Study of conductance fluctuations (1/fα noise) in metallic nanowires

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ABSTRACT

We have studied the conductance fluctuations in silver nanowires in the temperature range 4K to 375K. The nanowires with an average diameter of 15nm were electrochemically deposited using polycarbonate membrane as template. Principal motivation is to study low frequency defect relaxations in the nanowires that give rise to conductance fluctuations with a spectral power \( S(f) \propto 1/f^\alpha \). The Ag nanowires, stabilized at 400K with a current of few mA, show metallic temperature dependence. The \( S(f) \) was measured with a pseudo 4 probe ac technique with rms current of few tens of \( \mu \)A. We find that \( S_V(f) \) (which is \( \propto 1/f^\alpha \)) shows a rapid rise at around 220K as \( T \) is increased along with an enhancement in the exponent \( \alpha \). The exponent \( \alpha \approx 1 - 1.1 \) for \( T < 220 \) and it increases to \( \approx 1.4 \) at \( T = 375K \). In the same temperature range \( S(f) \) rises by an order of magnitude.

We analyze the data using a model assuming that there are two components to the 1/fα fluctuations - one arising from relaxation of local defects gives \( \alpha \approx 1 \). The other arises from the long-range diffusion of defects characterized by \( \alpha \approx 3/2 \). It is seen that for \( T < 220K \) the noise arises mainly from local defect relaxation and the temperature dependence of \( \alpha \) follows the Dutta-Horn model. Above this temperature the contribution from long-range diffusion dominates with the noise becoming thermally activated with an activation energy (\( \sim 300meV \)). Interestingly the activation energy is similar to but somewhat higher than that seen in micron sized films.

1. INTRODUCTION

The magnitude of conduction noise in nanowires is an important issue in studying the feasibility of using them in VLSI and ULSI circuits as interconnects. This can be appreciated if we do a rough calculation to estimate the conduction noise in a nano system. Assume we have a sample which is a cube of side 10nm. The conduction noise in this sample can be estimated using the empirical Hooge’s formula

\[
\frac{\langle \Delta R^2 \rangle}{R^2} = \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{S_V(f)}{V^2} df
\]

(1)

where

\[
S_V f = \frac{\gamma_H V^2}{N f}
\]

(2)

Taking the value of \( \gamma_H \) to be \( \approx 10^{-3} \) as is typical of a bulk metal and the bandwidth of operation to be from 1Hz to 1GHz, we get \( \sqrt{\frac{\langle \Delta R^2 \rangle}{R^2}} \approx 1.5\% \). For 1nm cube this value comes out to be \( \approx 5\% \) which sets the limit to best signal to noise ratio one can get in a practical device having such nanowires as components.

Another important factor that should be studied in these systems is their stability against damage when stressed thermally or by using an electric field. Low frequency conductivity noise with spectral power \( S_V(f) \sim 1/f^\alpha \) has been investigated as a tool to detect early electromigration (em) failure of metal films and wires.1–5 To our knowledge there has been only one study done on the stability of nanowires against electromigration damage.6 These authors had studied the critical current (\( J_{\text{fail}} \)) required for failure in gold wires of various diameters ranging from 600nm down to 10nm. They find that as the wire diameter is decreased \( J_{\text{fail}} \) remains more or less constant till about 30nm below which it starts decreasing. We would like to explore the reason for this decrease of stability of nanowires below a certain diameter and we use noise spectroscopy to provide an answer to this question.

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The spectral power of noise often can provide certain information on the existence of pathways that can give rise to long range diffusion leading to electromigration (em).\textsuperscript{7} We explain this as follows. Generally, an equilibrium defect fluctuation model like Dutta-Horn model\textsuperscript{9,10} is used to explain the observed $1/f$ spectra in metals. In this model the defect fluctuation is due to “localized” motion with a certain time scale $\tau$. A properly chosen energy distribution of the defects can give rise to $1/f$ noise. No long-range diffusion is associated with this defect motion. However, during electromigration or in a material which is damaged by em, the atomic motion leads to long-range diffusion. The underlying atomic process is distinct from the type of localized motion envisaged in a DH type model. Hence, there is no reason that the localized equilibrium defect fluctuations as envisaged in the DH model should also describe the dynamics of defects that give rise to long-range atomic transport caused by electromigration. In fact, it is natural to expect that the power spectrum should contain some signature of long-range diffusion in the form of resistance fluctuations with spectral power $S_V(f) \sim 1/f^{3/2}$. We argue that such a $1/f^{3/2}$ spectral power should be present in films with electromigration damage because the em damage opens up pathways that would support long-range diffusion of atoms. We have proved the existence of such a noise in Ag films of microscopic dimensions.\textsuperscript{7} We would like to use the same tool here to study em damage of nanowires and their stability.

Our investigations were done on silver nanowires 15nm in diameter. Resistivity and noise measurements were done in the temperature range 4.2K to 350K. The main observation that we make here is that there is a temperature $T^*$ beyond which the spectral power increases exponentially as a function of temperature and for $T \geq T^*$, $S_V(f)$ acquires a contribution from a term $\propto 1/f^{3/2}$. Existence of such a term signals that for $T \geq T^*$ there is onset of long range diffusion in the system which would make the nanowire unstable towards electromigration or thermal stress.

2. EXPERIMENTAL TECHNIQUES

The silver nanowires of approximate diameter 15nm were grown within polymeric templates (etched track membranes) using electrochemical deposition from silver salt. The wires used in this work have typical length $\approx 60 \mu$m. The membrane has an aerial pore density of $\approx 10^{10}/\text{cm}^2$. We carried out the noise measurement by retaining the wires within the polymeric membrane and attaching leads at the two ends using silver epoxy. Thus our measurement is a pseudo-4-probe measurement. The set up used for noise measurements is shown in figure 1.\textsuperscript{8} We have used a transformer preamplifier SR554 to couple the sample to the lock-in-amplifier. The carrier frequency was chosen to lie in the eye of the Noise Figure (NF) of the transformer or the preamplifier to minimize the contribution of the transformer noise to the background noise. The output of the preamplifier is fed to a

![Figure 1. The noise measurement setup](image)
Lock-in-amplifier (SR830). The output low-pass filter of the Lock-in amplifier has been set at 3 msec with a roll off of 24 dB/octave. For a 3 msec time constant the output filter of the lock-in with 24 dB/octave is flat to $f \leq 10$Hz. This determines the upper limit of our spectral range. The output of the Lock-in amplifier is sampled by a 16 BIT A/D card and stored in the computer. At each temperature the data are taken by stabilizing the temperature with $\Delta T/T \approx 4 \times 10^{-3}\%$. A single set of data are acquired typically for a time period of about fifty minutes or more at a sampling rate of 1024 points/sec. The complete data set of a time series at each temperature consisting of nearly 3 million points was decimated to about 0.1 million points before the spectral power $S_V(f)$ is determined numerically. The frequency range probed by us ranges from 1 mHz to 10 Hz which allows us to probe time scales of the order of $15$ msec to nearly 160 seconds. The frequency range is determined mostly by practical considerations. The lower frequency limit is determined by the quality of the temperature control. The sample resistance and the bridge output may show a long time drift. In general such a long time drift is subtracted out by a least square fit to the data. Taking these factors into consideration the lower spectral limit in our experiment has been kept at $10^{-3}$Hz.

3. RESULTS

The as grown wires were often not stable and their resistance was seen to drift as a function of time and current. As the wires were "trained" by passing a large dc current ($\sim 1$mA) through them for a few hours at somewhat higher temperatures (450K) the resistance came down by about 3-4 orders of magnitude. This is shown in figure 2. The wires thus trained were found to be very stable with time. They are also very reproducible on thermal cycling below the training temperature 450K. The sample resistance was measured using an ac bridge from 4.2K to 300K. The data is shown in figure 3. The $dR/dT$ was seen to be fairly linear down to 10K with a slight upturn below 10K. The value of $\alpha = \frac{dR}{dT} \sim 4 \times 10^{-3}/K$ was very close to the value seen for bulk silver. The residual resistivity ratio (defined as $\rho_{300K}/\rho_{4.2K}$) $\sim 3$ which is close to what one finds in silver films. The system can thus be characterized as weakly disordered.

During noise measurement the maximum bias used was always kept much lower than the bias that had been used to train the sample. This ensured that the sample resistance does not drift during the measurement. The noise magnitude was seen to scale as the square of the bias temperature range $T > 60K$ as shown in figure 4. (Note that noise scaling as the square of the applied bias does not ensure that the measured noise is coming from the sample and not from contacts. To check this rigorously one has to check the dependence of the measured noise on the sample size. This was not possible for us to do. But we removed and put the contacts several times and in each case the noise data was reproducible. This makes us reasonably confident that the measured noise is indeed coming from the sample and is not contact noise.)
In figure 5(a) we plot the relative variance of the conductance fluctuation $\frac{\delta G^2}{G^2} \equiv \frac{1}{V^2} \int_{f_{\text{min}}}^{f_{\text{max}}} S_V(f) df$ as a function of temperature down to 100K. In this paper we are focusing only in the region $T > 100K$. The relative variance of conductance fluctuations shows a peak at a temperature $T = 220K$. We define the peak temperature as $T^*$. $T^*$ divides the noise data into two temperature regions. As the temperature is increased from 100 K to 220K the noise magnitude increases by more than an order of magnitude. The relative variance for $T > T^*$ shows a shallow shoulder before beginning to rise again as the temperature is raised. This rise above 300K shows

**Figure 3.** The resistance of the sample as a function of temperature. The value of $\alpha = \frac{1}{R} \frac{dR}{dT}$ was $4 \times 10^{-3}$/K and the residual resistivity ratio was 3

**Figure 4.** The spectral power density of conductance fluctuations plotted as a function of the square for the bias applied across the sample.
an activated behaviour with an activation energy of $\sim 300$ meV. Here we would like to compare the data obtained in the nanowire with that measured for a macroscopic low noise silver film as shown in figure 5(b). In the case of the silver film there also exists a temperature $T^* \simeq 325 K$. This peak is not as sharp as in the case of nanowire but is clearly present.

In the lower temperature range ($100 K < T < 220 K$) the noise spectral power density is seen to be $1/f^{\alpha}$ in nature with the exponent $\alpha$ (defined as $-\delta S_{V}(f) / \delta f$) very close to 1. For temperatures above $T^*$ one can see the deviation from $1/f$ behaviour and the value of $\alpha$ changes gradually from 1 to $\sim 3/2$. This is shown in figure 6.

Figure 5. The relative variance of the conductance fluctuation $\frac{\delta G^2}{G^2}$ as a function of temperature for (a) silver nanowire (15 nm diameter, 60 µm long) and (b) silver film (12 µm × 15 mm × 900 nm).

Figure 6. The plot of the conductance fluctuation as a function of temperature at four representative frequencies.
Figure 7. The value of \( \alpha \) calculated from the noise power spectrum using DH model (shown by open circles) as well as the measured value of \( \alpha = \frac{-\delta S(V)}{\delta f} \) (shown by filled circles) as a function of temperature.

where we plot the noise as a function of temperature at four representative frequencies. We have plotted the data as \( f.S(V)(f)/V^2 \) to accentuate the deviation from 1/f dependence. We also note that such a departure of \( \alpha \) from 1 to higher values occurs for films for \( T > 350K \).

4. DISCUSSION

DH model of 1/f noise allows us to calculate the value of \( \alpha(T) \) as a function of temperature from the measured temperature dependence of the noise spectral power \( S(V)(f) \) using the relation:

\[
\alpha(T) = [1 - \frac{1}{\omega \tau_0} \left( \frac{\delta \ln S}{\delta \ln T} - 1 \right)]
\]  

(3)

This value of \( \alpha \) calculated from the noise power spectrum using DH model we call \( \alpha_{DH} \). If the predominant mechanism giving rise to noise in the system is activated defect relaxation with a distribution of the relaxation times, then the measured value of \( \alpha \) should be close to \( \alpha_{DH} \). It is seen that in our system the measured value of \( \alpha \) matches reasonably well with \( \alpha_{DH} \) for \( T < T^* \). However, \( \alpha \) deviates significantly from \( \alpha_{DH} \) for \( T > T^* \) as shown in figure 7. This shows that the noise in this temperature range has an component in addition to the one arising due the relaxation of local defects.

To explain the observation above \( T^* \) we use the following model. We assume that in the temperature range \( T > T^* \) the noise arises from two independent processes. The first process is due to relaxation of localized defects which persists at all temperatures. This type of noise can be explained quite adequately in the framework of DH model with \( S(V)(f) \propto 1/f \). The relative variance of the conductance fluctuations arising from these kind of processes we call \( \frac{\Delta G^2}{G^2} \). The second process arises due to the long range diffusion, the relative variance of which we call \( \frac{\Delta G^2}{G^2} \). The spectral power density of this kind of process is characterized by \( S(V)(f) \propto 1/f^{3/2} \).

Thus we write the observed spectral power as

\[
S(V)(f) = \frac{A}{f} + \frac{B}{f^{3/2}}
\]  

(4)
Figure 8. The contribution of the $1/f$ component and that of the $1/f^3/2$ component to the relative variance of conductance fluctuations. The measured relative variance of conductance fluctuations is also plotted in the same graph.

and,

$$\frac{\langle \Delta G^2 \rangle}{G^2}_A = \frac{1}{V^2} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{Adf}{f}$$  \hspace{1cm} (5)$$

$$\frac{\langle \Delta G^2 \rangle}{G^2}_B = \frac{1}{V^2} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{Bdf}{f^{3/2}}$$  \hspace{1cm} (6)$$

so the total relative variance is

$$\frac{\langle \Delta G^2 \rangle}{G^2} = \frac{\langle \Delta G^2 \rangle}{G^2}_A + \frac{\langle \Delta G^2 \rangle}{G^2}_B$$  \hspace{1cm} (7)$$

Figure 8 shows the contribution of the $1/f$ component and that of the $1/f^3/2$ component to the relative variance of conductance fluctuations. The measured variance is also plotted in the same graph. It is seen that for $T < T^*$ the contribution of $(\langle \Delta G^2 \rangle_{\text{obs}})_B$ to the observed noise is negligibly small, the entire noise contribution is coming form the relaxation of local defects as quantified by $(\langle \Delta G^2 \rangle_A)$. As the temperature is raised beyond $T^*$ the contribution of $(\langle \Delta G^2 \rangle_{\text{obs}})_B$ increases by more than 2 orders of magnitude to become equal to the contribution of $(\langle \Delta G^2 \rangle_A)$. This shows that for temperatures above $T^* = 220K$ there are comparable contributions from both processes. The appearance of long range diffusion for $T > T^*$ would make it unstable towards thermal, stress or electromigration damage. We feel that the instability seen in nanowires of diameter $< 100nm$ arises because the temperature $T^*$ ( where the instability sets in) is below 300K. In contrast in microns Ag films $T^* \approx 325K$. As a result such instability does not occur at room temperature. A more detailed study to investigate the dependence of $T^*$ on the diameter of the wire is under progress.

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