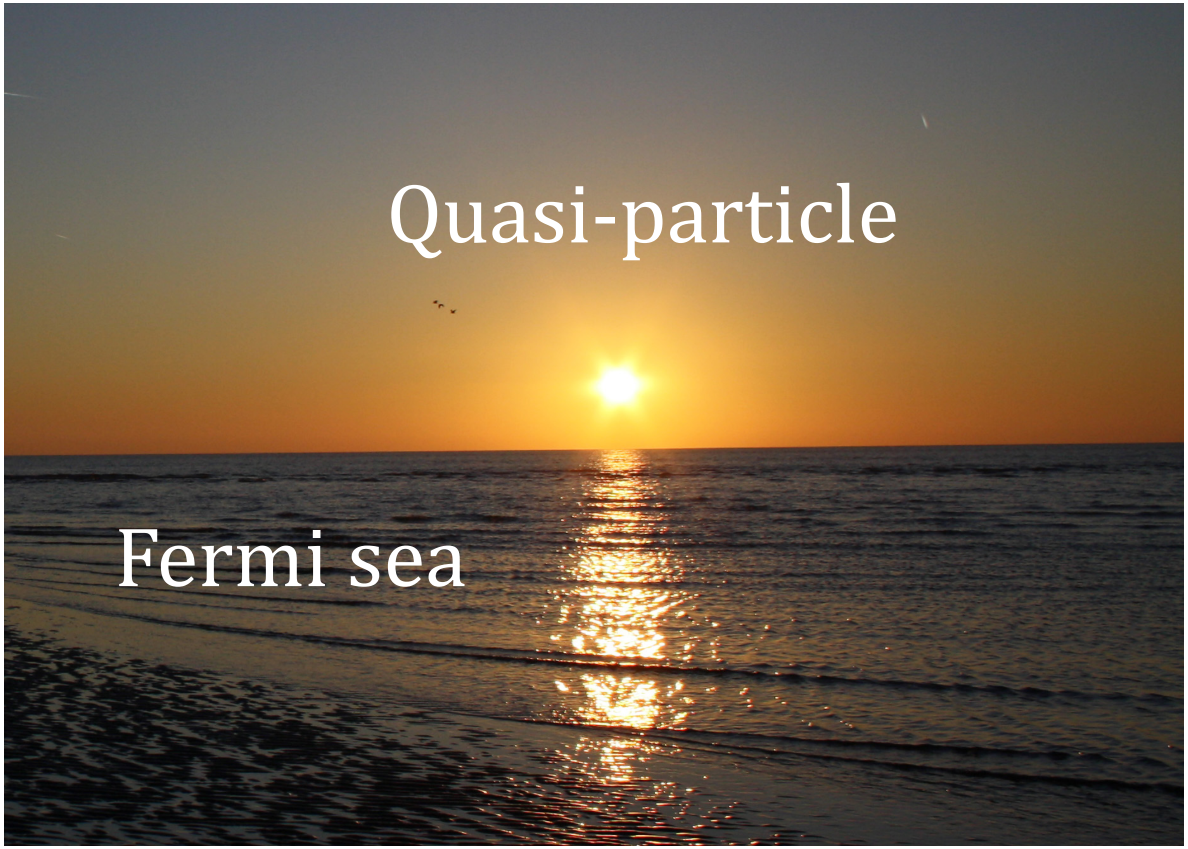
$$\beta = \frac{\hbar}{k_B T}$$

0



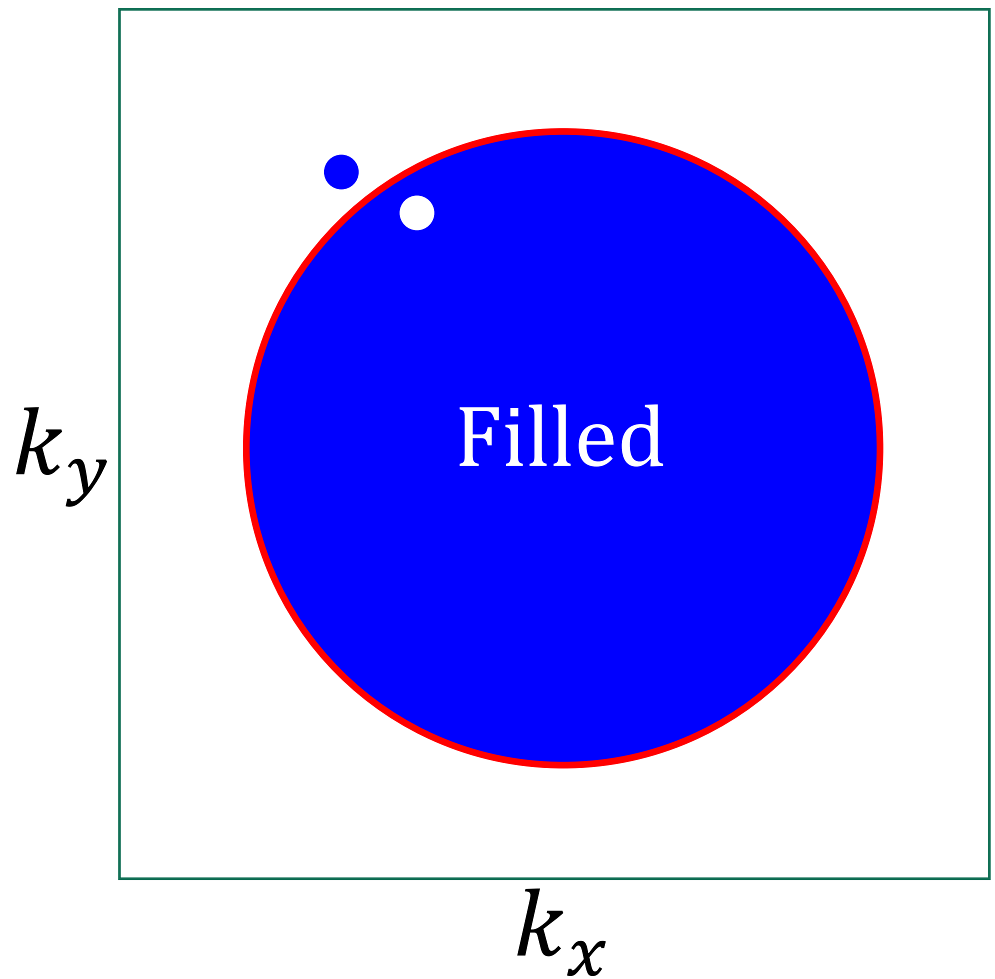
From Sondhi, Girvin, Carini, Shahar, RMP (1997)

Conventional (Fermi liquid) metal

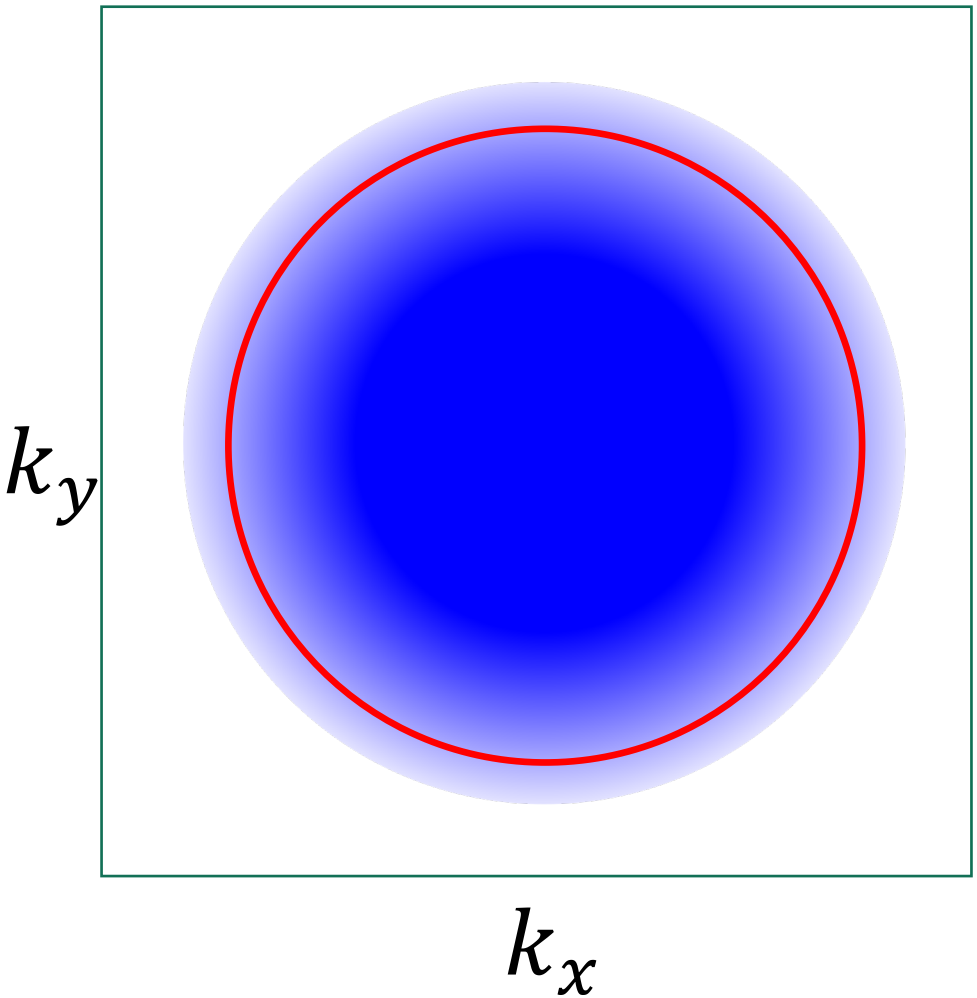


Conventional (Fermi liquid) metal

Fermi surface is quantum mechanical:
no classical analogue

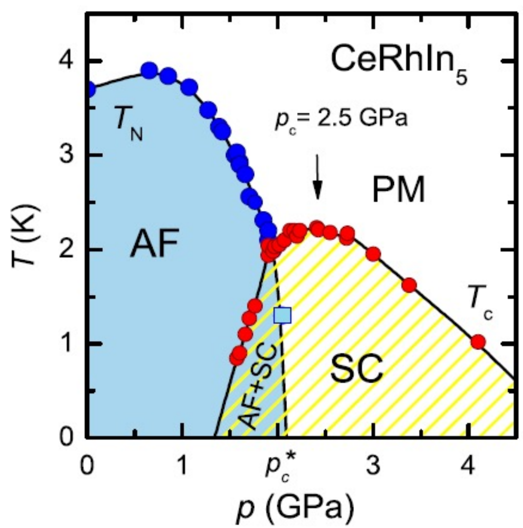


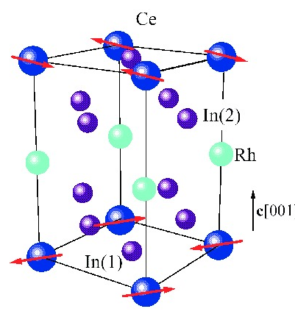
Critical metal: non-Fermi liquid?



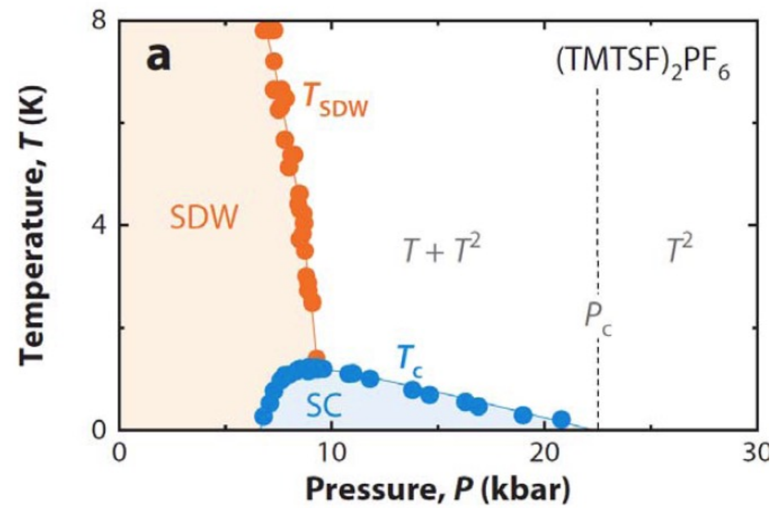
Quantum criticality in unconventional superconductors?

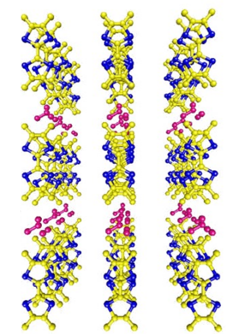
Heavy Fermions



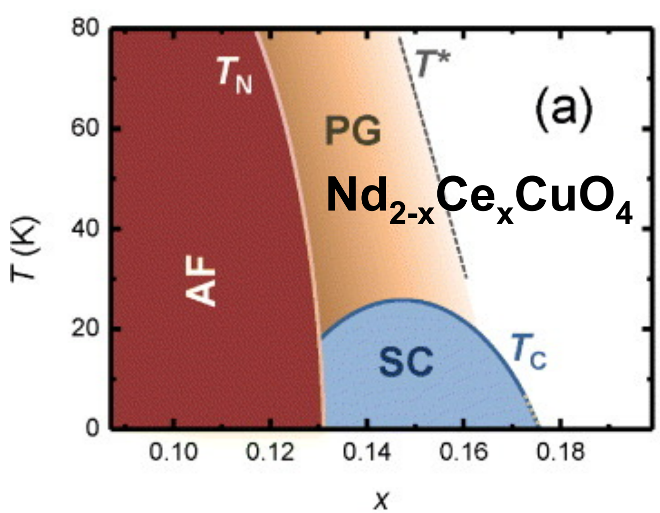


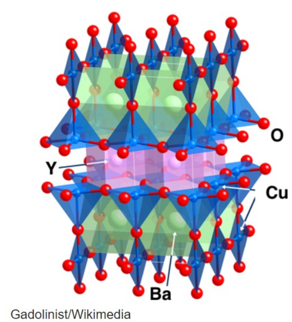
Organic supercondutors



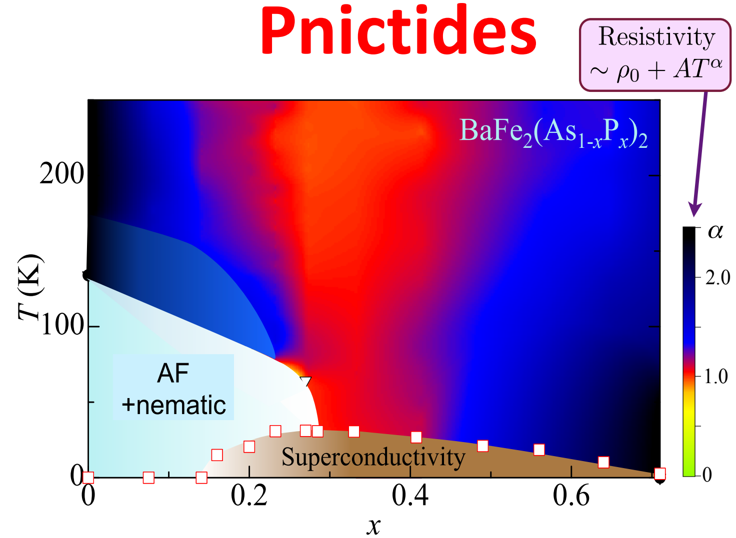


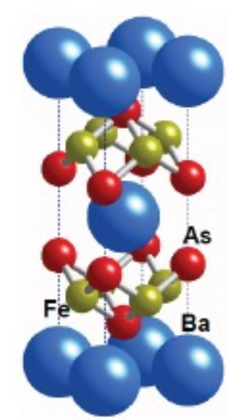
Cuprates





Pnictides

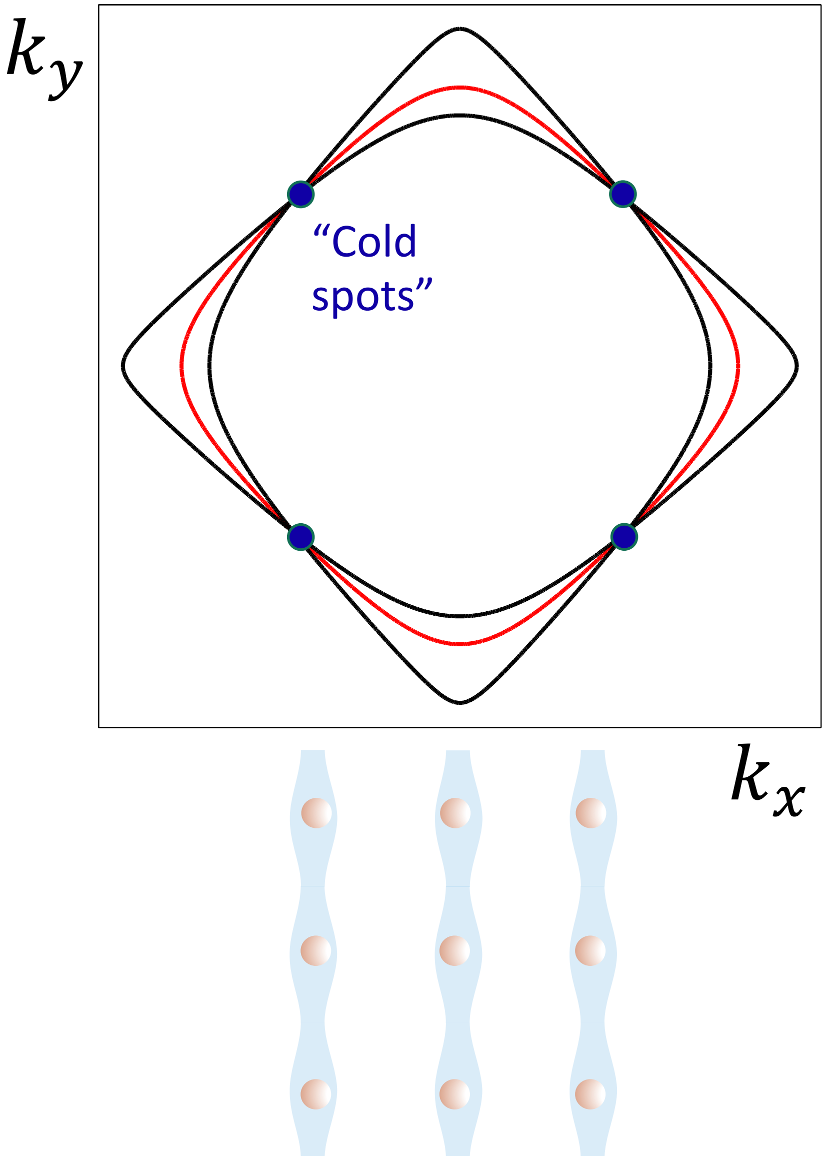




Kasahara,...,Matsuda (2010)

Two types of metallic quantum critical points

Ising-nematic QCP



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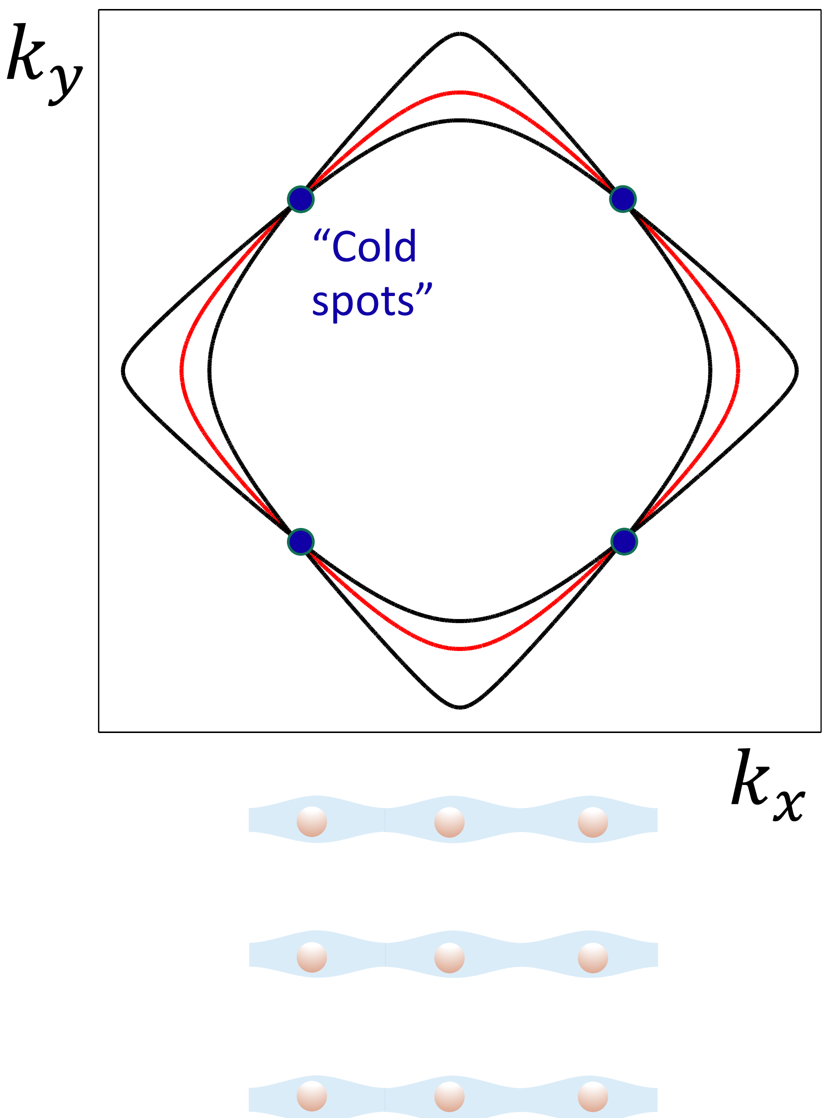
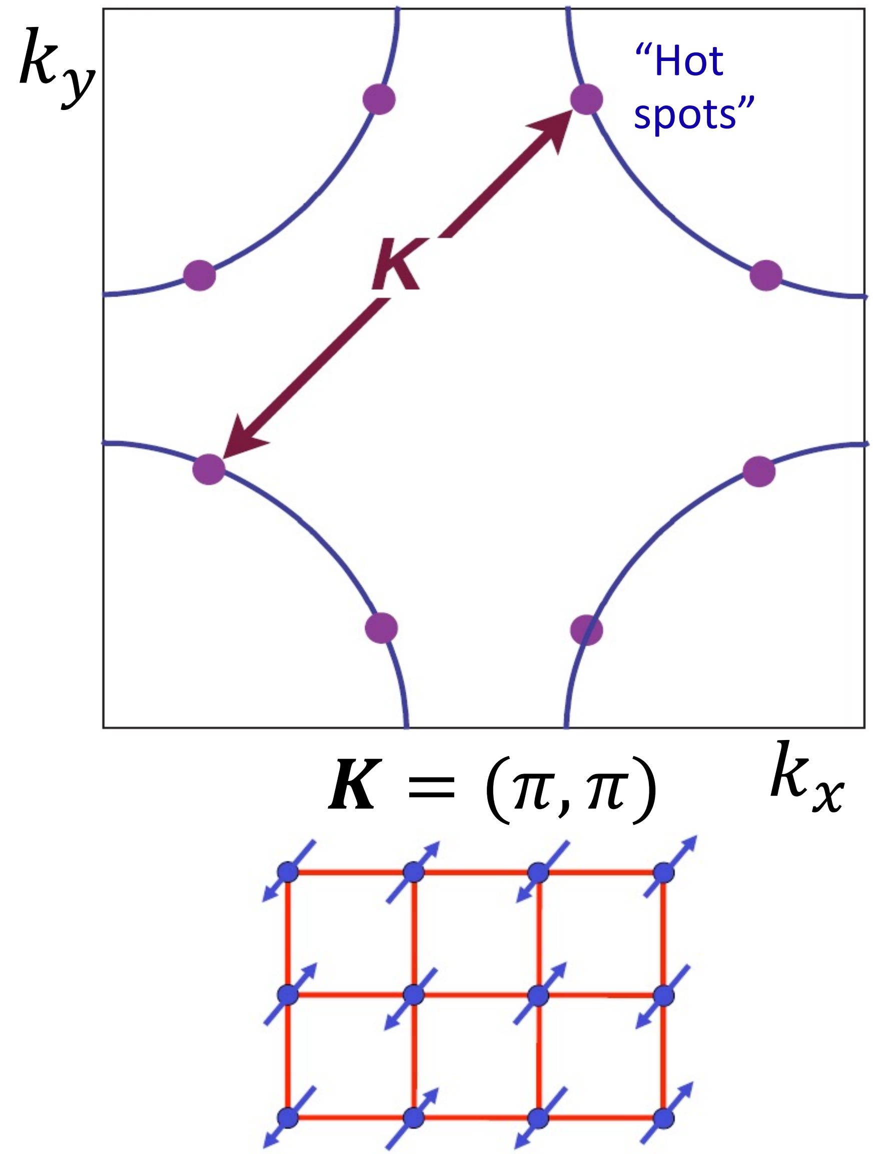
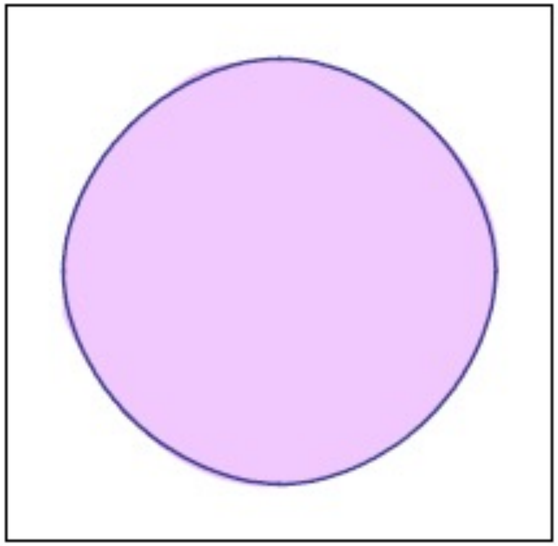


Antiferromagnetic QCP

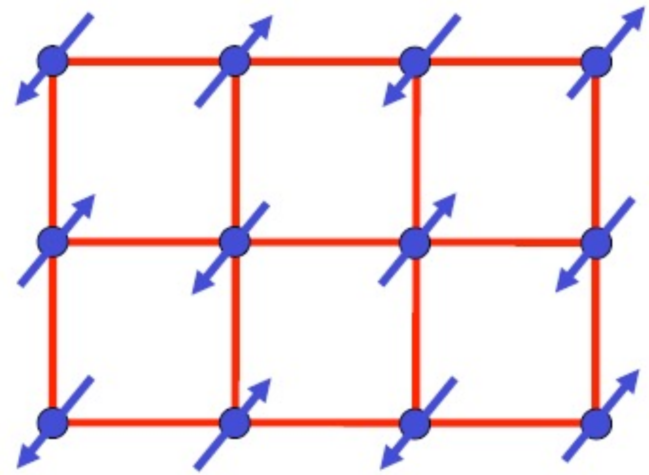


Models for metallic quantum criticality



Fermions

+



Order parameter ϕ

$$S = S_{\text{fermions}} + S_{\phi} + S_{\text{int}}$$

$$S_{\text{fermions}} = \int d^2k d\tau\, \psi_k^+ (\partial_\tau + \varepsilon_k) \psi_k$$

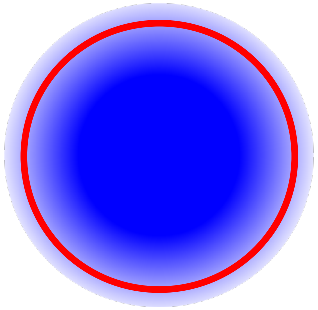
$$S_{\phi} = \int d^2x\, d\tau\, (\nabla\phi)^2 + r\phi^2 + (\partial_\tau \phi)^2/c^2 \ldots$$

$$S_{\text{int}} = \alpha \int d^2x\, d\tau\, \phi \psi^+ \psi$$

α – "Yukawa" coupling

Metallic Quantum Criticality: Open Questions

- Critical exponents?
- Destruction of Fermi Liquid theory?
- QCP "masked" by enhanced superconductivity/other order?



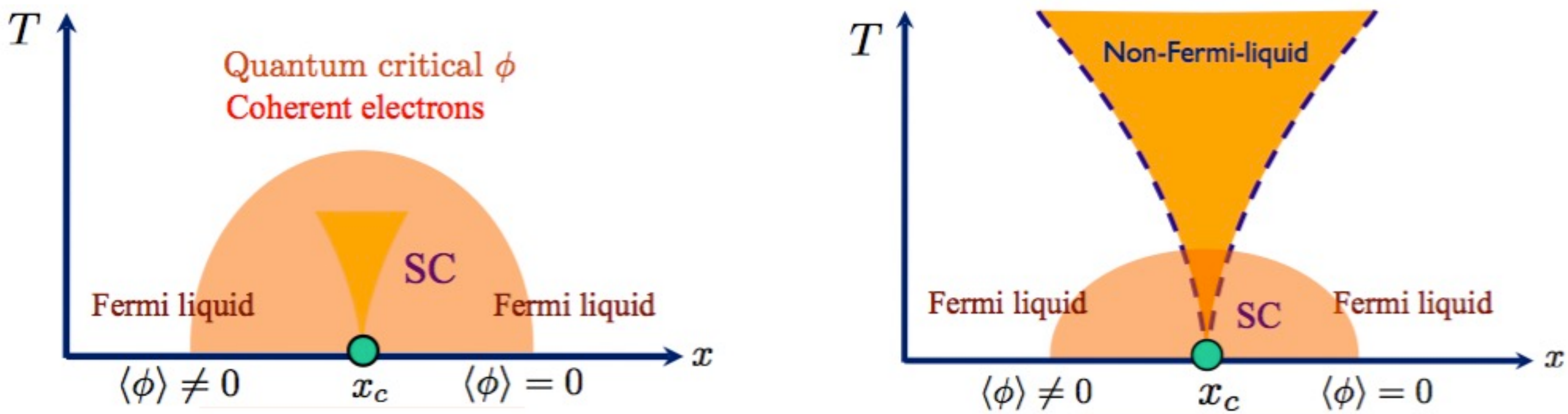


Figure from: Max Metlitski, David Mross et. al. (PRB, 2014)

Strongly coupled problem!

Herz, Millis, Abanov, Chubukov, Sachdev, Metlitski, Mross, Senthil, S-S. Lee, Raghu, Kachru, Metzner, Holder, Hartnoll…

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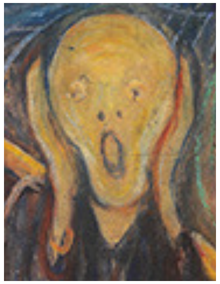
Determinant Quantum Monte Carlo (QMC)

Effective bosonic action: $e^{-S_{\text{eff}}[\phi]} = e^{-S_0[\phi]}\det(M[\phi])$

M-fermion action matrix

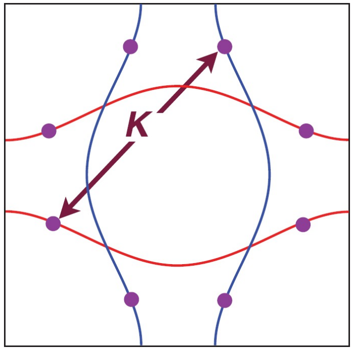


$e^{-S_{\text{eff}}[\phi(\vec{x},\tau)]}$ can be negative (or complex): **"Sign Problem"**

Many actions describing QCPs in metals are sign problem free:

$$\mathbf{Im}\left(e^{-S_{\text{eff}}}\right) = 0,\ \mathbf{Re}\left(e^{-S_{\text{eff}}}\right) \geq 0$$

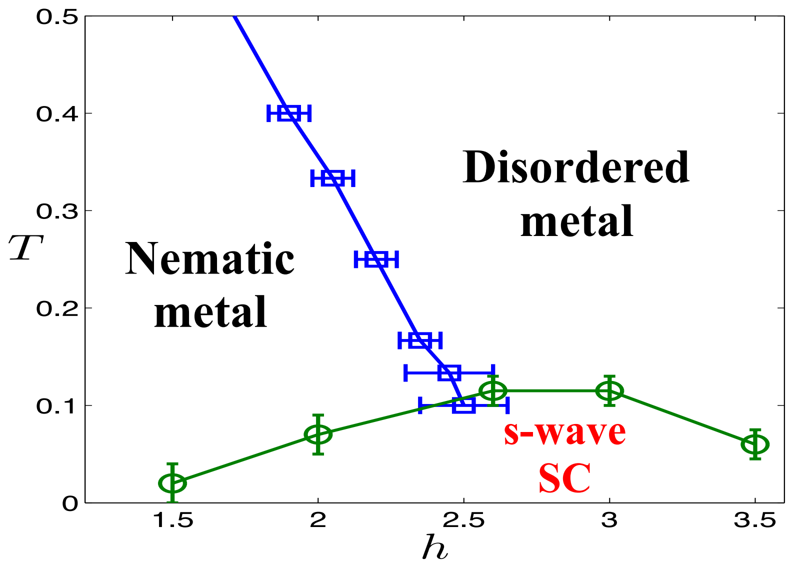
- **Ising Nematic criticality:**
  $\det(M) = \det(M_\uparrow)\det(M_\downarrow) = |\det(M_\uparrow)|^2 \geq 0$
- **SDW criticality:**
  Two bands, inter-band "hot spots" :
  Effective "time reversal" $\Longrightarrow$ sign free

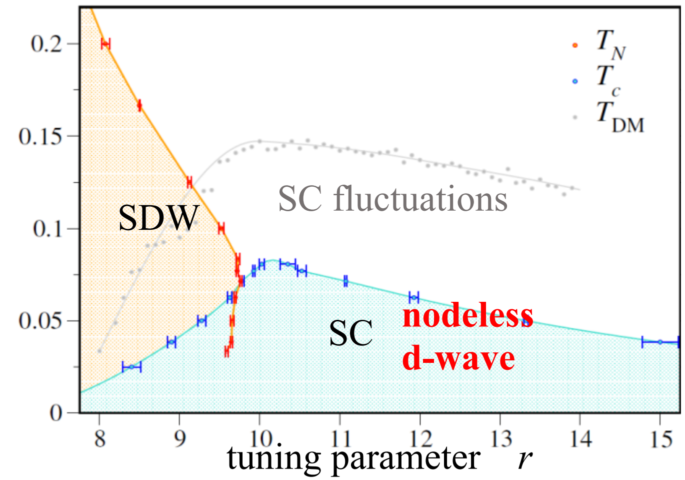


EB, Metlitski, Sachdev, Science (2012)

Is superconductivity enhanced near the QCP?

Yes.





Do any other types of order emerge generically near the QCP?

No. There's no "pseudogap" emerging near the QCP, either.

Description of the quantum critical regime?

Weak coupling:

Strongly renormalized bosons weakly renormalized (FL) fermions

Stronger coupling:

Signatures of non-Fermi Liquid behavior

Results

Ising nematic critical point

Divergent nematic susceptibility:

$$\chi \propto \frac{1}{T + A(h - h_c) + Bq^2}$$

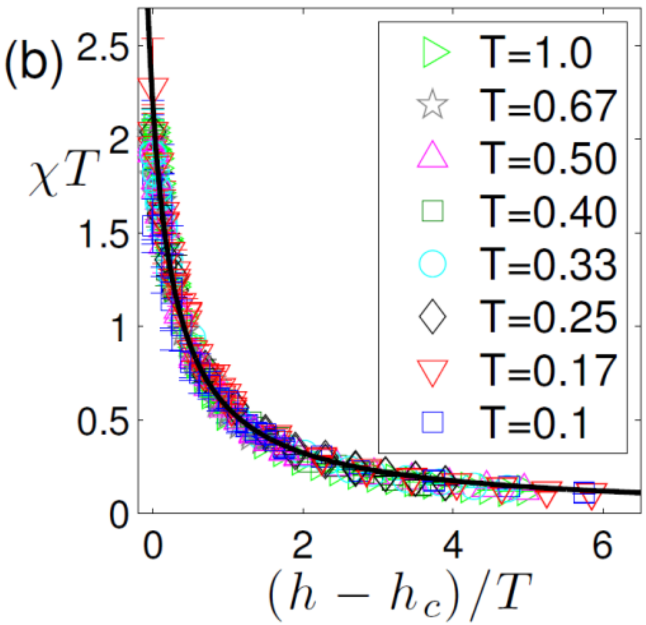
ω_n dependence: Landau damped,
q dependence of coefficient is complex

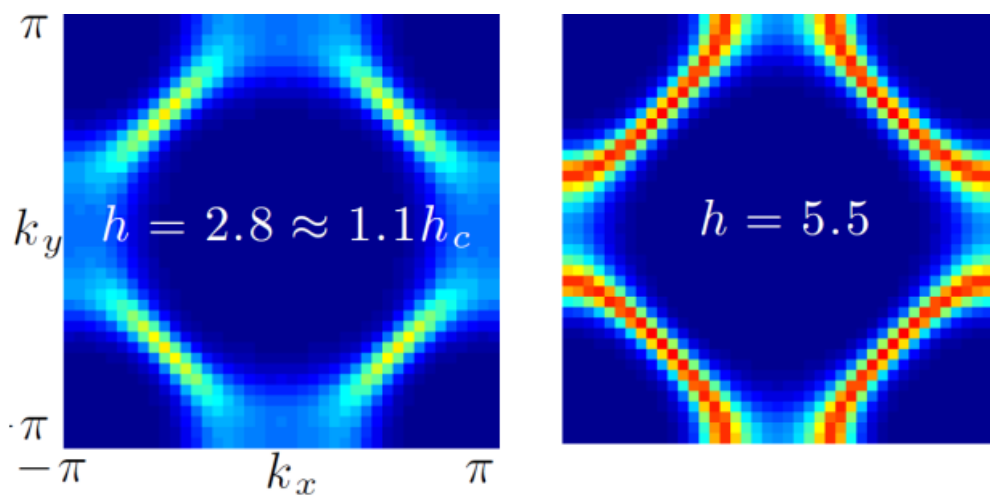
Low energy electronic spectrum:

$$G\left(\tau = \frac{\beta}{2}\right) \approx \int_{-T}^{T} d\omega A(\mathbf{k}, \omega)$$

Non-Fermi liquid behavior
away from "cold spots"

Unexpected behavior:
$\mathrm{Im}\Sigma_{k_F}(i\omega_n, T) \approx const$



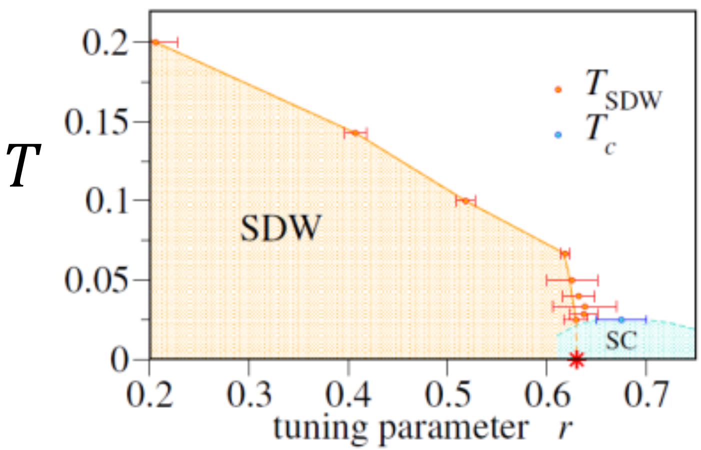


Schattner, Lederer, Kivelson, EB, PRX (2016); PNAS (2017)

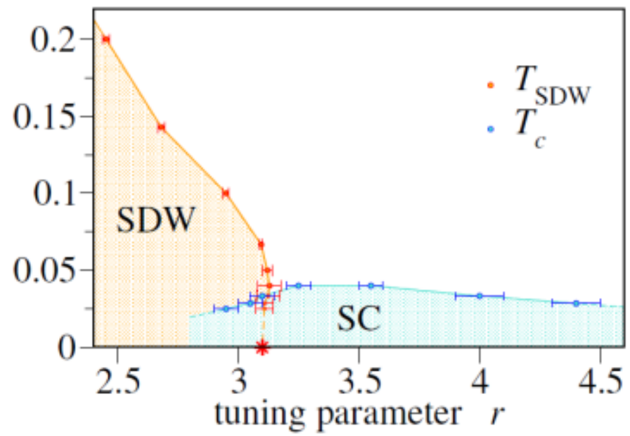
Results

Easy-plane AFM critical point: phase diagram

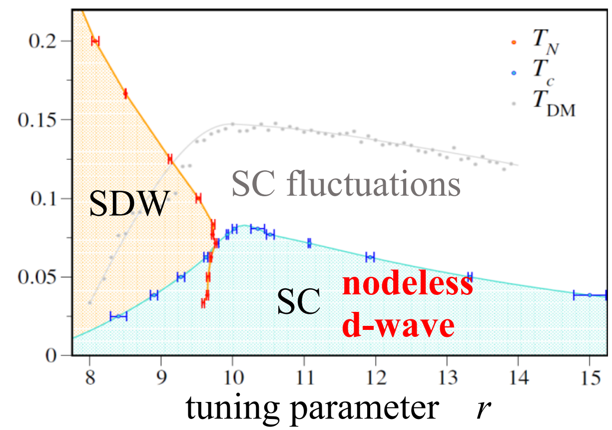
$\lambda = 1.5$



$\lambda = 2$



$\lambda = 3$

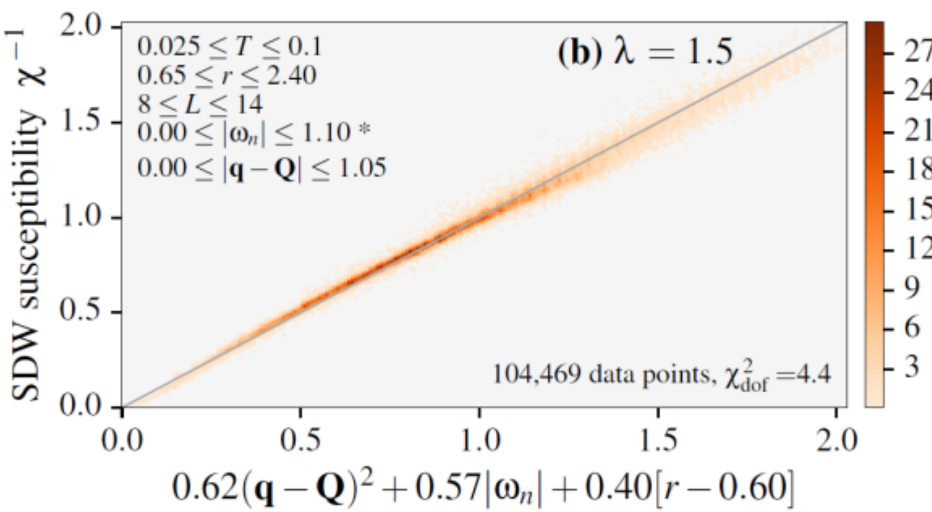


QCP covered by nodeless d-wave SC dome

Magnetic susceptibility above T_c:

$$\chi \propto \frac{1}{|\omega_n| + Aq^2 + B(r - r_c) + C(T)}$$

O(3) AFM transition: similar SC T_c,
χ has similar form (C. Bauer, 2020)



Schattner, Gerlach, Trebst, EB (PRL 2016, PRB 2017); Bauer, Schattner, Trebst, EB (PRR 2020)

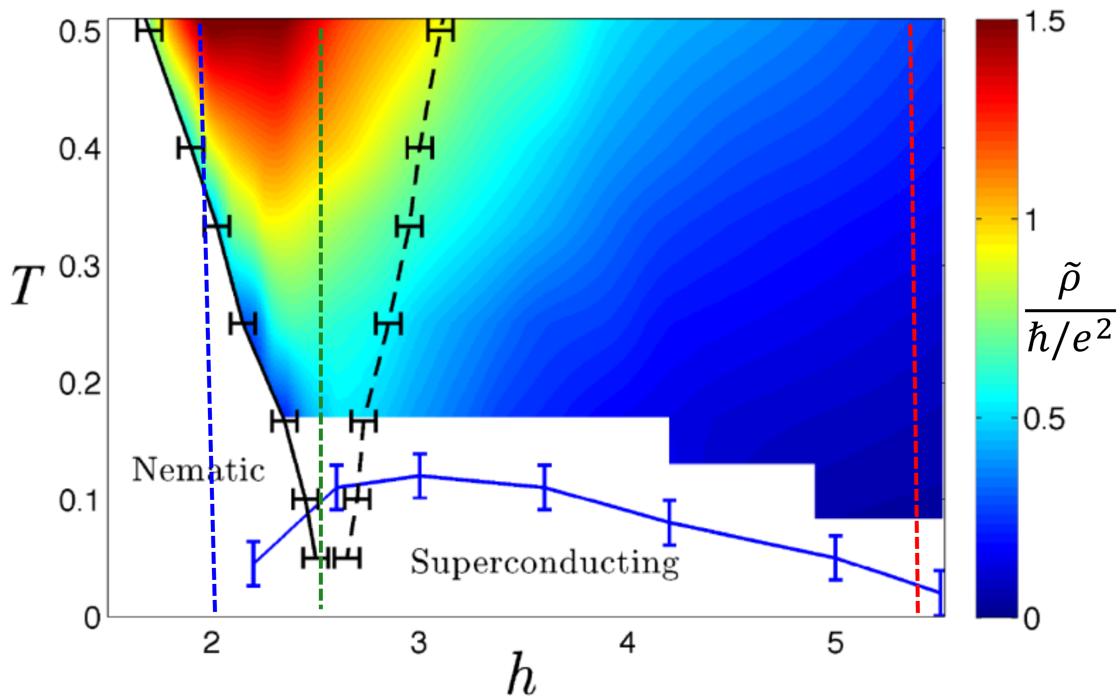
Transport

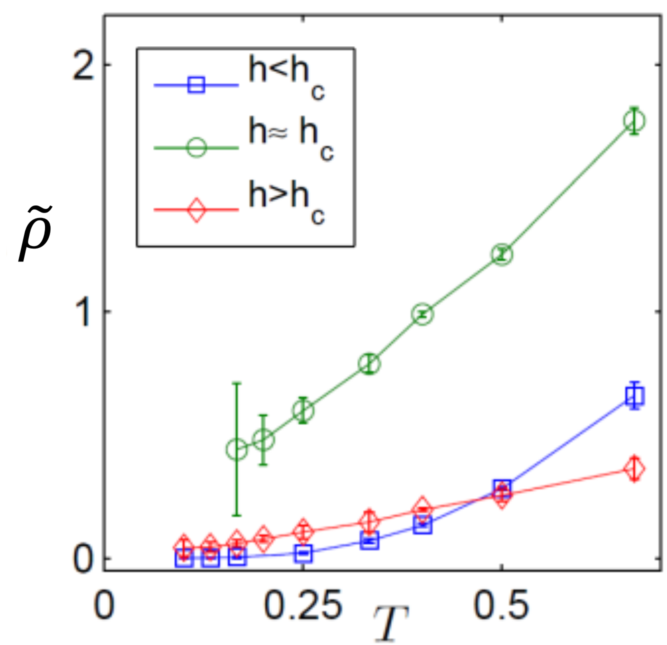
Ising nematic critical point

"Resistivity proxy": $\tilde{\rho} \approx \frac{\int_0^T d\omega\, \omega^2 \sigma(\omega)}{T\left[\int_0^T d\omega \sigma(\omega)\right]^2}$

If $\sigma(\omega)$ is a Lorentzian: $\tilde{\rho} = \rho_{dc}$

Qualitatively similar results for AFM QCP

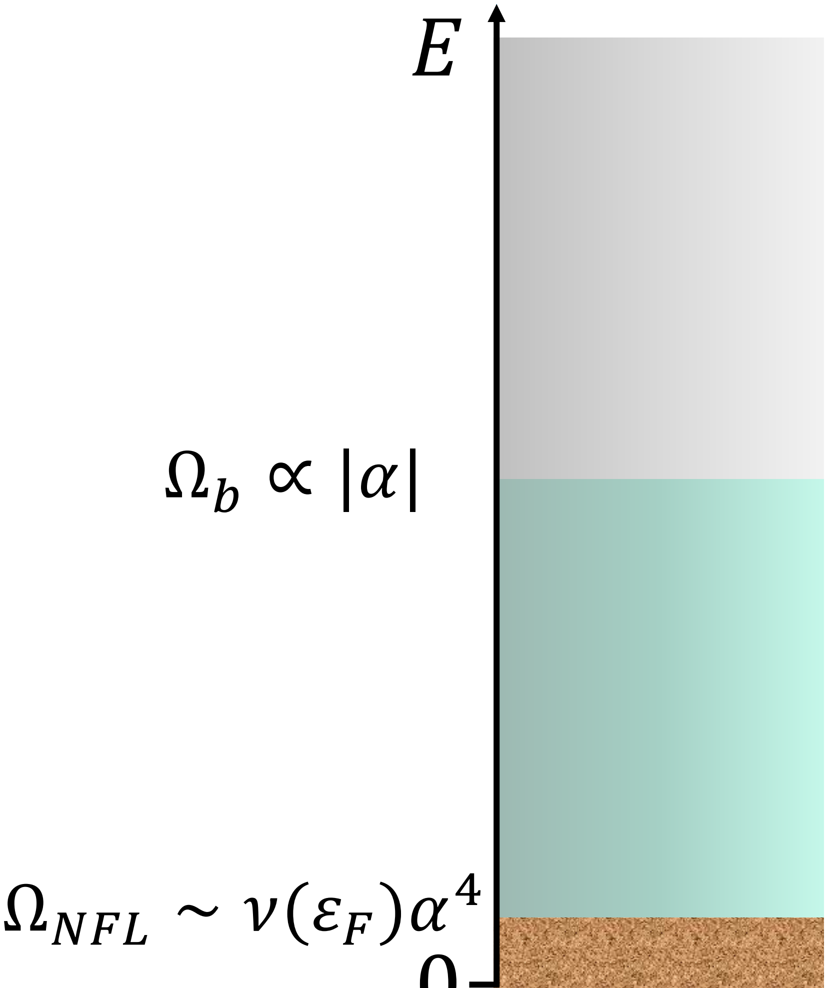




S. Lederer, Y. Schattner, EB, S. Kivelson, PNAS (2017)

Weak coupling, d=2 (Ising nematic)

$\nu(\varepsilon_F)\alpha^2 \ll 1$



Wilson-Fisher

$z = 1$, FL

Landau damped bosons,
coherent electrons
$z = 3$

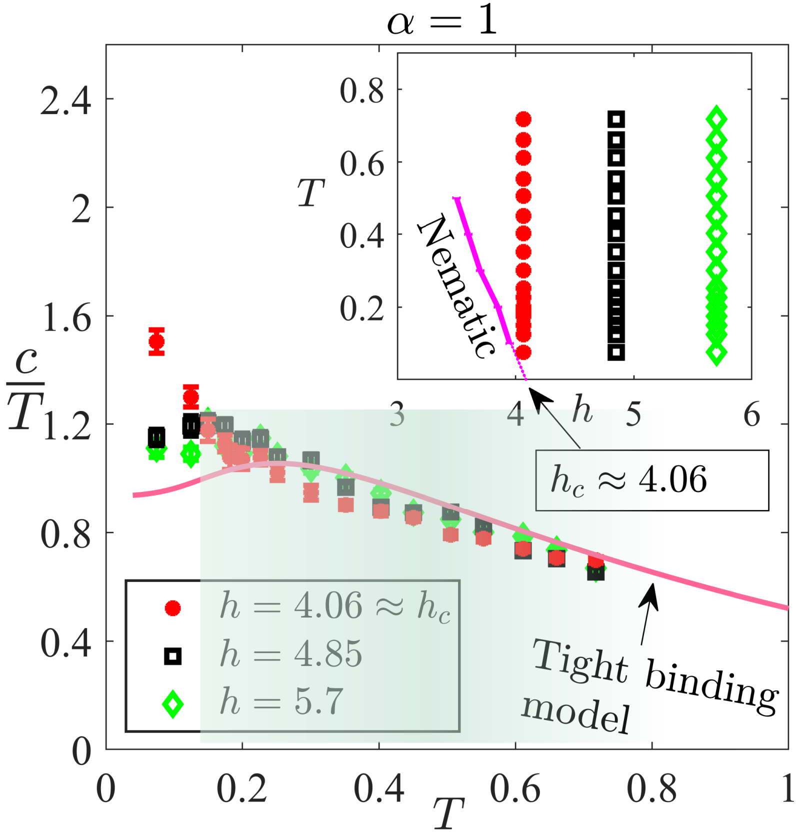
NFL? $(\Sigma(\omega = \Omega_{NFL}) \sim \omega)$

Superconductor? $(T_c \sim \Omega_{NFL})$

Specific heat: Ising nematic QCP

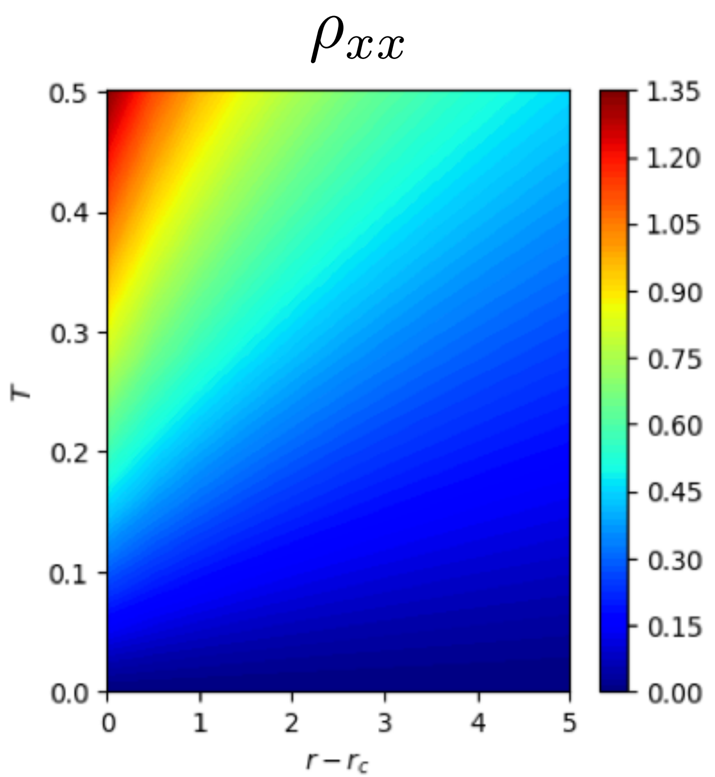
Broad "coherent electron regime" above $T_{NFL} \sim T_c$

- $m \sim m^*, Z \sim 1$
- Non-Fermi liquid scattering rate

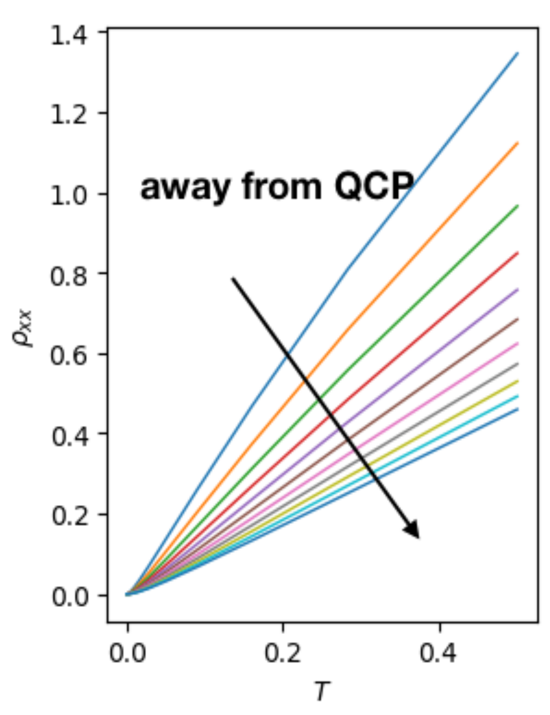


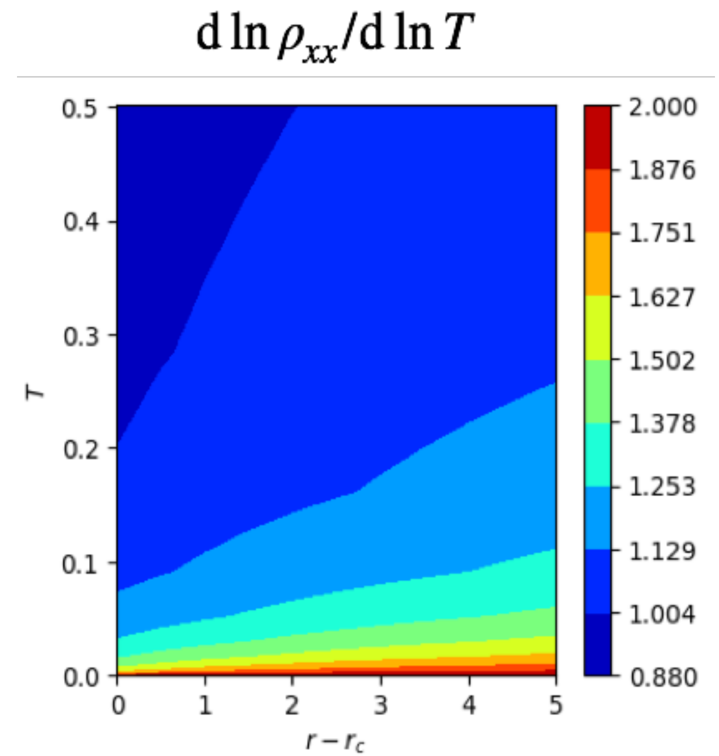
Grossman, Hofmann, Holder, EB, PRL (2021)

Analytical transport calculation: coherent electron regime



$\propto 1/\xi^2$





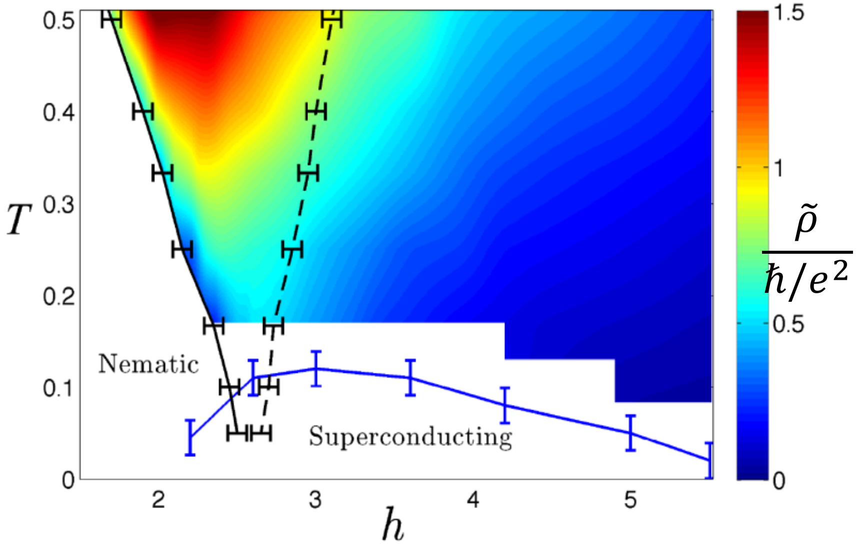
- Non-zero resistivity due to Umklapp proceeses
- Quasi-linear resistivity for $T > T_0$ ($T_0 \sim |\boldsymbol{q}_0|^z$)

Xioayu Wang and EB, PRB (2019)

What have we learned?

Metallic quantum criticality is accessible via sign problem-free Quantum Monte Carlo simulations.

- **Generic properties:**
  - ***QCP "preempted" by high-T_c superconductor!***
  - ***Quantum critical regime above T_c:***
    - ***Rapid growth of correlations***
    - ***Breakdown of Fermi liquid behavior***
    - ***Anomalous transport***
- **What's missing…**
  - ***No "competing orders" other than SC***
  - ***No "Pseudogap"***



Thank you.