The good, the bad, and the strange: Experimental and theoretical status of linear in T resistivity

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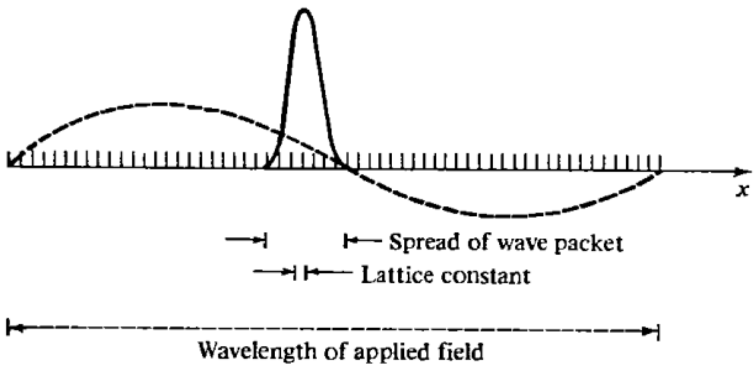
Semiclassical theory of transport in metals

$$k_F l \gg 1$$

$$l/a \gg 1$$



Drude formula: $$\rho = \frac{m}{ne^2\tau} = \frac{3\pi}{2}\frac{h}{e^2 k_F}\frac{1}{k_F l}$$

Limit of validity: Mott-Ioffe-Regel limit

$$\rho \ll \frac{3\pi}{2}\frac{h}{e^2}\frac{1}{k_F} \equiv \left(\frac{3\pi}{2k_F a_B}\right)\rho_Q$$

"Quantum of Resistivity"

$$\rho_Q = \frac{h}{e^2}a_B = 136.6\mu\Omega\text{cm}$$

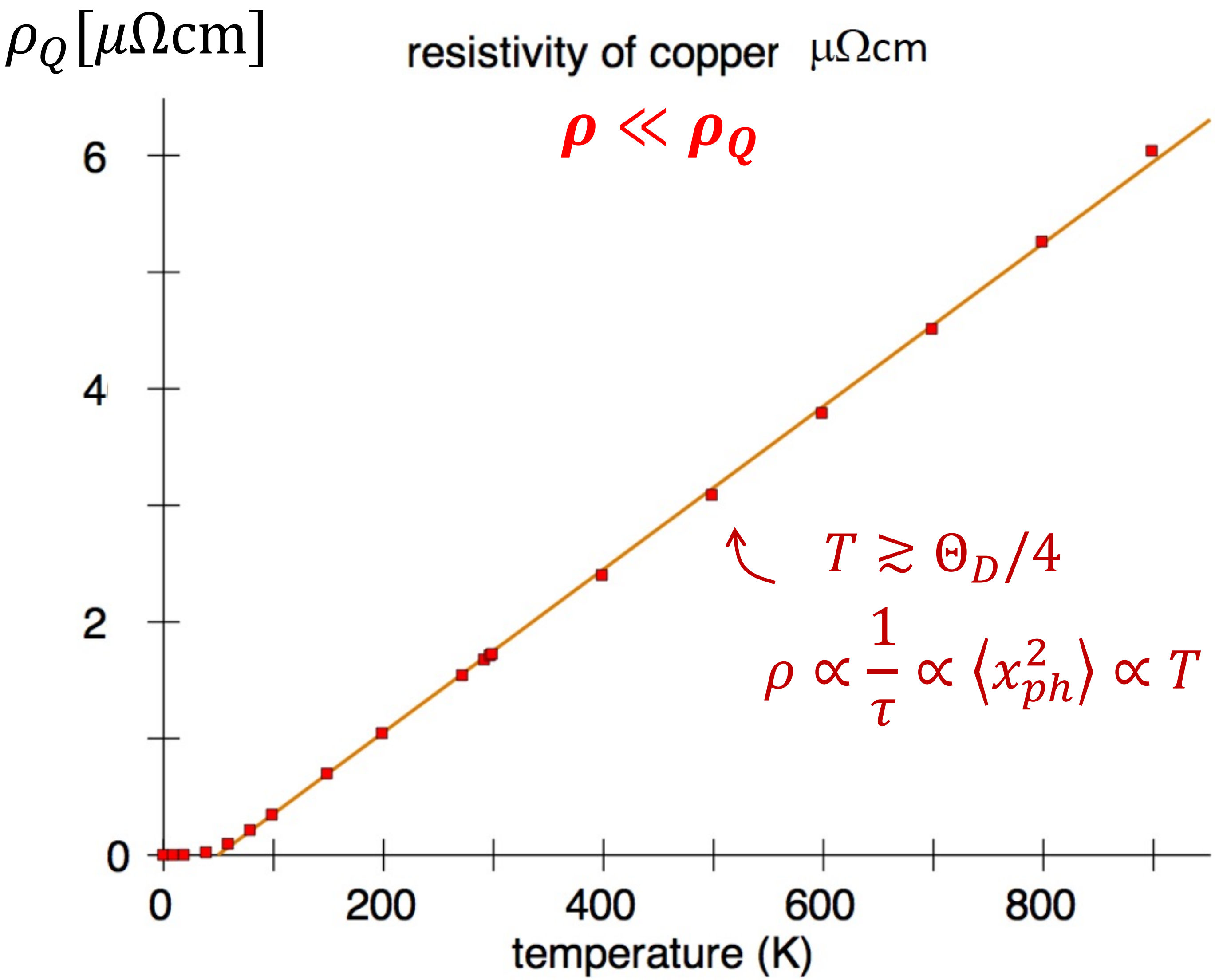
Outline

- Experimental survey: resistivity in good, bad, and strange metals
- Evidence for universal "Planckian" bound on relaxation time?
- Theoretical models

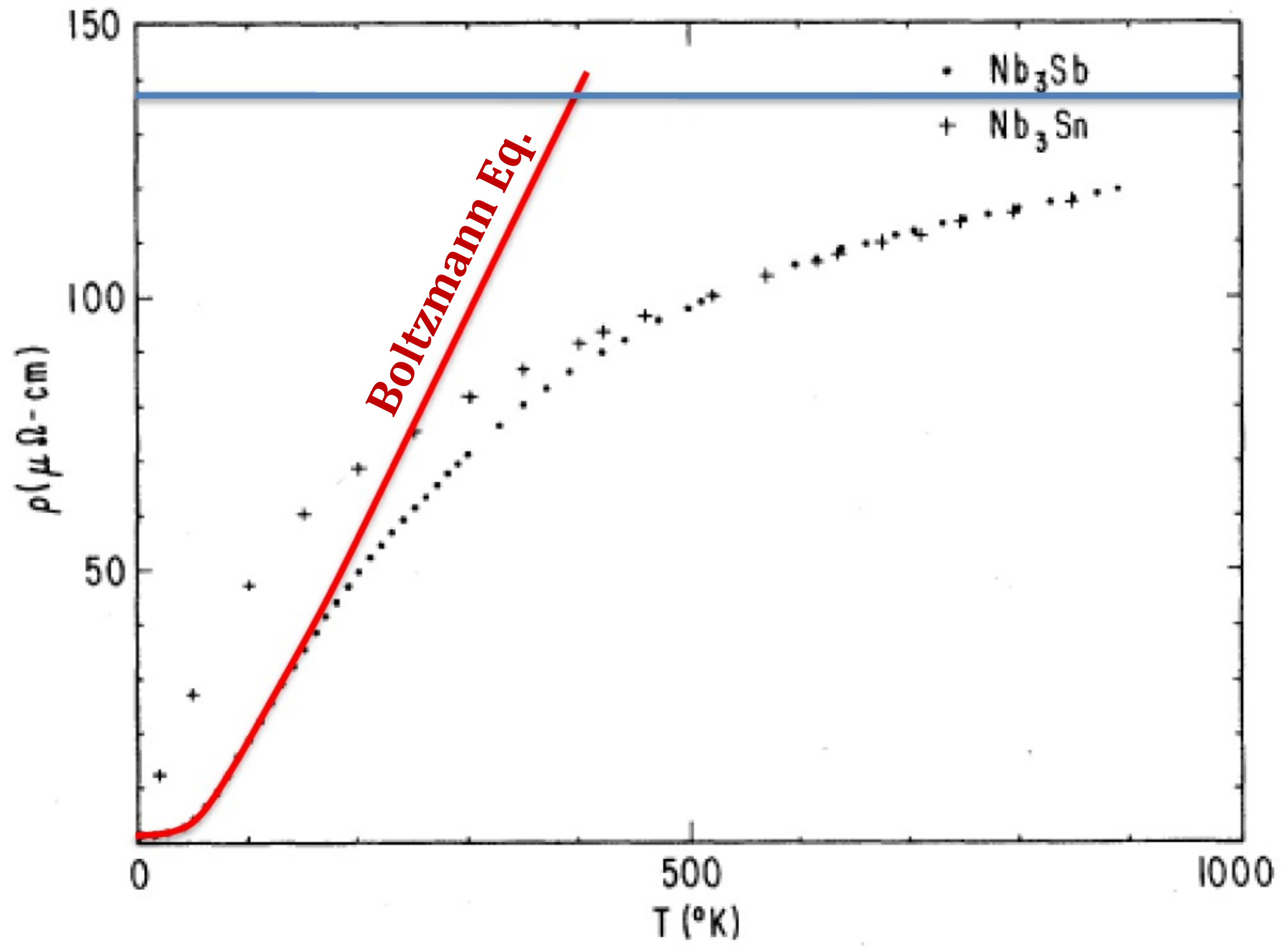
  - Bad metals and resistivity crossover from a large-N limit

  - Strange metal in $Sr_3Ru_2O_7$ and Planckian bound

Resistivity of a good metal



Resistivity saturation

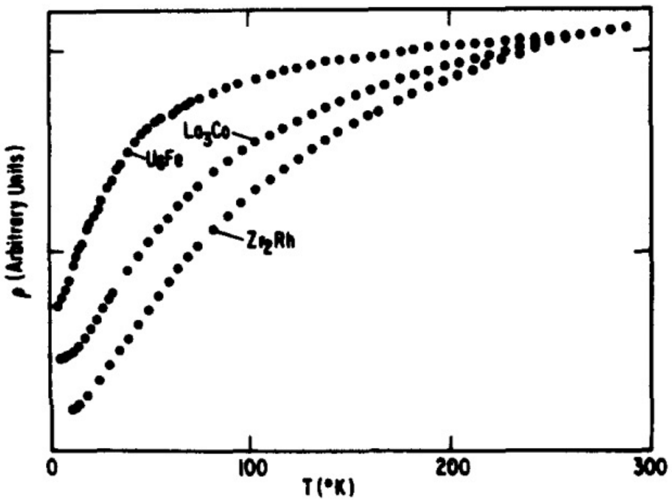


$$\rho_Q = \frac{h}{e^2} a_0$$

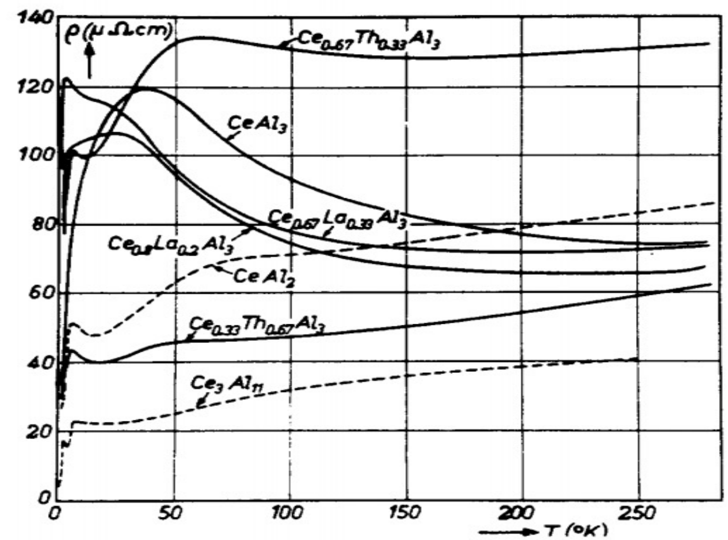
$$\ell \sim a_0 \sim a \sim k_F^{-1}$$

(Mott-Ioffe-Regel limit)

Resistivity saturation (2)



Fisk and Webb, Sol state comm. (1973)



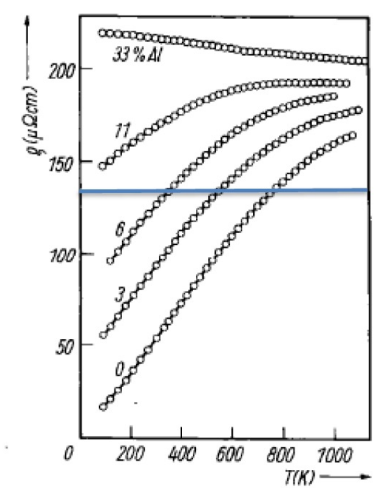
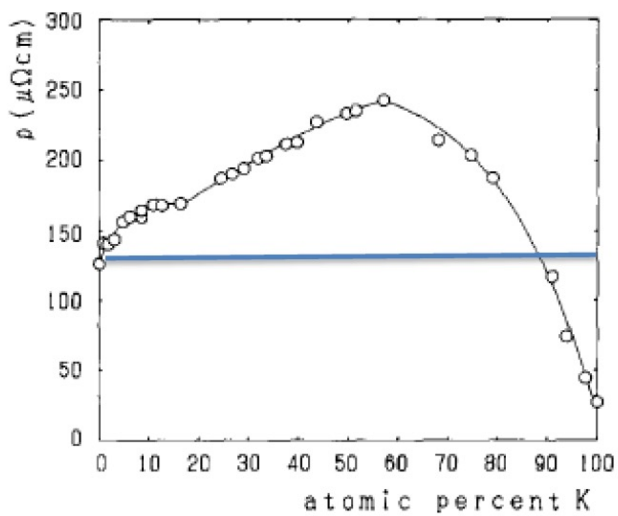


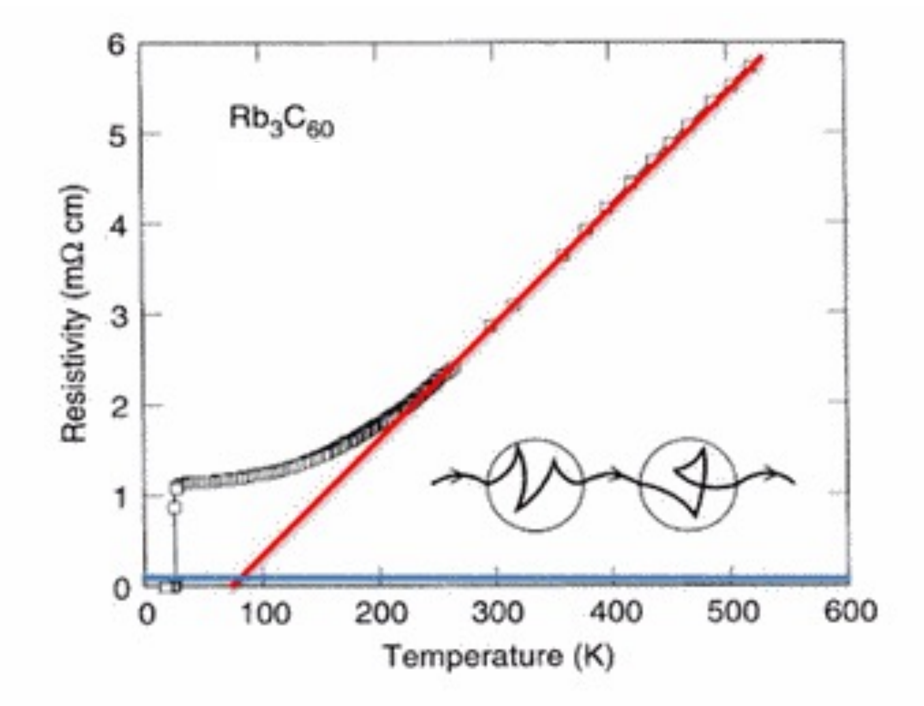
Fig. 3. Resistivity versus temperature for Ti and TiAl alloys containing 0, 3, 6, 11, and 33% Al



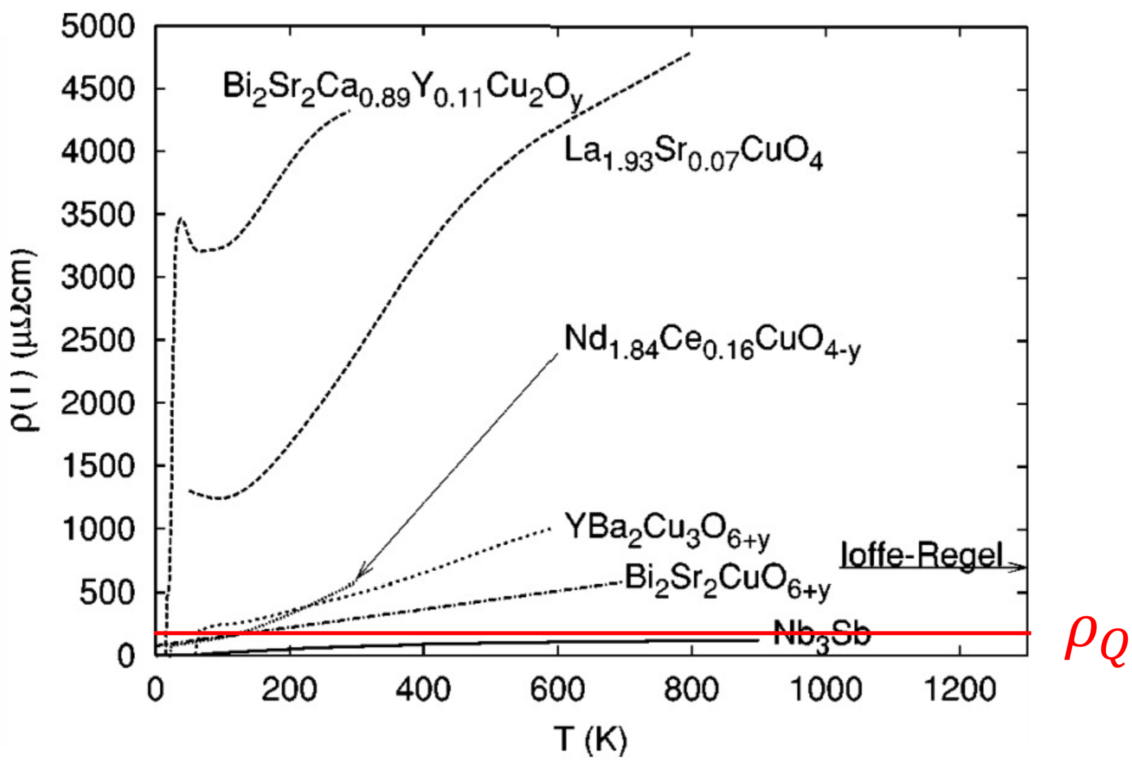
g 3. The concentration dependence of the electrical resistivity, ρ, of liquid K–Hg alloys at 573 K.

"Bad Metals"

Palstra et al. (1994)



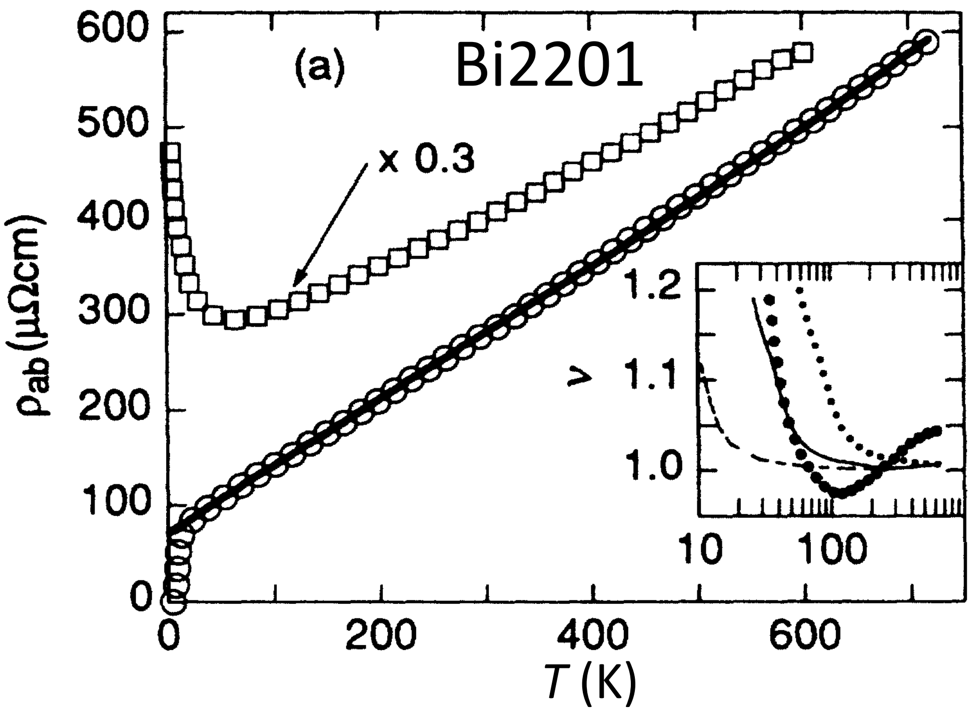
From Caladra and Gunnarson (2003)



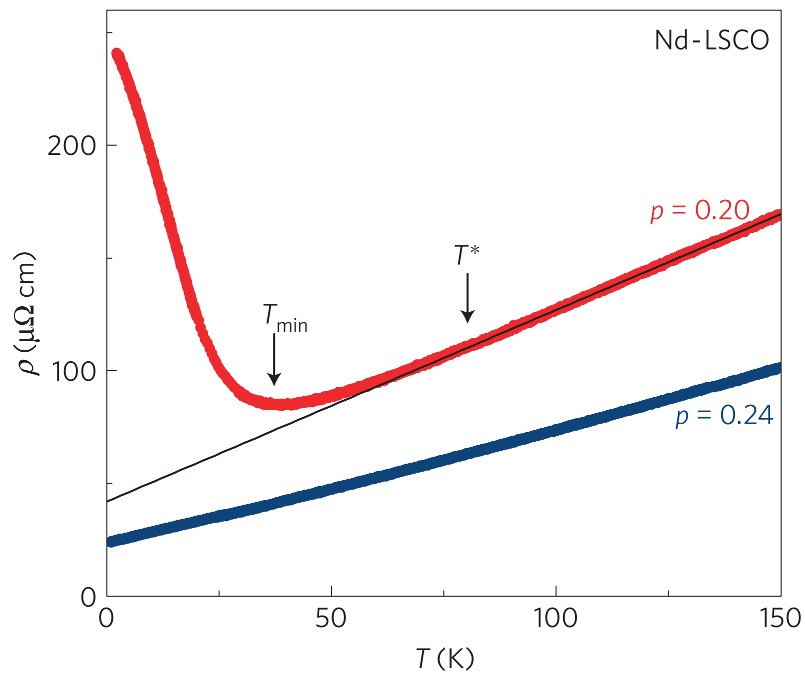
Bad metals: $\rho(T) \gtrsim \rho_Q, \frac{d\rho(T)}{dT} > 0$

Emery and Kivelson, PRL (1995)

Strange metals: Linear resistivity as $T \to 0$?



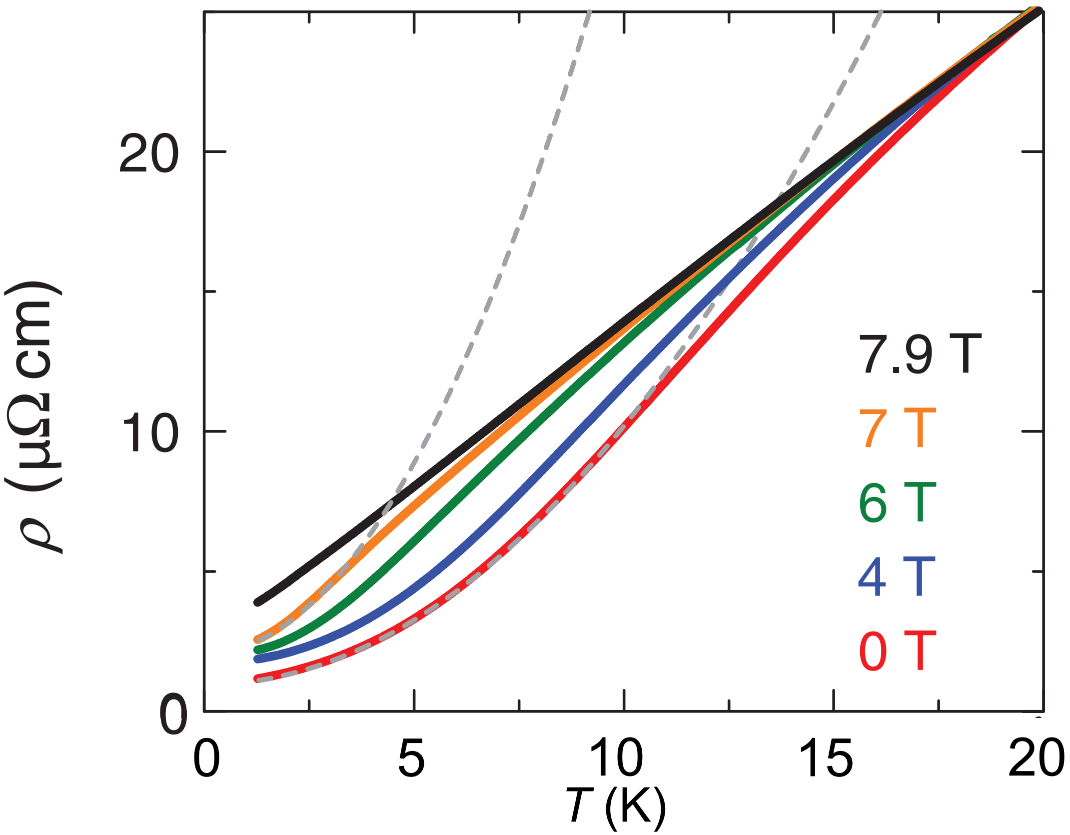
S. Martin et al. *(PRB 1990)*



Daou et al. *(Nature 2009)*

$\rho \propto T$: What sets the slope?

$Sr_3Ru_2O_7$



$$\rho = \frac{m}{e^2 n\tau}$$

(average $\frac{n}{m}$ from low T quantum oscillations)

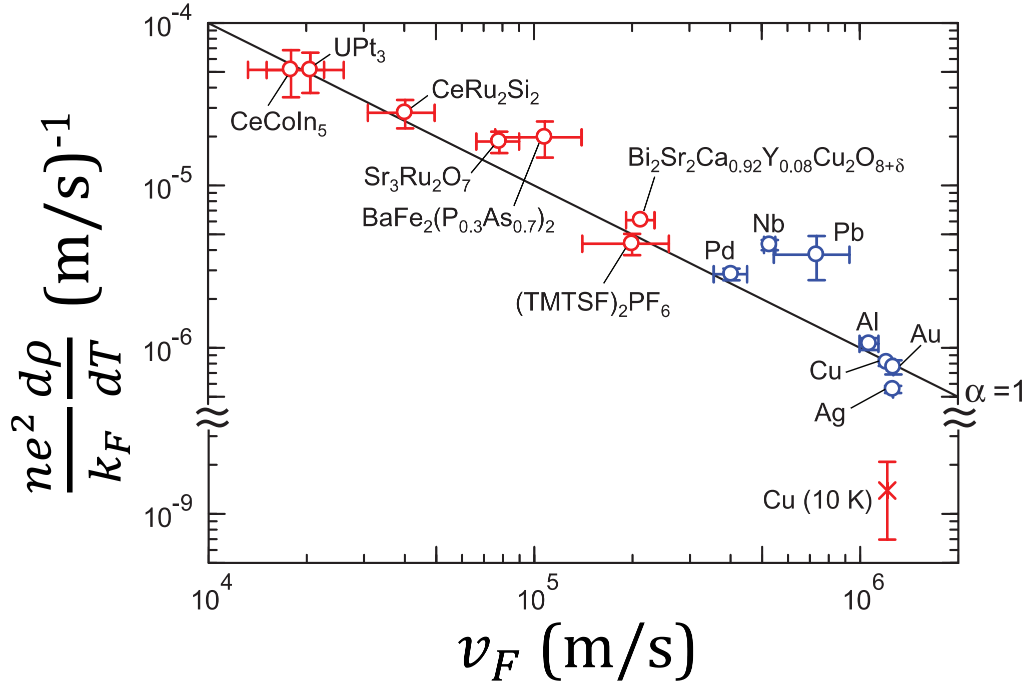
$$\frac{1}{\tau} = \frac{\alpha k_B T}{\hbar}, \alpha \approx 1.5$$

Bruin, Mackenzie et al. (2013)

$\rho \propto T$: What sets the slope?

$$\frac{d\rho}{dT}\cdot\frac{e^2 n}{k_F}=\frac{\alpha}{v_F}$$

$$\frac{1}{\tau}=\frac{\alpha k_B T}{\hbar}$$



- Cuprates (e/h doped): $\boldsymbol{\alpha = 0.7 - 1.4}$

  Legros, Taillefer, Proust et al. (2018); *Grissonnanche, Ramshaw et al. (2020)*
- $PdCrO_2$: $\boldsymbol{\alpha \approx 0.9}$

  Hicks, Mackenzie et el. (2015)
- Twisted bilayer graphene: $\boldsymbol{\alpha = 1 - 1.5}$

  Cao, Chowdhury, Jarillo-Herrero et al. (2020)

Bruin, Mackenzie et al. (2013)

"Planckian bound" on dissipation?

"Planckian Bound" $\frac{1}{\tau} \leq \frac{\alpha k_B T}{\hbar}$ with $\alpha = O(1)$

(Sachdev, Zaanen, Hartnoll,...)

Related proposed bounds:

$\frac{\eta}{s} \geq \# \frac{\hbar}{k_B}$ *(Kovtun, Son, Sarinets, 2004)*

$D_c \geq \# \frac{\hbar v_F^2}{k_B T}$ *(Hartnoll, 2015)*

- Proper definition of $\frac{1}{\tau}$?
- Electron-phonon systems at high T: $1/\tau \propto \lambda T$. Where's the bound?
- Apparent violations, e.g. e-doped cuprates at high T

Recent critique: M. Sadovskii (Physics-Uspekhi, 2021)

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Large-N electron-phonon Model

N identical electron flavors c_a

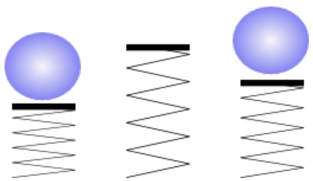
N^2 identical optical (Einstein) phonon flavors X_{ab}

$$H = \sum_{\mathbf{k},a} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}a} c_{\mathbf{k}a} + \sum_{j,a,b} \frac{P^2_{jab}}{2M} + \frac{M\omega_0^2}{2} X^2_{jab} + H_{\mathrm{int}}$$

Form of H_{int}:

$$\frac{\alpha}{\sqrt{N}} \sum_{i,a,b=1..N} X_{iab} c^\dagger_{ia} c_{ib}$$

Model 1: "Holstein"



$$\frac{\alpha}{\sqrt{N}} \sum_{i,j,a,b=1} X_{i,j;ab} c^\dagger_{ia} c_{jb}$$

Model 2: "SSH"

Energy scales

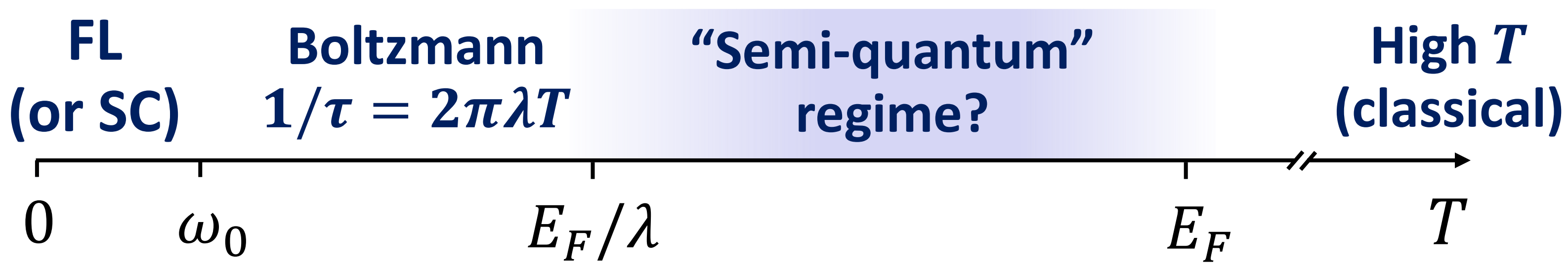
Assume $N \gg 1$: solve to leading order in $1/N$

$\mathrm{Nb_3Sn}$: 5 electronic bands, 12 phonon modes

$\mathrm{A_3C_{60}}$: 3 electronic bands, 189 phonon modes

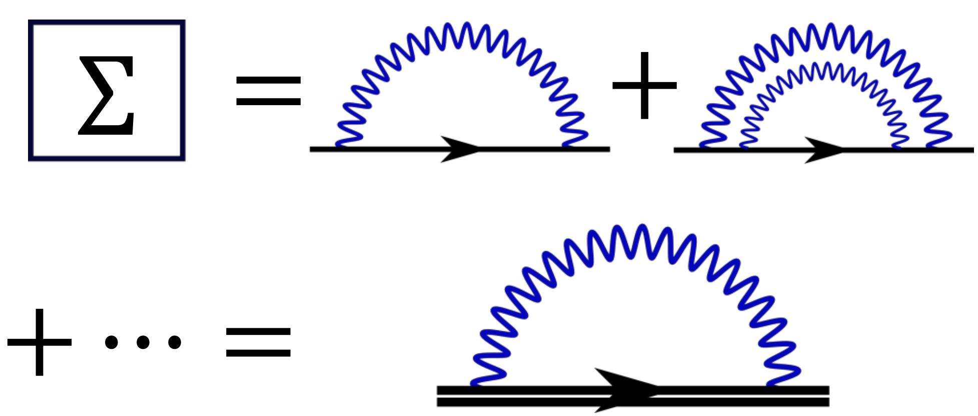
Dimensionless el-ph coupling: $\lambda = \frac{\alpha^2 \nu_0}{M\omega_0^2} > 1$ (ν_0: DOS at E_F)

Temperature regimes:

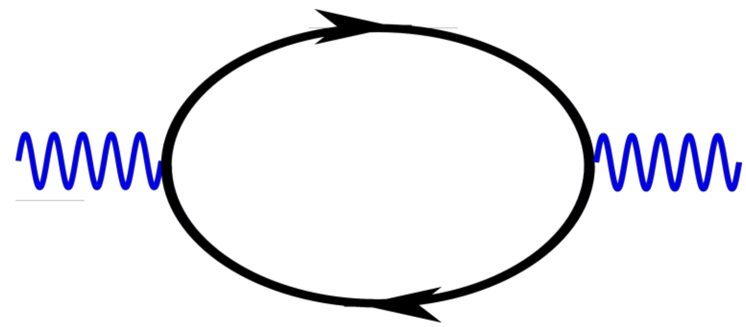


Large N limit

The electron propagator is strongly renormalized:



Phonon renormalization is subleading:

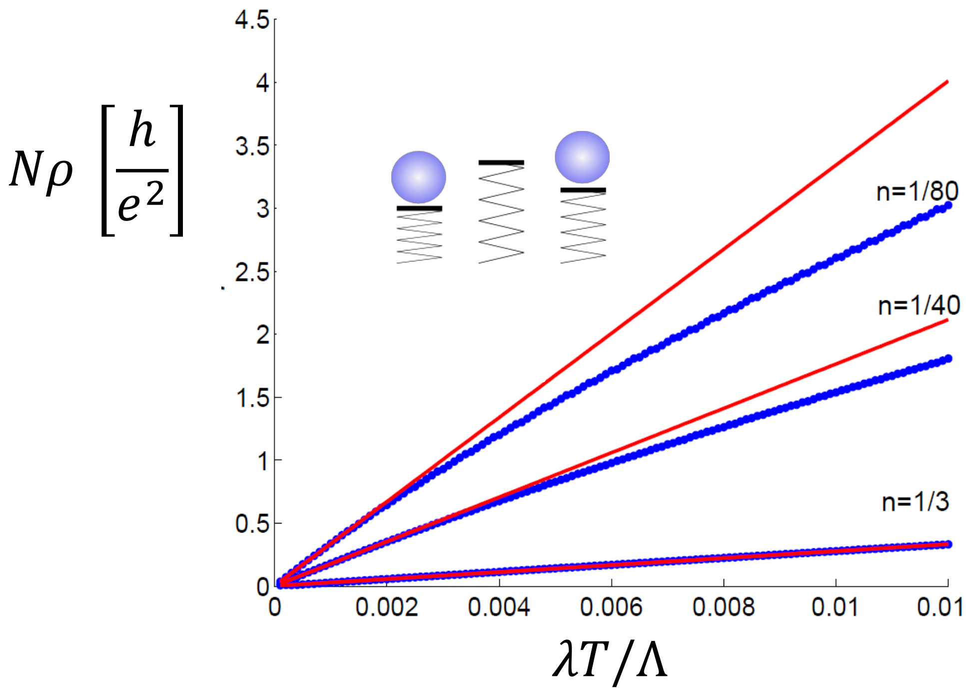


$O(1/N)$

Strong scattering of electrons, weak feedback on lattice (no lattice instability and no small polarons at large λ)

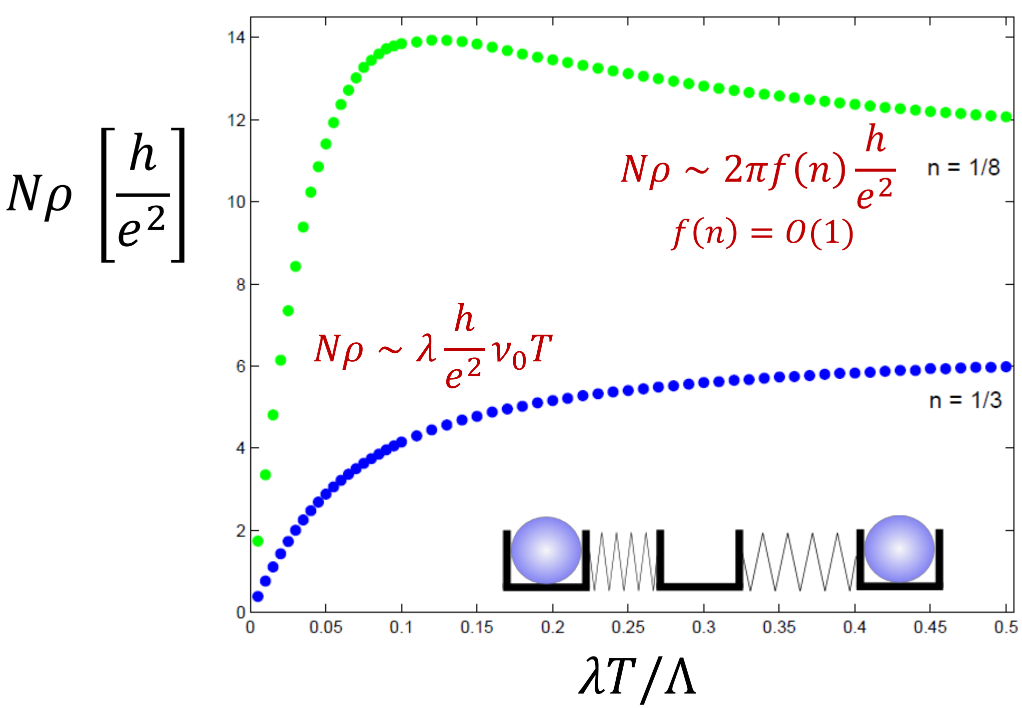
Results ($d = 2$)

Holstein



Crossover when $N\rho \sim \rho_Q$,
But no saturation. $\rho \propto T$

SSH



Resistivity Saturation!

Y. Werman and EB, PRB (2016)

Y. Werman, S. Kivelson and EB, npj Quantum Materials (2017)

Interpretation: Einstein relation

$$\sigma = \chi D$$

$\lambda T \gg \Lambda$ (= low T bandwidth):

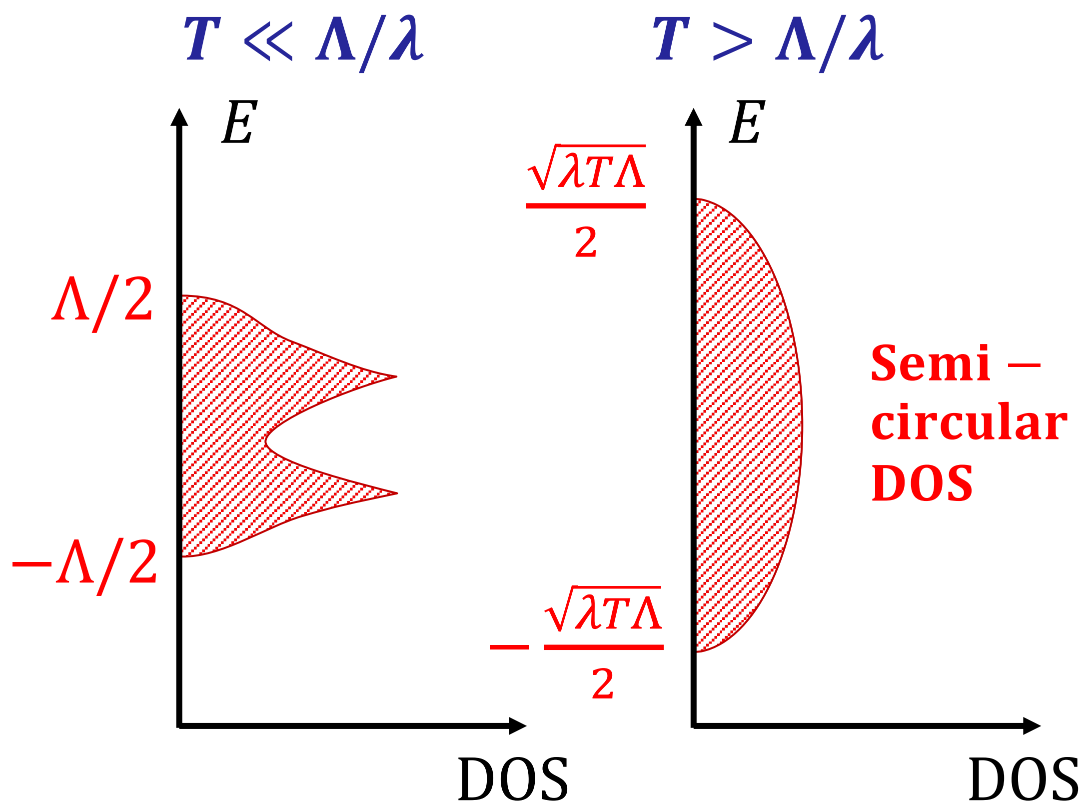
Compressibility: $\chi \sim N\sqrt{\frac{1}{\lambda T \Lambda}}$

Diffusion constant:

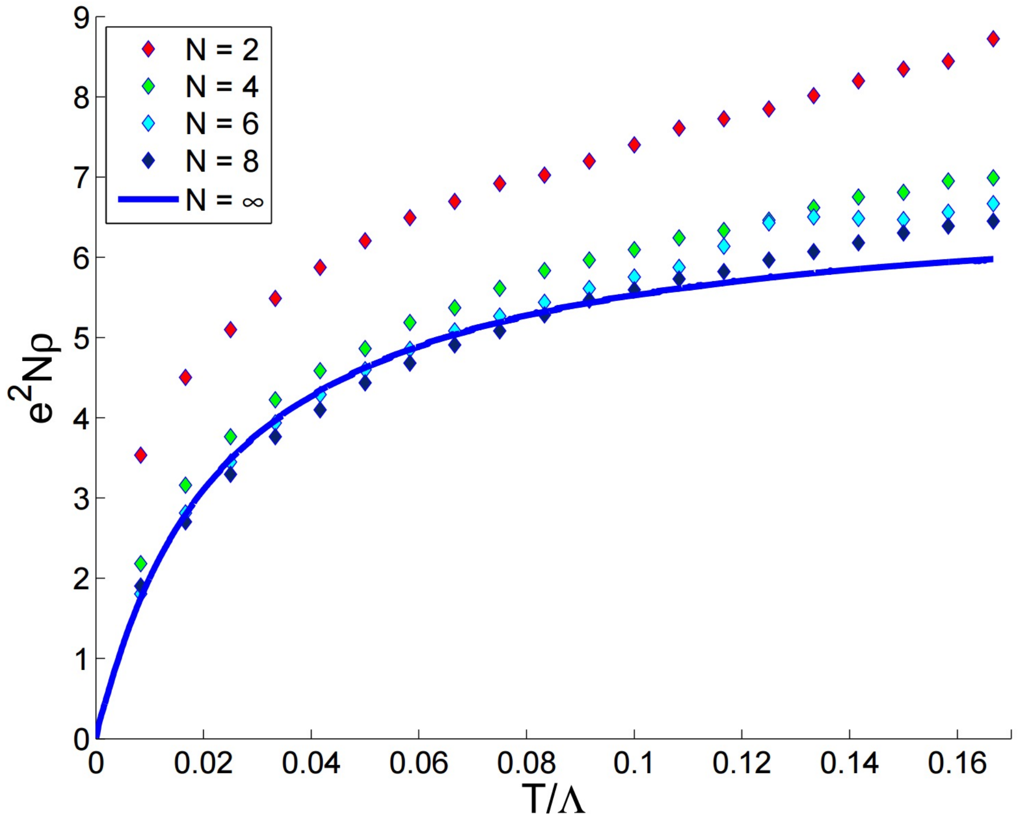
$D \sim \Lambda^2 \sqrt{\frac{1}{\lambda T \Lambda}}$ (*Holstein*)

$D \sim \sqrt{\lambda T \Lambda}$ (*SSH*)

Phonon assisted hopping



Finite N: Monte Carlo calculation
$(\omega_0 = 0)$



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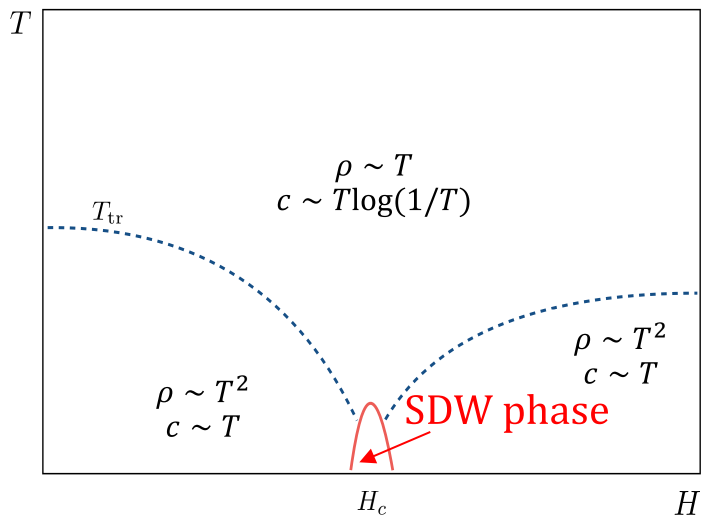
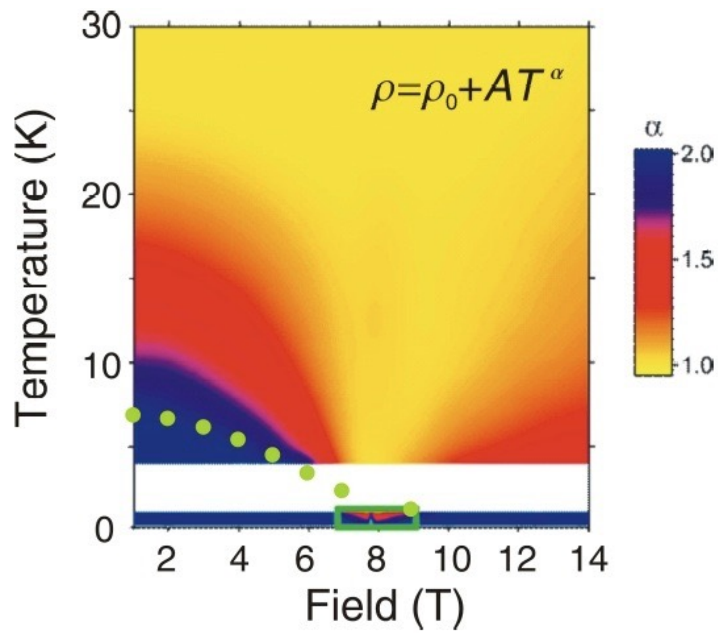
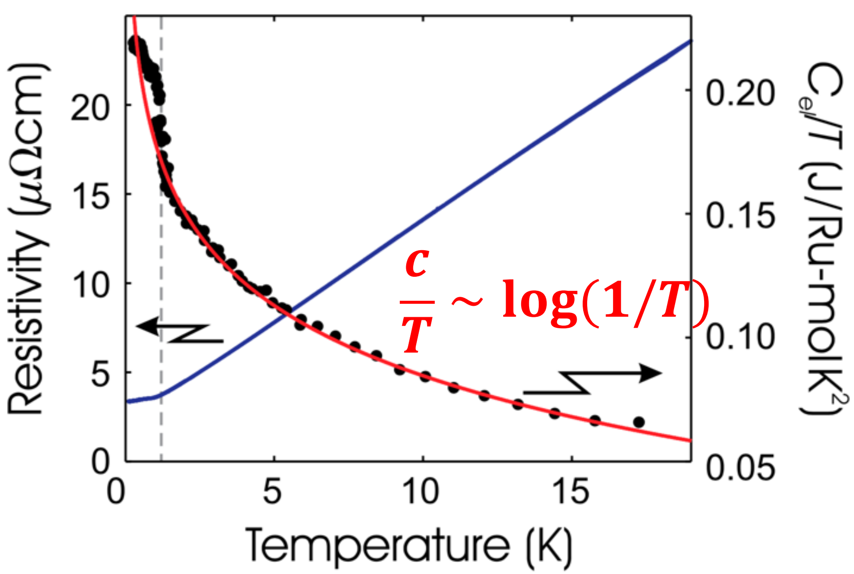
Anomalies in $Sr_3Ru_2O_7$

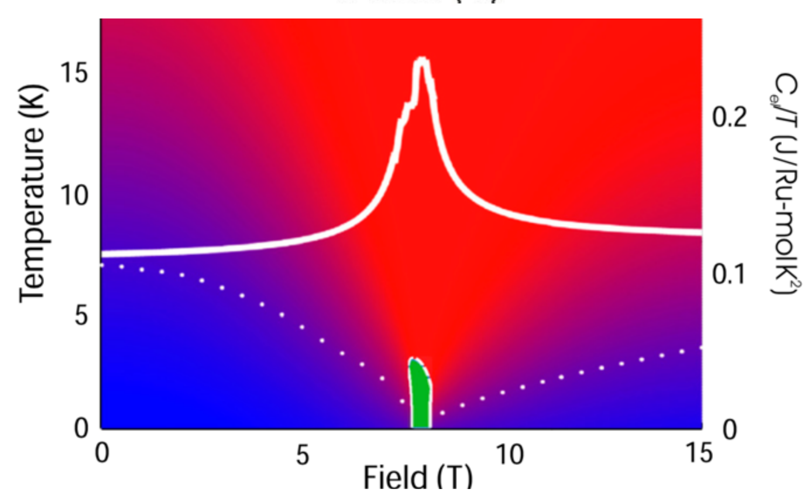
At first sight, "canonical" quantum critical behavior

Grigera et al. (2001); Rost, Mackenzie et al. (2011)

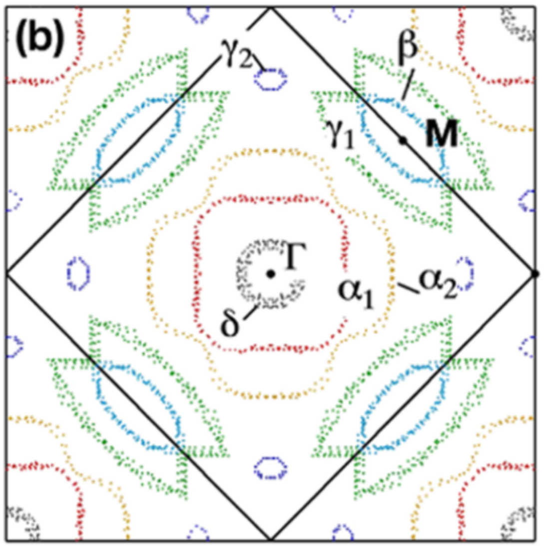




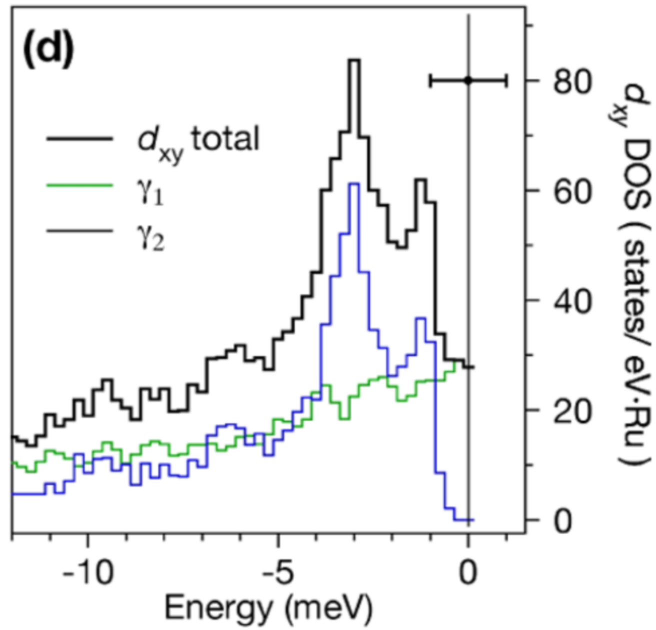




Fermiology of $Sr_3Ru_2O_7$



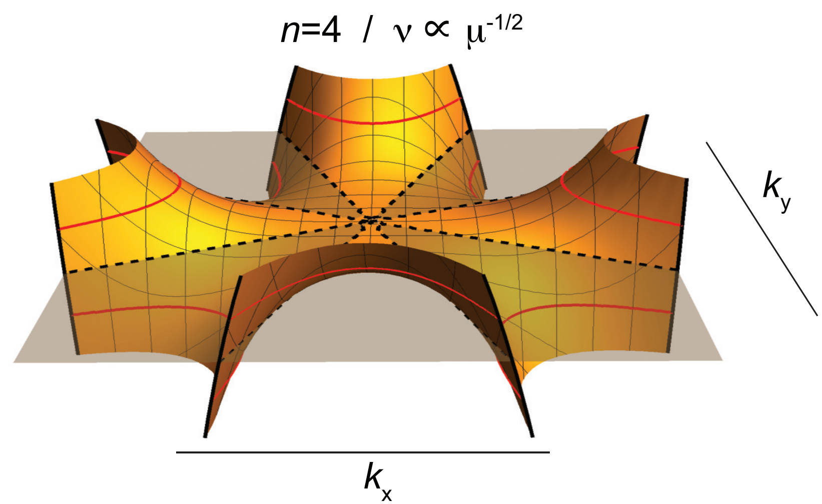
[Tamai et al PRL '08]



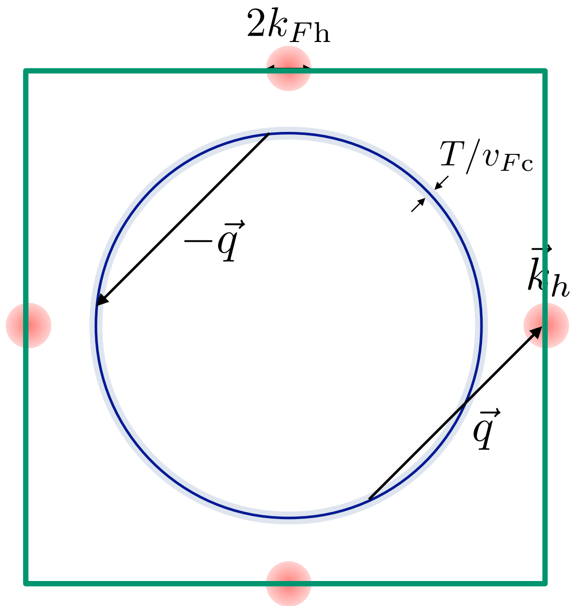
Close to multi-critical van Hove singularity!

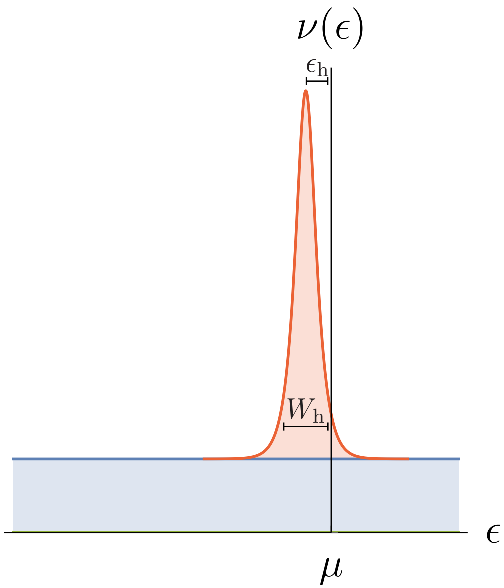
$$\varepsilon(\boldsymbol{k}) = ak^2 + bk^4\cos 4\varphi$$
$$a \approx 0$$

Efremov, Betouras et al. (2018)



"Cold" and "Hot" Fermions





Key scattering process: $cc \to ch$

For $T > max(W_h, \epsilon_h)$: $\Sigma'_c(\omega) \sim \left(\frac{k_{Fh}}{k_{Fc}}\right)^2 \omega \log(\frac{1}{\omega})$

Marginal Fermi liquid! *Varma et al. (1989)*

Scattering rate: $\frac{1}{\tau_c} \sim \left(\frac{k_{Fh}}{k_{Fc}}\right)^2 \frac{k_B T}{\hbar}$

Mousatov, EB, Hartnoll, PNAS (2020)

Approach to the critical field

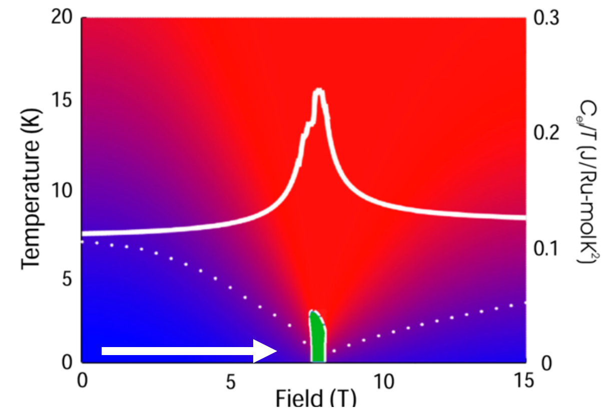
c/T and $A = \rho/T^2$ both increase as $H \to H_c$

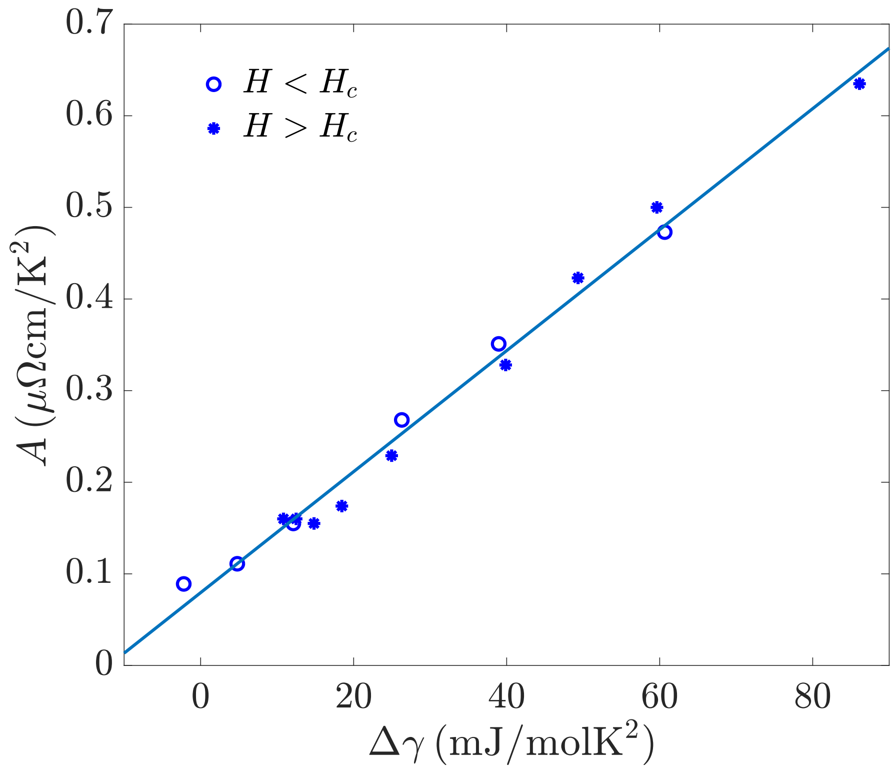
$$\rho = \frac{m_{\star c}\Gamma_c}{n_c e^2} \sim \frac{m_{\star h}}{n_c e^2}\frac{(k_B T)^2}{\hbar E_{Fc}}$$

$$\Delta\gamma \sim k_B^2 m_{\star h}$$

$$\Rightarrow \quad A \sim \Delta\gamma$$

Different from Kadowaki-Woods: $A \sim \gamma^2$





Mousatov, EB, Hartnoll, PNAS (2020)

Planckian limit?

Marginal Fermi liquid *Varma et al. (1989)*

Self-energy: $\Sigma = \Sigma' + i\Sigma''$

$$\Sigma'(\omega, T) = -\lambda\omega\log\left(\frac{\Lambda}{\max(|\omega|, T)}\right)$$

$$\Sigma''(\omega, T) = \frac{\pi\lambda}{2}\max(|\omega|, T)$$

Quasi-particle lifetime: $G^{-1}(k, \omega) = \omega - \varepsilon_k - \Sigma(\omega)$

$$\frac{1}{\tau} = \frac{\Sigma''(0, T)}{1 - \partial\Sigma'(0, T)/\partial\omega} \sim \frac{\lambda T}{1 + \lambda\log\left(\frac{\Lambda}{T}\right)} < T$$

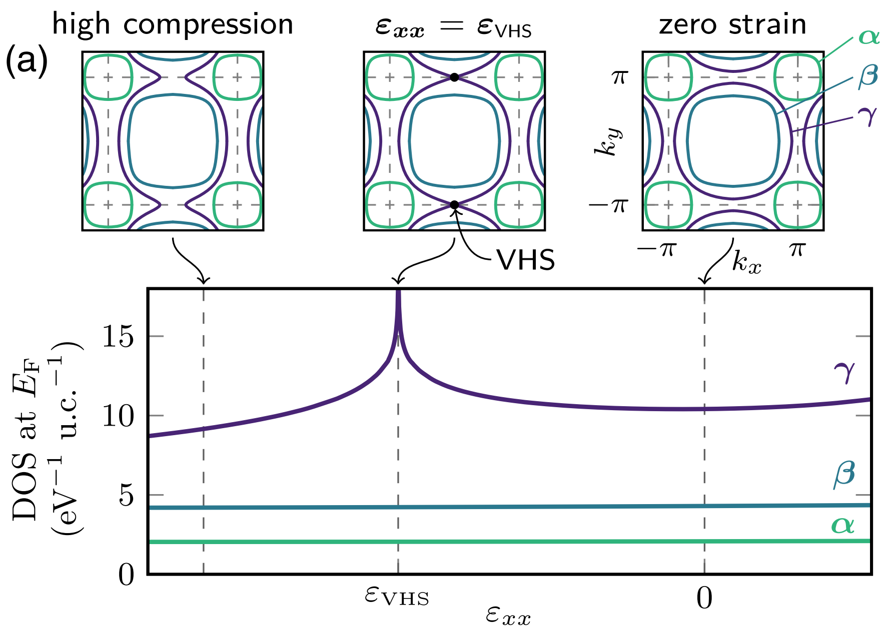
Independent of the coupling constant!

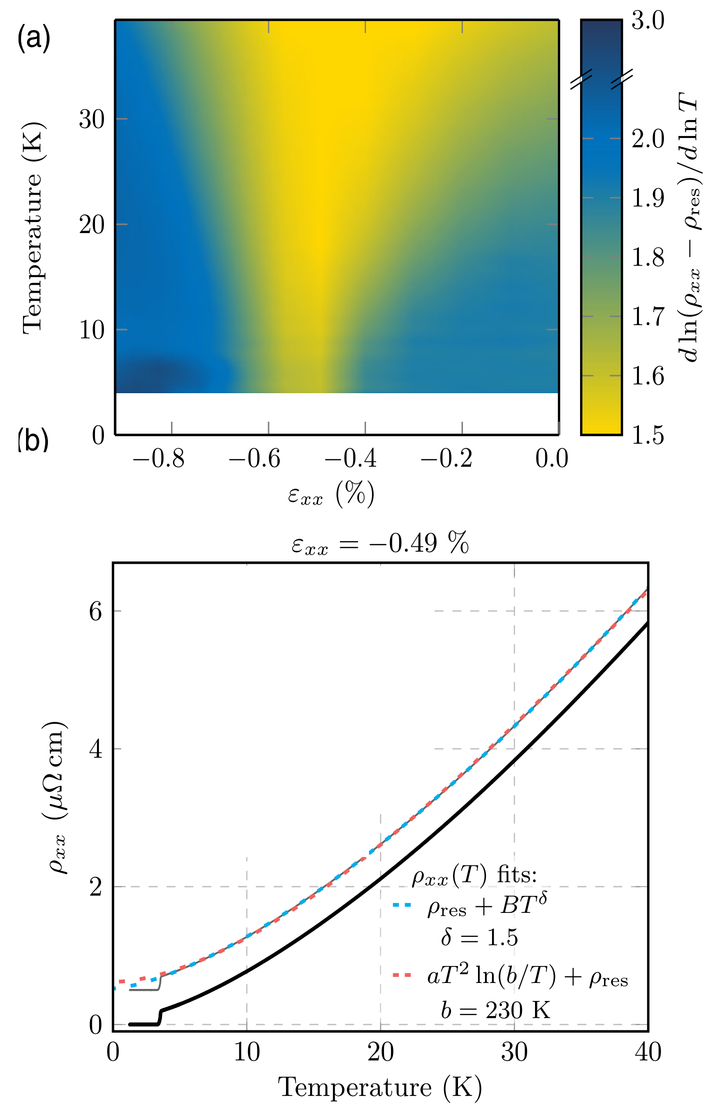
'Quantum critical' $\Sigma \sim \lambda\omega^a, a < 1$

Low T: $\boldsymbol{\frac{1}{\tau} \sim T}$, independent of λ (assuming ω/T scaling)

Turning through the van Hove singularity in Sr_2RuO_4

Barber, Hicks et al. (2018)





Resistivity consistent with $\rho \sim T^2 \log(1/T)$

Prediction: $\kappa \sim \frac{1}{\sqrt{T}}$ $\left(L = \frac{\kappa\rho}{T} \sim \sqrt{T}\log(1/T)\right)$

Stangier, EB, Schmalian, in preparation

Summary

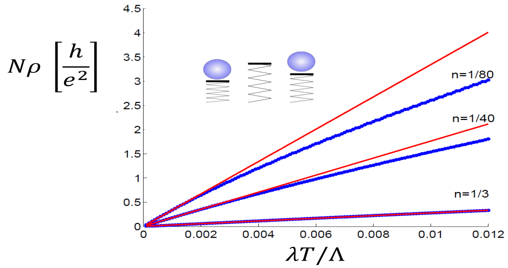
- **Bad metals** $\rho > \rho_{MIR} = \frac{h}{e^2 k_F}, \frac{d\rho}{dT} > 0$

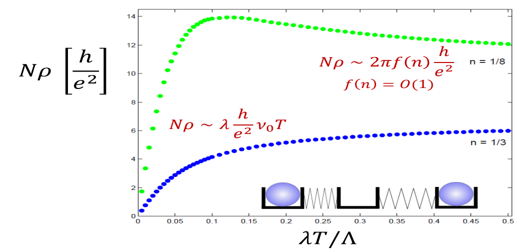
  vs. **Strange metals** $\rho \sim T^x$ at low T, $x < 2$ (often $x = 1$)

- **Lessons from solved models** (large-N):

  Resistivity crossover at $\rho \sim \rho_{MIR}$, with

  or without saturation.



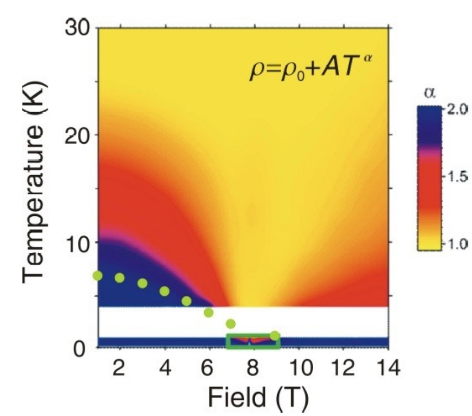


- **Planckian bound** $\frac{1}{\tau} \leq \# \frac{k_B T}{\hbar}$ **:** a useful notion,

  although much remains to be clarified.

  - Precise definition?

  - Counter-examples?



Thank you.