

# What can the Wiedemann-Franz law tell us about strange metals?

**Erez Berg**

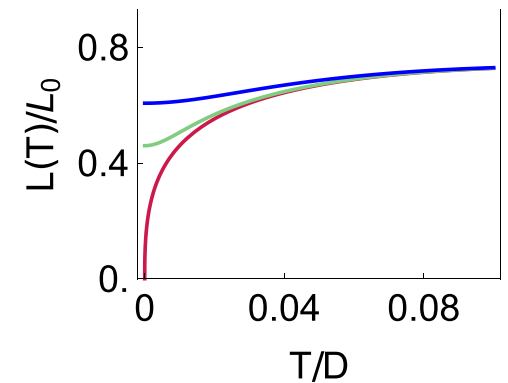
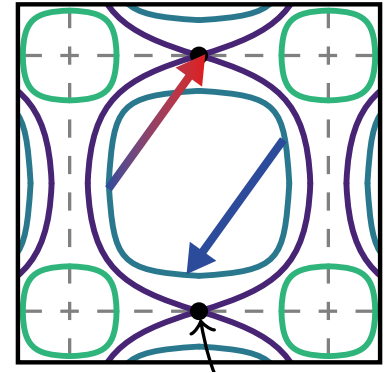
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***Thanks to: Clifford Hicks, Andy Mackenzie, Brad Ramshaw, Elina Zhakina, Steve Kivelson, Boris Spivak, Gaël Grissonnanche***



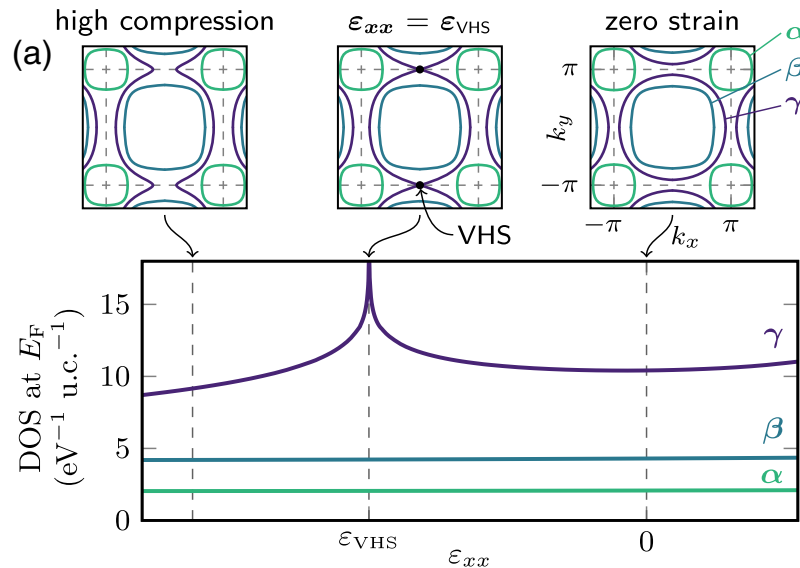
# Outline

- A (not so strange) quantum critical metal:  
Tuning to a Van Hove singularity
- Weidemann-Franz and its breakdown
- What can we learn from WF about  
scattering in strange metals?

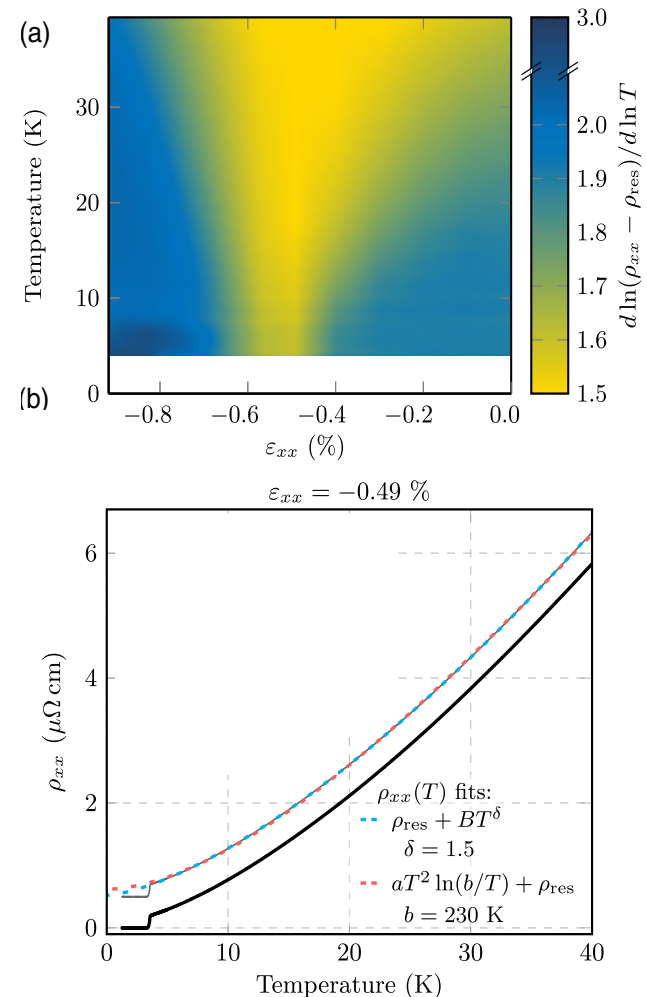


# Turning through the Van Hove singularity in $\text{Sr}_2\text{RuO}_4$

*Barber, Hicks et al. (2018)*

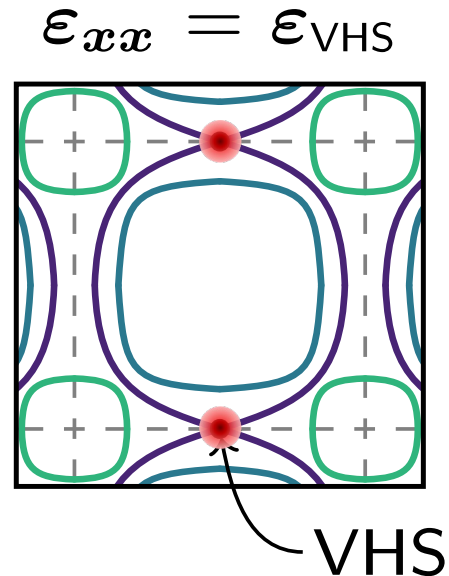


Resistivity at  $\epsilon_{\text{VHS}}$  consistent with  
 $\rho \sim T^2 \log(1/T)$



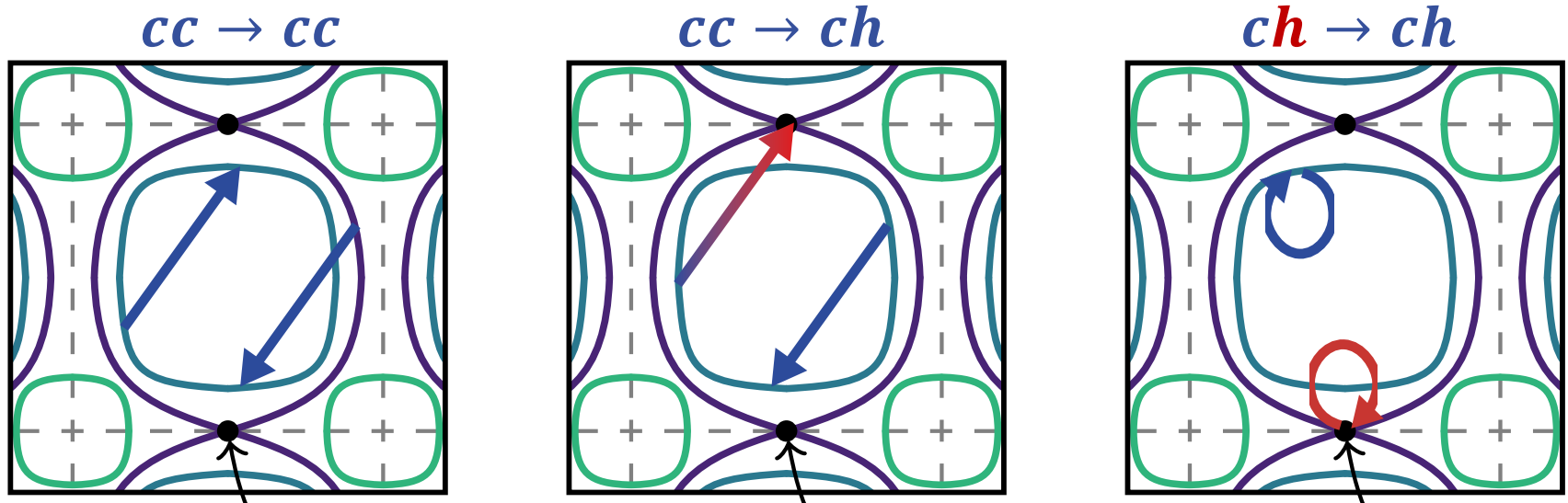
# Turning through the Van Hove singularity in $\text{Sr}_2\text{RuO}_4$

Why no “short circuiting”?



# Scattering processes: “Cold” and “Hot” fermions

Scattering processes:



$$\rho \sim T^2 \log(1/T)$$

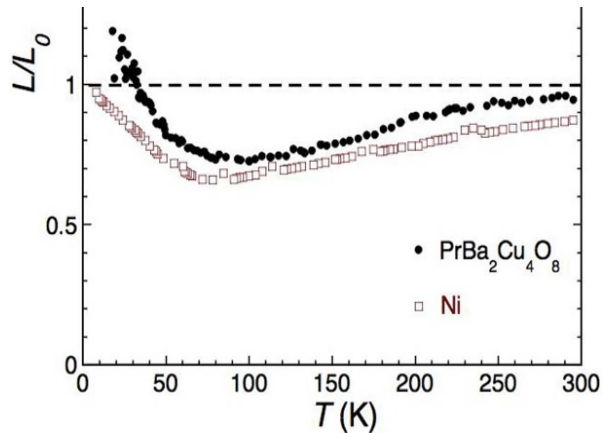
**Stronger peak in DOS gives marginal FL with  
 $\rho \propto T$ ,  $c \propto -T \log T$ : Explains  $\text{Sr}_3\text{Ru}_2\text{O}_7$ ?**

*C. Mousatov, EB, S. Hartnoll, PNAS (2020)*

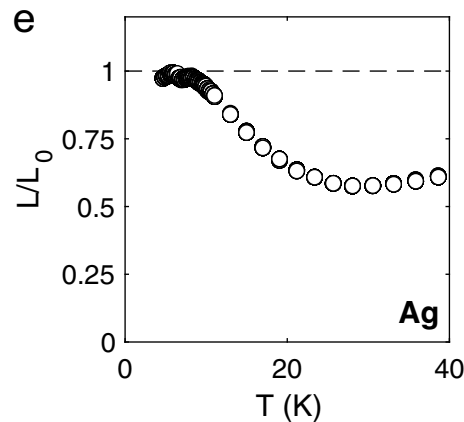
# Reminder: Wiedemann-Franz

*G. Wiedemann and R. Franz (1853)*

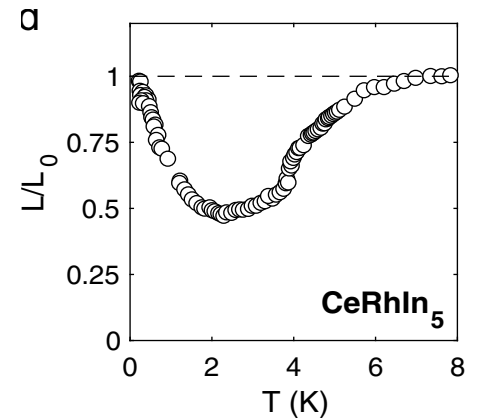
$$L = \frac{\kappa}{T\sigma} \quad L_0 = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2$$



*Bangura, Hussey et al. (13')*



*Jaoui, Behnia et al. (18')*



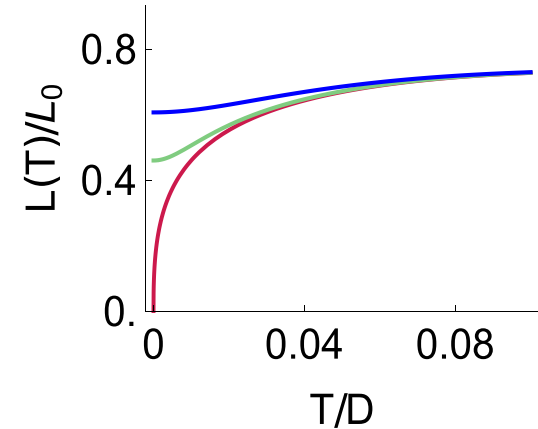
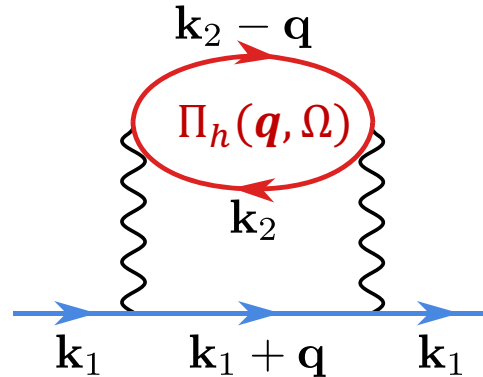
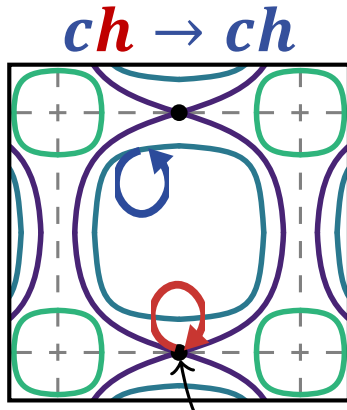
*Paglione, Canfield et al. (05')*

**$L \rightarrow L_0$  when the scattering  
is effectively *elastic***

$$J_e = e \int_{\mathbf{k}} n_{\mathbf{k}} v_{\mathbf{k}}$$

$$J_Q = \int_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \mu) n_{\mathbf{k}} v_{\mathbf{k}}$$

# Breakdown of WF Law near Van Hove singularity



$$\text{Im}\Pi_h(\mathbf{q}, \Omega) = -\frac{m}{2\pi} \min \left[ \frac{2m\omega}{|q_x^2 - q_y^2|}, 1 \right]$$

$$\frac{\kappa}{T} \sim \frac{1}{T^{3/2}} \quad \sigma \sim \frac{1}{T^2 \log(1/T)}$$

$$\Sigma''_c(\mathbf{k}, \omega) \sim \max \left[ T^{\frac{3}{2}}, \omega^{\frac{3}{2}} \right]$$

*Gopalan, Gunnarson, Andersen (92')*

$$\frac{L}{L_0} \sim \sqrt{T} \log(1/T)$$

Clean 2D FL:  $\frac{L}{L_0} \sim \frac{1}{\log(1/T)}$

*Lyakhov,  
Mishchenko (03')*

Clean FL, el-ph:  $\frac{L}{L_0} \sim T^2$

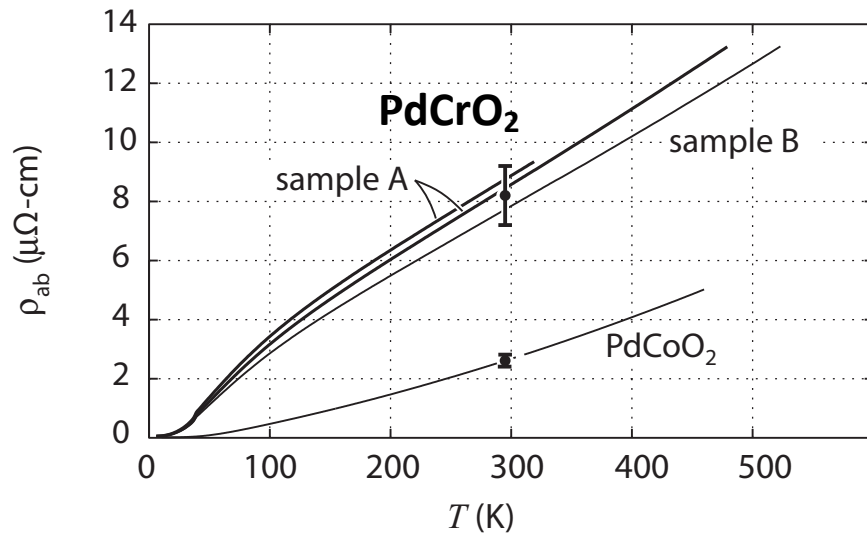
*Lavasani, Bulmash,  
Das Sarma (19')*

**Analogous violations in hydrodynamic  
and quantum critical metals**

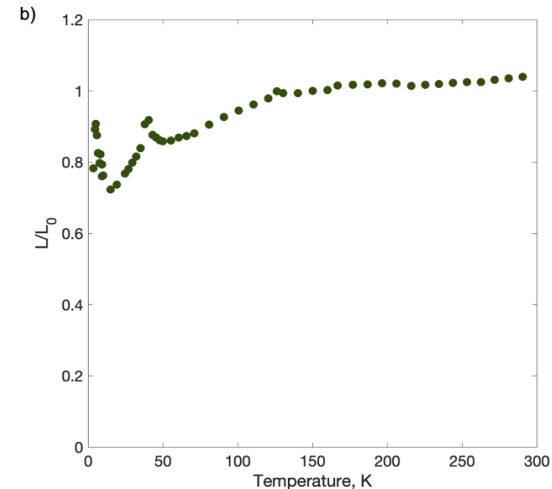
*Mahajan, Barkeshli, Hartnoll (13')*

*Veronika Stangier, EB, J. Schmalian, PRB (2022)*

# What does the Lorenz ratio tell us about strange metals?



*Hicks, Yelland et al. (15')*



*Zhakina, Mackenzie et al. (unpublished)*

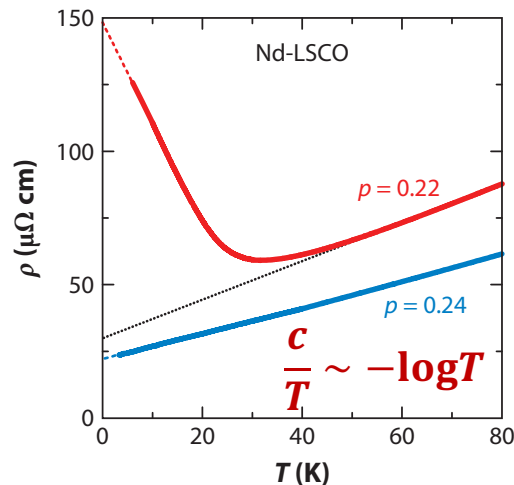
$\rho \propto T$  from **quasi-elastic** scattering (probably phonons)

Difference between  $\text{PdCrO}_2$  and  $\text{PdCoO}_2$ ? Specific heat?

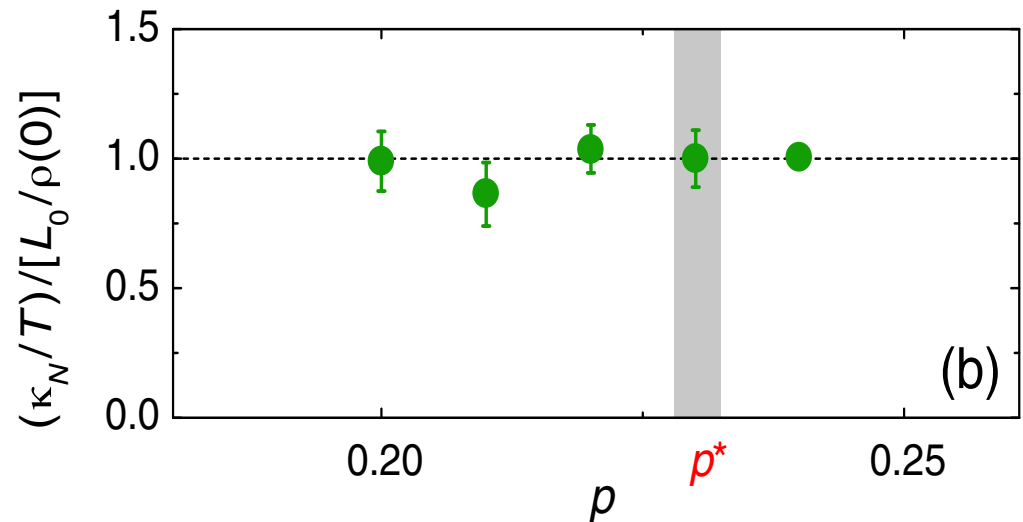
Why Planckian?



# What does the Lorenz ratio tell us about strange metals?



*Proust, Taillefer (18')*



*Michon, Taillefer et al. (18')*

For  $p \geq p^*$ , WF obeyed also by  $\sigma_{xy}/\kappa_{xy}$

*Grissonnanche, Taillefer et al. (19')*

**Strange/marginal Fermi liquid or conventional metal with strange scatterers?**

*See Kivelson talk*

# Lorenz ratio of a marginal Fermi liquid

## Marginal Fermi liquid

$$\Sigma(k, \omega) \sim \lambda \left[ \omega \log \left( \frac{\Lambda}{\omega} \right) + \frac{i\pi}{2} \max(T, \omega) \right]$$

$$\rho \propto T \quad c \propto T \log \left( \frac{1}{T} \right)$$

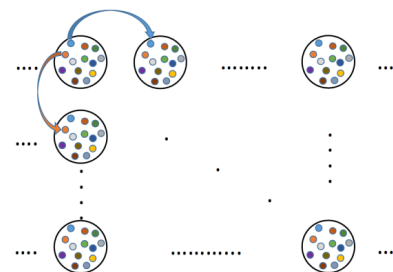
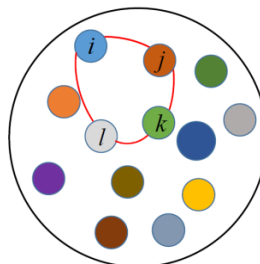
*Varma, Littlewood, Abrahams, Schmitt-Rink, Ruckenstein (89')*

**Planckian!**  $\frac{1}{\tau} \sim \frac{\Sigma''(k, \omega)}{d\Sigma'/d\omega} \sim \frac{T}{\log(\Lambda/T)}$  ( $\lambda$  independent)

***Lorenz ratio?***

# Lorenz ratio of a marginal Fermi liquid

**Model:** lattice of Sachdev-Ye-Kitaev (SYK) dots



$$H_c = \sum_{i=1, \mathbf{k}}^N \varepsilon_{\mathbf{k}} c_{\mathbf{k}i}^\dagger c_{\mathbf{k}i} + \sum_{ijkl=1, \mathbf{r}}^N \frac{U_{ijkl}}{N^{3/2}} c_{\mathbf{r}i}^\dagger c_{\mathbf{r}j}^\dagger c_{\mathbf{r}k} c_{\mathbf{r}l} + \sum_{ij=1, \mathbf{r}}^N \frac{W_{ij, \mathbf{r}}}{N^{1/2}} c_{\mathbf{r}i}^\dagger c_{\mathbf{r}j}$$

$U_{ijkl} = 0$ ,  $\overline{U_{ijkl}^2} = U^2$  **Translationally invariant** in every realization

“Kondo lattice”: Two bands  $c, f$  with bandwidths  $W_f \ll W_c$

$$H = H_c + H_f + \sum_{ijkl=1, \mathbf{r}}^N \frac{V_{ijkl}}{N^{3/2}} c_{\mathbf{r}i}^\dagger c_{\mathbf{r}j} f_{\mathbf{r}k}^\dagger f_{\mathbf{r}l}$$

*D. Chowdhury, Y. Werman, EB, T. Senthil, PRX (2018)*

*See also: A. Patel, J. McGreevy, D. Arovas, S. Sachdev, PRX (2018)*

# Lorenz ratio of a marginal Fermi liquid

Solvable in large N limit

$$\Sigma_c = \text{[red ellipse with blue line]} + \text{[dashed semi-circle with blue line]}$$

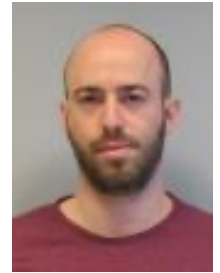
$$\Sigma_f = \text{[red ellipse with red line]} + \text{[blue ellipse with red line]}$$

$$\sigma(\Omega, T) = \frac{1}{\Omega} \left[ \text{[blue loop with wavy lines]} + \text{[blue loop with red ellipse and wavy lines]} + \dots \right]$$

*D. Chowdhury, Y. Werman, EB, T. Senthil, PRX (2018)*

*See also: A. Patel, J. McGreevy, D. Arovas, S. Sachdev, PRX (2018)*

# Lorenz ratio of a marginal Fermi liquid



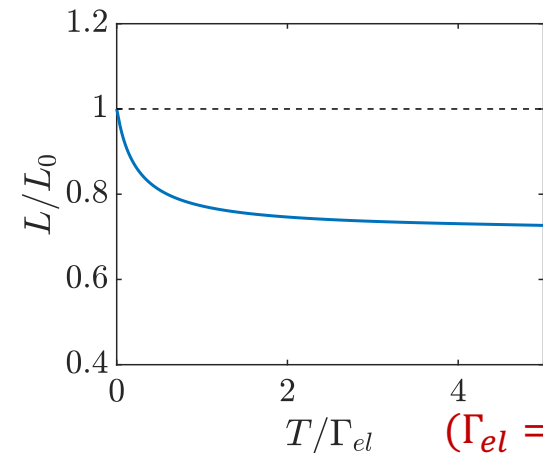
Evyatar Tulipman

**Clean limit ( $w = 0$ ):**

$$\rho(T) = AT$$

$$\frac{L}{L_0} = 0.71306 \dots$$

*Patel, McGreevy, Arovas, Sachdev (18')*



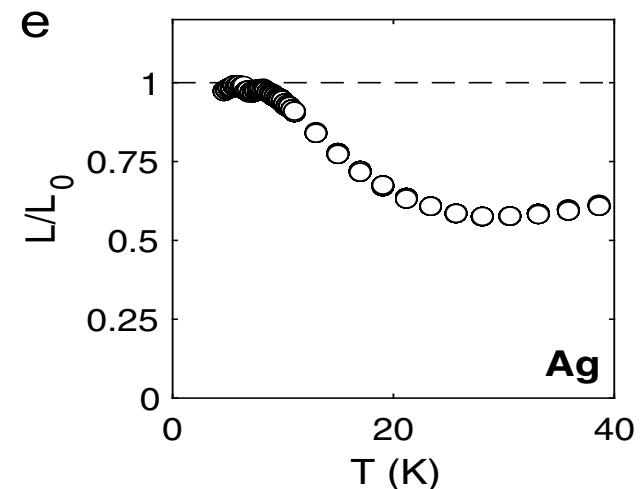
$$(\Gamma_{el} = \pi v_0 |w|^2)$$

**Disordered ( $w > 0$ ):**

$$\rho(T) = \rho_0 + AT$$

$$\frac{L}{L_0} = 1 - \alpha T + \dots$$

**Sharp signature of MFL!**

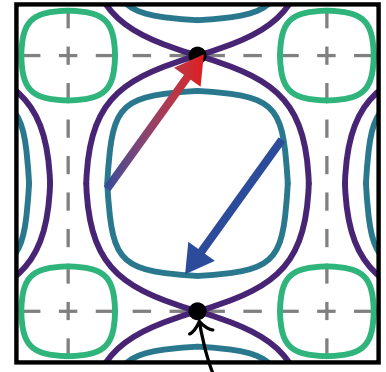


# Summary

The Wiedemann-Franz law and its violation can serve as a probe for the nature of scattering

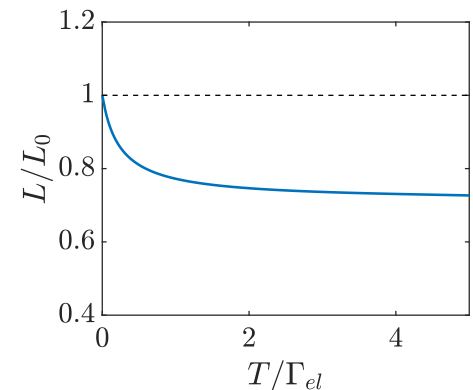
- Clean metal near a VHS

$$\frac{L}{L_0} \sim \sqrt{T} \log(1/T)$$



- Disordered marginal Fermi liquid

$$\frac{L}{L_0} = 1 - \alpha T + \dots$$



Thank you!