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Quantum shot noise

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In this chapter we review recent developments in both theoretical and experimental aspects of quantum shot noise. We discuss the properties of shot noise in systems of various sizes from mesoscopic, ballistic, systems to classical diffusive conductors. In particular we consider the effect of the elctron's Fermi statistics on shot noise. In addision, we discuss shot noise as a tool for charge measurements. The Fractional Quantum Hall system which gives rise to an appearance of new elementary excitations which carry a smaller charge, hence resulting in a suppressed shot noise.

Key words: shot noise, Fermi statistics, ballistic transport, quantum point contact, diffusive conductors, Fractional Quantum Hall effect, fractional charge.

1. Introduction

Shot noise refers to time-dependent fluctuations in an electrical current. This is a direct consequence of the *particle*-like nature of the electron, namely, the electron's *discreteness* and the stochastic nature of its transport through the conductor [1]. Shot noise generally provides information not necessarily given by the time-averaged current which, according to the Landauer formula, probes the transmission of the electron wave. We are going to address here only shot noise and not the vast field of electronic noise generated by various sources, such as conductance fluctuations, recharging of traps, and other sources of $1/f$ noise.

In truly stochastic (Poisson-like) electron emission processes, the time average, or ensemble average, of the squared current fluctuations, $\langle(\Delta i)^2\rangle_{\Delta f}$, measured in a frequency range Δf, is given by the classical *shot noise* expression [1]:

$$\langle(\Delta I)^2\rangle_{\Delta f} = S_i(f)\cdot\Delta f = 2Ie\cdot\Delta f, \tag{1}$$

where $S_i(f)$ the *white* spectral density and I the average (DC) current. In contrast to electron emission devices, where electrons are injected thermally or via tunnelling, in a classical conductor, where electrons scatter many times and thermalize, shot noise is not observed [2–4]. In mesoscopic conductors, however, where electrons do not scatter or scatter only elastically, shot noise is suppressed. The reason for this suppression is fundamental and results from the fermionic nature of the electrons. Contrary to this, light intensity fluctuations are strong

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and difficult to suppress, reflecting the fact that light is also carried by indivisible quanta, the photons, which obey Bose–Einstein statistics. Driven by practical applications the field of quantum optics was developed in the 1970s to address the problem of ‘quiet light’. Shot noise in mesoscopic systems, on the other hand, is a new field. Only recently, following intense theoretical activity [5–7], have experiments revealed the nature of shot noise in ballistic and diffusive conductors [8–11]. Moreover, recent observations of a fractional charge in the Fractional Quantum Hall (FQH) regime [12, 13] have proven that shot noise measurements can indeed provide information not easily accessible via conductance measurements. It is quite possible that in 20 years time quantum conductors will find realistic applications, hence push shot noise measurements, that provide microscopic and time-dependent information on electrical transport, to continuously evolve and be more common in spite of the difficulties in conducting the experiments.

2. Shot noise in a ballistic conductor

2.1. Mesoscopic conductance

The conductance of a sample whose size is larger than the coherence length of electrons, l_ϕ, is well understood [14, 15]. On a length scale smaller than l_ϕ the interference of partial electronic waves, that sample the full extent of the conductor, means that it is no longer possible to define a ‘local conductivity’. Then the whole conductor has to be considered as a single quantum system, with a conductance calculated by the ratio between the net current flow to the electrochemical potential differences between the leads immediately adjacent to the coherent domain. This approach was first proposed by Rolf Landauer [16] in 1957 for one-dimensional conductors and became a most significant concept in mesoscopic physics. Landauer suggested considering the leads as black body sources which incoherently emit carriers toward the coherent domain and perfectly absorb carriers which are elastically scattered by it. Sample conductance is then proportional to the sum of transmission probabilities, t_n, through all possible channels at the fermi energy:

$$G = 2e^2/h \sum_n t_n. \tag{2}$$

This relation, and its extension to geometries with multiple contacts [17], is in very good agreement with experiments when interactions between the electrons can be neglected. This approach was successfully utilized to explain Universal Conductance Fluctuations in diffusive conductors, Aharonov–Bohm conductance oscillations in conducting rings, conductance quantization in ballistic Quantum Point Contacts, thermal transport coefficients, and conductance quantization in the Quantum Hall regime.

2.2. Noise

The simplest example of current fluctuations is Johnson–Nyquist or thermal noise, which exists in any conductor with conductance G at finite temperature T. The zero-frequency spectral density of current fluctuations is given by: $S_{\mathrm{th}}(0) = 4k_{\mathrm{B}}TG$. This equilibrium noise originates from microscopic current fluctuations (the DC current is zero) due to the finite temperature and is independent of the electronic charge. When current flows, current fluctuations, due to charge granularity and the stochastic nature of the transport, may arise. W. Schottky [1], in 1918, considering noise in a vacuum diode, had already recognized that thermionic emission of electrons over the barrier in the heated cathode has a very low probability, t. Electron emission thus obeys a poissonian statistics with resultant current fluctuations. These fluctuations in the number of electrons, coined shot noise, is given by the well known poissonian expression $\langle \delta n^2 \rangle = \langle n \rangle$, with spectral density of current fluctuations $S_i = 2eI$. This noise, essentially a *partition noise*, is a direct result of the small transmission probability through the barrier.

In mesoscopic systems both noise sources, thermal and shot, coexist. The first calculations of current *fluctuations* in mesoscopic conductors were made, for the simplest mesoscopic system—a ballistic Quantum Point Contact (QPC)—by Khlus [5] and independently later by Lesovik [6] and Yurke and Kockenski [7], essentially utilizing the Landauer approach. Generally, current fluctuations in a QPC result both from 'emission noise' of the reservoirs in thermal equilibrium (thermal noise) and from the randomness of the particles' transmission through the barrier (partition noise).

Let us consider partition noise in the simplest example of particles either transmitted ($p_i = 1$) through a barrier with transmission probability t, ($t = \langle p_i \rangle$), or reflected by it ($p_i = 0$) with reflection probability $1-t$. Suppose there are n_i independent particles attempting to pass the barrier. The distribution of transmitted particles is binomial with an ensemble average $\langle n_t \rangle = n_i t$ and

$$\langle n_t^2 \rangle = n_i t(1-t) + n_i^2 t^2. \tag{3}$$

The ensemble average of the squared *fluctuations* of the transmitted particles with $\delta n_t \equiv n_t - \langle n_t \rangle$, is:

$$\langle \delta n_t^2 \rangle = \langle n_t^2 \rangle - \langle n_t \rangle^2 = n_i t(1-t). \tag{4}$$

This is true for all particles, being distinguishable or not, being fermions or bosons, as long as their incoming flux does not fluctuate in time. For such noiseless incoming flux the fluctuations of the outgoing particles are sub-poissonian, namely, their binomial fluctuations are smaller by $(1-t)$ from the poissonian limit; eventually vanishing for unity transmission ($t = 1$).

Under the condition of a noiseless incoming flux, the fluctuations in the number of transmitted electrons, given by eqn (4), can be expressed via the zero-frequency spectral density, $S_i = 2\langle (e\delta n_t)^2 \rangle / \Delta\tau$, with $e\delta n_t$ the charge fluctuation from the average transferred charge $e\langle n_t \rangle$ transmitted during time $\Delta\tau$:

$$S_i = \frac{2e^2 \langle \delta n_t^2 \rangle}{\Delta\tau} = \frac{2e\langle n_t \rangle (1-t)}{\Delta\tau} = 2eI(1-t). \tag{5}$$

Equation (5) is actually the result obtained by Khlus [5] and Lesovik [6] for a QPC conductor, showing clear suppression of shot noise by $(1-t)$. In the general case, finite temperature introduces fluctuations in the number of incident particles from the reservoirs. The above described noise, associated with partition, has thus to be combined with the inherent noise in the incident flux. The noise in the incoming flux, in turn results from the equilibrium thermal fluctuations in the emitting reservoir and thus strongly depends on the quantum statistics of the incident particles [16]. In the reservoirs, the fluctuations in the occupation number of particles in a particular state α are given by [18]:

$$\langle \delta n_\alpha^2 \rangle = \langle n_\alpha \rangle (1 \pm \langle n_\alpha \rangle), \tag{6}$$

where $\langle n_\alpha \rangle$ is the average equilibrium occupation of particles in incident state α. The plus sign is for bosons in *thermal equilibrium* for which the fluctuations are super poissonian and grow with decreasing temperature. The minus sign is for fermions, reflecting the binomial Fermi–Dirac distribution with sub-poissonian fluctuations, vanishing at zero temperature. Substituting n_α for n_i and combining eqn (3) with eqn (6), assuming all n_α are not correlated, one obtains an expression accounting for both *partition* and *thermal* fluctuations in the particles' flux, assuming transmission only from one side:

$$\langle \delta n_t^2 \rangle = \sum_\alpha n_\alpha t(1 \pm n_\alpha t), \tag{7}$$

where the sum is over all incident states. For fermions at zero temperature the occupation number for incident particles is 1 and eqn (7) reduces to eqn (4) with sub poissonian noise. This unique *quantum suppressed* shot noise for fermions is sometimes called Quantum Shot Noise (QSN). In constrast to this, boson occupation numbers at low temperatures can be large and consequently the fluctuations in their flux will also be large, giving rise to a super-poissonian noise. It is worth noting that a more general expression can be found for particles with anyonic (fractional) statistics [19, 20] interpolating between bosons and fermions. Fractionally charged quasiparticles, recently detected utilizing shot noise measurements (as will be detailed later) are expected to obey such an exotic statistics.

The above discussion neglected electrons which are injected backward from the other reservoir. In fact at low applied voltages one has to take into account carriers emitted in both directions from the two reservoirs. The reader will find the complete derivation in references [6, 14, 21, 22], where different but equivalent approaches were used. For a multichannel quantum conductor the complete zero-frequency spectral density of quantum shot noise is [21, 22]:

$$S_i = 4k_{\mathrm{B}}TG + 2eI\frac{\sum_n t_n(1-t_n)}{\sum_n t_n}\left(\coth\left(\frac{eV_{DS}}{2k_{\mathrm{B}}T}\right) - \frac{2k_{\mathrm{B}}T}{eV_{DS}}\right), \quad (8)$$

where t_n is the transmission of an individual quantum channel and eV_{DS} is the potential energy difference between the chemical potentials of the two reservoirs. Note that eqn (8) reduces to the Johnson–Nyquist noise $S_i = S_{\mathrm{th}} = 4k_{\mathrm{B}}TG$ for $T \gg eV_{DS}/2k_{\mathrm{B}}$. In the other extreme $eV_{DS}/2k_{\mathrm{B}} \gg T$, the noise increases linearly with voltage (or current) and the suppression factor with respect to the poissonian–Schottky result is

$$\sum_n t_n(1-t_n)\Big/\left(\sum_n t_n\right).$$

2.3. Experimental observation of noise suppression

While theory made progress there had been no experimental verifications of noise suppression in ballistic conductors until very recently. Recent experiments were performed on one of the simplest ballistic systems: a short and narrow quantum wire in the form of a QPC. A QPC is usually defined electrostatically in the plane of a two dimensional electron gas (2DEG) by means of a negative voltage applied to metallic gates evaporated on the sample's surface. As the voltage applied to the gates decreases so does the effective width of the wire and the number of quasi 1D channels (modes) in the QPC. The conductance goes through a step-like decrease [23, 24] as each new channel is reflected by the QPC in accordance with eqn (2). As long as the transmission coefficient in a channel is unity and the number of participating channels remains constant the conductance is also a constant given by an integer multiple of the quantum conductance. This behavior strongly affects the shot noise generated by a QPC in accordance with eqn (8). At a zero temperature the shot noise can be expressed via the simpler form:

$$S_i = 2eV_{DS}\frac{2e^2}{h}\sum_n t_n(1-t_n), \quad (9)$$

with $V_{DS}\frac{2e^2}{h}$ being the current in each doubly degenerate channel with unity transmission. Obviously, the expected shot noise at constant bias voltage is zero at conductance plateaus ($t_n = 1$) and maximal when the transmission probability through a certain channel is $t_n = \frac{1}{2}$.

First attempts to measure noise generated by a partially transmitting QPC were performed by Li *et al.* [25] and by Washburn *et al.* [26]. The measurements were restricted to frequencies below 100 kHz and performed under a relatively large applied voltages in order to obtain a sufficiently large shot noise signal. Although some indications of noise suppression were found, the authors failed to observe the expected linear increase in S_i with current (characteristic to shot noise) due to strong $1/f$ or telegraphic conductance noise that accompanied the measurements at these low frequencies. The main difficulty in measuring shot noise is its strength. For example, even for a relatively high $V_{DS} = 1$ mV, i.e. a voltage which is comparable to the fermi energy of the 2DEG, the peak noise is $S_i = 6.2 \times 10^{-27}$ A^2 Hz^{-1} ; a rather small signal. New technical solutions were highly desirable in order to increase the sensitivity in a reliable way.

The first decisive improvement came with the high-frequency technique developed by Reznikov *et al.* [8]. At high enough frequencies shot noise dominates the $1/f$ noise even at high V_{DS}, however, the large background

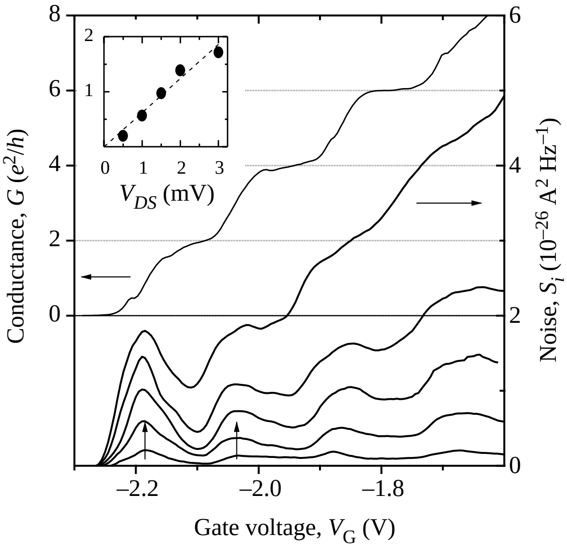

Fig. 1. Noise spectral density S_i measured at $V_{DS} = 0.5, 1, 1.5, 2, 3$ mV (bottom to top), and normalized linear conductance G measured at $V_{DS} = 0.5$ mV. Noise maxima are around $t_n = \frac{1}{2}$, where they are theoretically expected. Inset: Dependence of the first peak height (same scale as in the main figure) on injection voltage. The dashed straight line is the predicted behavior. (Reznikov *et al.* [28]).

noise of any amplifier used is still dominant. Even a cryogenic amplifier, used in the above work [8], has an equivalent input noise temperature of 40 K (with 50 Ω input impedance), or $S_i = k_B T/50 \cong 10^{-23}$ A^2 Hz^{-1}; more than three orders of magnitude larger than that of the shot noise. The low input impedance, however, allows measurement within a wide-frequency band Δf. It turns out that wide-band measurements improve the signal-to-noise ratio (S/N), relative to the ratio of shot noise signal to the amplifier's noise, by a factor of $(\Delta f \tau)^{1/2}$, where τ is the time of measurement [27]. Measuring shot noise in a wide frequency band $\nu = 8$–18 GHz ($\Delta f = 10$ GHz), with $\tau = 1 - 10$ s, improves the S/N by a factor of $(1 - 3) \times 10^5$, and leads to an acceptable $S/N \approx 10^2$. Moreover, the DC voltage across the QPC, V_{DS}, was modulated at a low frequency $f < 1$ kHz and the amplified noise signal was measured synchronously with the modulation using lock-in technique. The measuring time τ was determined by the averaging time of the lock-in amplifier. An important technical consideration was to reduce the value of the capacitance between the source and drain of the QPC in order to prevent shorting the shot noise signal through this capacitance (bypassing thus the amplifier)[28]. To accomplish this a narrow mesa (some 5 μm wide) was used in order to minimize the stray capacitance between the leads.

Typical DC conductance curves along with the noise, S_i, were measured as a function of QPC gates voltage V_G, at $T = 1.5$ K, and are shown in Fig. 1. The linear conductance is quantized in units of $2e^2/h$, after subtracting the series resistance of the ohmic contacts. The measured noise, at a constant V_{DS}, is consistent with eqn (9), as indeed seen in Fig. 1. The noise peaks at $t_n \approx \frac{1}{2}$ with magnitude of the first peak (at $t_1 = \frac{1}{2}$) close to $(1/4) \cdot 2e \cdot (2e^2/h) \cdot V_{DS}$. This noise amplitude grows almost linearly with the DC current as seen in the inset of Fig. 1. This linear dependence of the measured noise is crucial in order to substantiate the origin of the noise. In the pinch-off regime of the QPC, when $t_1 \ll 1$, the noise is expected to behave classically ($S_i = 2Ie$). Indeed the noise amplitude is found to be linearly dependent on currents smaller than 50 nA [8], however, for currents larger than 100 nA the noise signal saturates and eventually becomes current independent. Even though the authors attributed this behavior to an onset of correlated transport due

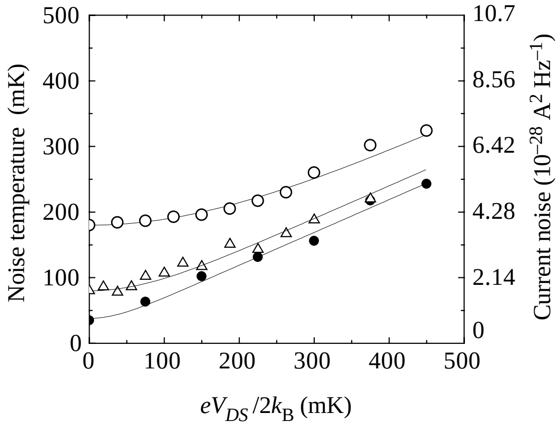

Fig. 2. Spectral density of the QPC current fluctuations, also expressed as noise temperature, for transmission $t_1 = 0.5$ and temperature T =38 mK (full circles), 80 mK (open triangles), and 180 mK (open circles), as a function of the average voltage expressed in relevant temperature units. The solid lines are predictions of eqn (8) (Kumar *et al.* [9]).

to coulombic interactions, it now seems most likely that this noise suppression results from partial electron transport above the barrier with an effective large transmission and thus suppressed noise [29].

A different approach, leading to an even more sensitive measurement, was implemented later by Kumar *et al.* [9]. It allowed to measure simultaneously both thermal noise and shot noise and thus test shot noise suppression at finite temperatures (eqn (7)). In this work the authors utilized amplifiers with a high input impedance (instead of the 50 Ω input impedance amplifier used in [8]). It turned out that although the high input impedance reduces the available frequency band, the better matching between the impedance of the sample and that of the amplifier actually improves the S/N by a factor of $(r/50)^2\ 10^5$, where r is the QPC resistance. Although the measurement band was reduced to some 10 kHz, a measurement time of 600 s allowed an improvement of the S/N by a factor of 8×10^2 all in all giving rise to a sensitivity some 50 times better than in [8]. Moreover, in [9] the noise signal was measured independently by two ultra-low noise amplifiers and the cross correlation (CC) spectrum was calculated with the aid of a spectrum analyzer. This cross correlation technique allows the noise of the amplifiers and the thermal noise of the leads to be averaged out since they are uncorrelated. The measured signal therefore is proportional only to the correlated part, namely, the sample noise S_i and the current noise of the amplifiers, S_i^a, which is introduced back into the measurement circuit by the amplifiers themselves. The latter can be determined and thus accurately subtracted, allowing thus an *absolute* measurements of the *total* sample noise S_i. After a calibration procedure the equilibrium noise (without current) was measured in a temperature range 30 to 600 mK. This allowed a check of the Johnson–Nyquist relation $S_{\text{th}} = 4k_B T G$ and led to an accurate measurement within a few percent. Then, at a fixed temperature, the current was swept and S_i was measured, verifying the predicted behavior. Absolute noise measurements were limited only by statistical errors.

It is important to mention that although the measurement frequency in [9] was small $1/f$ noise was still negligible, opposite to the previous results [25, 26]. There are two reasons for that: first, the very small applied voltage, $V_{DS} \sim 10\ \mu$V, reduced the relevance of $1/f$ noise which scales with V_{DS}^2 as opposed to shot noise which is linear in the applied voltage; and second, the special care to reduce the effect of radiofrequency radiation emanating mostly from the external 'hot' circuit. It was shown [30] that excitation of the sample by an intentionally applied driving voltage or by unintentional external radiation is the main reason for the switching behavior that gives rise to $1/f$ noise. Careful filtering reduced the black body radiation temperature to below 300 mK and $1/f$ noise to a negligible value at frequencies above 100 Hz.

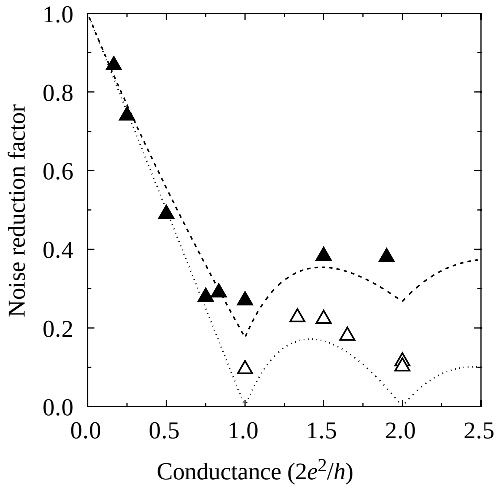

Fig. 3. Noise reduction factor, $S_i/2Ie$, versus conductance for $B = 0$ (filled triangles) and $B = 0.23T$ (open triangles). Predictions for no mode mixing without (dotted curve) and with (dashed curve) the calculated heating effects for $B = 0$ (Kumar *et al.* [9]).

Figure 2 shows the resultant noise for a conductance e^2/h, corresponding to a transmission $t_1 = \frac{1}{2}$ of the first channel. The measured noise changes from Johnson–Nyquist noise at zero applied voltage to noise that depends linearly, within the experimental accuracy, on V_{DS} for $eV_{DS}/2k_B \gg 1$. Note also that in the measurement frequency window, for central frequency in the 4 kHz to 9 kHz range, the noise was found to be white, as expected. It should be stressed that the three solid curves are not fits but the prediction of eqn (8) using $t_1 = \frac{1}{2}$ with no adjustable parameters.

In both experiments [8, 9] noise peaks were less pronounced when transport took place via higher channels, namely $t_n > 1$. This fact was attributed to both electron overheating or mode mixing. At sufficiently low temperature the thermalization length of electrons with phonons becomes larger than the size of the sample, allowing the heat generated within the QPC to dissipate only via diffusion of hot electrons into the sample's leads [9]. An estimate of the resulting electron temperature leads to a reasonable agreement between the experimental data and eqn (8) (see Fig. 3). The effect of overheating may be strongly reduced by application of a magnetic field such that $\omega_c\tau > 1$ (ω_c being the cyclotron frequency and τ the momentum relaxation time), as shown in the same figure. In this case electrons near the QPC tend to move along equipotential lines. The non-diagonal thermal conductivity leads to a spatial separation of the incoming cold electrons from the outgoing hot electrons. The incoming electrons are thus quieter and the resultant noise is much closer to the zero temperature prediction. Noise measurements in strong magnetic field will be discussed later.

2.4. Quantum interference of electrons

Shot noise can be used as a tool to probe statistical properties of particles. An interesting example of such an experiment is presented by the recent work of Liu *et al.* on 'quantum interference' between different electrons [31]. Interference between two *different* electrons is manifested due to the Pauli exclusion principle leading to the fermi statistics. The idea behind Liu *et al.*'s experiment is presented in Fig. 4. The beam-splitter stochastically partitions a noise-free incident flux into two output ports or, in other words, the electrons' antisymmetrized input state of the two electrons in the two incoming ports $|\Psi_-\rangle$ (for bosons the wavefunction is symmetric $|\Psi_+\rangle$) is subjected by the beam-splitter to a unitary transformation, U, such that the probability amplitude for two particles to be found in the same port, $\langle\Psi_{2,0}|$ (see Fig. 4), is 1 for bosons and 0 for fermions. This result is a consequence of constructive interference between the direct and exchange terms for bosons

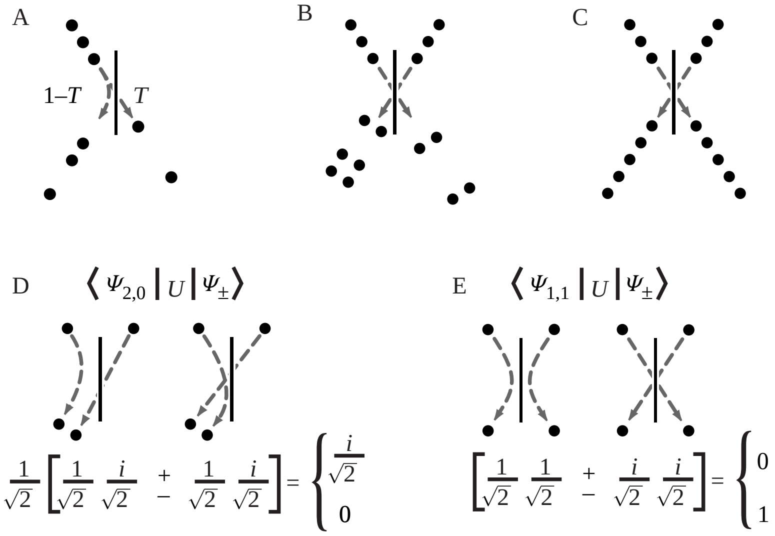

Fig. 4. A, An ideal beam-splitter with transmission t stoichastically partitions a constant flux into two ports. B, Collision of quantum paticles (for $t = \frac{1}{2}$) having a symmetric two-particle wave function (bosons) results in two particles always in one of the ports. C, Collission of quantum particles (for $t = \frac{1}{2}$) having an antisymmetric two-particle wavefunction (fermions with the same spin projection) always results in one particle at each port. D, The symmetrized (upper sign) or antisymmetrized (lower sign) input state $|\Psi_\pm\rangle$ is propagated by the beam-splitter unitary transformation, U, and projected onto the output state $\langle\Psi_{2,0}|$ corresponding to the probability amplitude of both particles transmitted to the left port. This results in a constructive (destructive) interference for fermions (bosons). E, For the output state corresponding to each particle in a different port, $\langle\Psi_{1,1}|$, (note, an additional minus sign asociated with the reflection process results in a destructive (constructive) interference for fermions (bosons)).

and destructive interference for fermions. Similarly, for the output state where one particle is in each port, $\langle\Psi_{1,1}|$, bosons will interfere destructively and fermions constructively. For a semi-transparent beam-splitter with transmission probability $t = \frac{1}{2}$, the transmission matrix for the two-particle wavefunction is

$$\frac{1}{\sqrt{2}}\begin{vmatrix} 1 & i \\ i & 1 \end{vmatrix}$$

leading to electrons appearing each in a *different* port or alternatively pairs of bosons appearing in the *same* (either) port.

In the experiment of Liu *et al.* [31] the above idea was implemented. A QPC on the left-hand side of the beam-splitter produces an electron beam, which is partitioned by a beam-splitter resulting in shot noise that is monitored in either of the two output QPC's. Adding another, similar, electron beam, produced by another QPC on the right of the beam-splitter, led to a decrease in the shot noise signal in both output ports due to reordering of the electrons fluxes.

3. Shot noise in diffusive systems

The equation for excess noise in a multichannel conductor (eqn (8)) is also applicable for a mesoscopic diffusive system—a system with dimensions larger than the elastic scattering length but smaller than the phase breaking length l_ϕ. In such systems the ensemble average over impurity configurations had to be performed [2], leading to:

$$\left\langle \frac{\sum_n t_n(1-t_n)}{\sum_n t_n} \right\rangle = \frac{1}{3}. \tag{10}$$

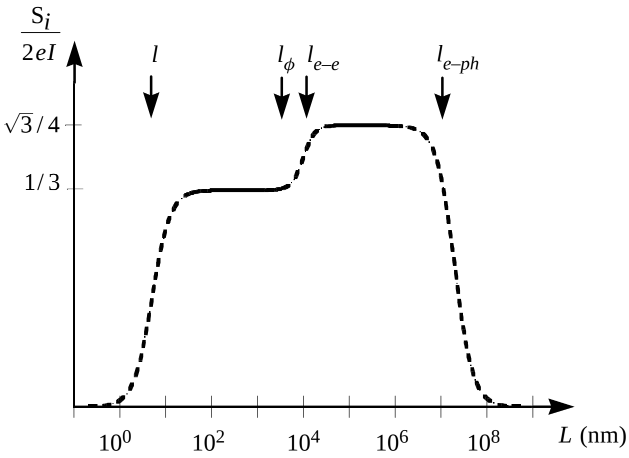

Fig. 5. Theoretical predictions, shown in schematic form, for $S_i/2eI$ versus sample length L in the regime $eV_{DS} \gg k_B T$ for a diffusive wire. Solid lines give the theoretical predictions discussed in the text. Dashed lines sketch the expected behavior in the cross-over regimes. The arrows pointing to the L axis depict qualitatively the relative magnitude of the length scales l, l_ϕ, $l_{\mathrm{e-e}}$ and $l_{\mathrm{e-ph}}$; typical values are 50 nm, 1 μm and 10 mm respectively. The last three length scales are voltage dependent and have been estimated for $V_{DS} = 100\ \mu$ V.

Note that weak-localization and universal-conductance-fluctuation correction terms should be added to the above expression for noise. These terms, however, are small and thus are less important. At zero temperature the expected shot noise is three times smaller than that given by the Schottky formula, namely $S_i = \frac{2}{3}eI$.

It is important to note that eqn (10) is obtained for a system smaller than l_ϕ, however, the necessity for this criterion has been questioned [32]. The situation became clear after Nagaev [33] obtained the *same* result employing only Bolzmann's equation, suggesting that electron coherence plays little role in noise suppression. The situation here is similar to the calculation of conductance of a diffusive system. While the leading term (Drude) can be obtained classically, weak-localization and universal-conductance-fluctuation corrections, which are much smaller, are quantum in origin. When the system size increases, exceeding the inelastic electron–electron scattering length, the spectral density of the noise is predicted to increase [4, 10, 34, 35]. The reason for this behavior is the redistribution of energy between electrons—which can be viewed as overheating of the electron sub-system. When the length of the diffusive conductor increases even further, becoming larger than the electrons' energy relaxation length, $l_{\mathrm{e-ph}}$ (determined by electron–phonon interactions), the phonons cool the electron sub-system and the noise decreases as shown schematically in Fig. 5. In order to observe shot noise in diffusive conductors, one has to apply a total voltage (across the total sample's length) such that the voltage drop on a length of order l_ε is of order $k_B T/e$. The different regimes were explored experimentally by Steinbach *et al.* [10] measuring silver thin film resistors with different lengths ranging from 1 to 7000 μm.

3.1. Frequency dependence

We have discussed thus far only the zero-frequency spectral density of the noise. In contrast to the zero-frequency limit, the spectral density at a finite frequency, f, is non-zero even in the absence of an applied voltage and at zero temperature. This unavoidable noise is due to quantum fluctuations of the system in its ground state, namely, the zero-point fluctuations. These fluctuations are proportional to $\frac{1}{2}hf$ [36–38]. Shot noise, however, which is defined as the additional noise due to the driven current has a different frequency spectrum. The highest possible frequency for which non-zero shot noise exists (again, at zero temperature) is set by the applied voltage $f_{up} = eV_{DS}/h$. Detailed calculations of the total spectral density in the case of a

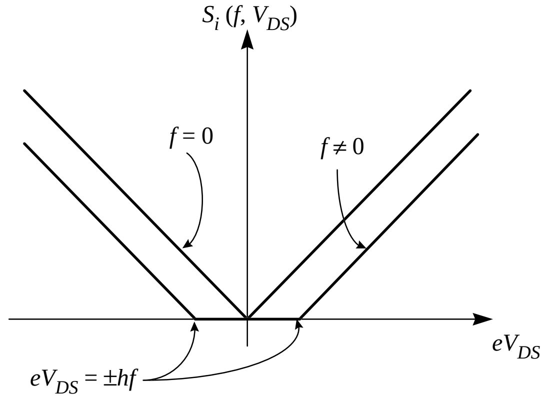

Fig. 6. Theoretical prediction for the behavior of the spectral density of current fluctuations at zero and finite frequency, $S_i(f, V_{DS})$, as a function of the excitation, eV_{DS}, at zero temperature. For $f > 0$ noise apears above a threshold excitation given by $eV_{DS} = hf$.

single channel QPC [39] show that the spectral density of the shot noise decreases linearly from $2e\frac{e^2}{h}Vt(1-t)$ at zero frequency to zero at f_{up}. In other words one expects a zero spectral density of shot noise at a given frequency, f, for any voltage such that $eV < hf$, as shown in Fig. 6. The main assumption in this calculation is that the width (in time) of the electrons' wavepacket, constructed from electrons with energies between μ and $\mu + eV_{DS}$ (given by h/eV_{DS}), is larger than the dwell time in the QPC, t_{d}. Measuring the spectral density of shot noise in the QPC is technically very difficult because of the small noise signals involved and the large impedance mismatch. However, in diffusive mesoscopic conductors, where higher currents can be injected [11] and the impedance is closer to 50 Ω, measurement is easier. Although there is no theory for the spectral density of shot noise in such systems, one would expect to obtain results similar to those obtained in a ballistic QPC.

Schoelkopf *et al.* [11] utilized a modulation technique similar to that in [8] but with a relatively narrow frequency band compared to f_{up}, about 0.5 GHz wide. Rather than scanning the frequency band across a wide frequency spectrum at a fixed applied voltage, the central frequency (of the band), f, was kept constant and the applied voltage was changed. The derivative of the noise with respect to the applied voltage was measured by means of adding to the applied voltage a small (low-frequency) alternating voltage ΔV. This derivative should vanish at voltages smaller than hf/e and should obtain a constant value (reflecting the linear dependence of the spectral density on the applied voltage) when the applied voltage exceeds hf/e (see Fig. 6). The width of the transition region in the derivative is determined by the larger value between ΔV and $k_{\mathrm{B}}T/e$ ($\Delta V \approx 80$ μV in this experiment). The experimental findings for three different central frequencies, 1.5, 10 and 20 GHz, shown in Fig. 7, are consistent with the theoretical predictions.

4. Measurement of a fractional charge

This section is devoted to the utilization of shot noise measurement as a tool for measuring charge. Since Milliken's oil-drop experiment [40] it is well known that the electrical charge is quantized in units of electron charge, e. The indivisibility of the charge quantum for free particles reflects the fundamental gauge invariance properties. Only bounded particles such as quarks in high-energy physics, which are beyond the scope of this review, are indirectly found to have a fractional charge. Charge quantization in systems of strongly interacting electrons, however, need not always be in units of e. In fact different charges may appear in such systems, for example, the well known Cooper pairs of electrons (twice the charge) in superconductors. Another less intuitive example is the prediction of fractionally charged quasiparticles in low-dimensional systems such as

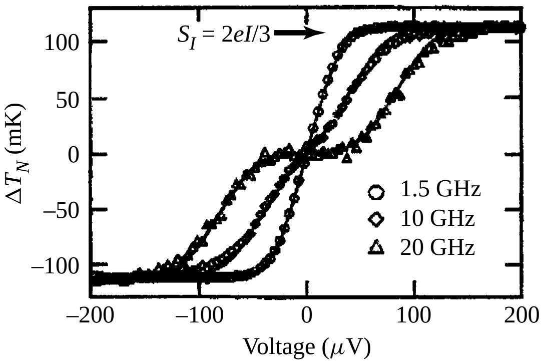

Fig. 7. Measured differential noise at several frequencies of 1.5, 10 and 20 GHz, with mixing chamber temperature 40 mK. Solid lines show the predictions by Yang [39] for an electron temperature of 100 mK, accounting for a voltage modulation of 60 μV peak to peak (Schoelkopf *et al.* [11]).

1D commensurable organic conductors [41] or 2D systems in the Fractional Quantum Hall (FQH) regime which will be discussed below. Obviously the total charge of an isolated electronic system *must* be quantized in units of e. However, if a system is divided arbitrarily into two parts one expects no sign of charge quantization in each part, unless the link between the two sub-systems is very weak. For the same reason, and with the same restriction, the (polarization) charge on a capacitor plate is not quantized and takes any continuous value.

For 2D conductors fractionally charged quasiparticles were first introduced in a theoretical work by Laughlin [42], put forward in order to explain the FQH effect. The FQH effect is a phenomenon that occurs at low temperatures in a 2DEG subjected to a strong perpendicular magnetic field, B. The energy spectrum of such a system consists of highly degenerated Landau levels with degeneracy per unit area $n_0 = B/\phi_0$, with $\phi_0 = h/e$ the flux quantum (h being Planck's constant). Whenever the magnetic field is such that an integer number ν (the filling factor) of Landau levels are occupied, that is $\nu = n_s/n_0$ equals an integer (n_s being a 2DEG areal density), the longitudinal conductivity of the 2DEG vanishes while the Hall conductivity equals $\nu e^2/h$ with very high accuracy. This phenomenon is known as the Integer Quantum Hall (IQH) effect[43]. A similar phenomenon, called the FQH effect, occurs at fractional filling factors, namely, when the filling factor equals a rational fraction with an odd denominator q. In contrast to the IQH effect, which could be understood in terms of non-interacting electrons, the FQH effect cannot be explained in such terms and results from interactions among the electrons under a strong magnetic field. Laughlin had argued that the net effect of these interactions, in the FQH regime, is equivalent to having a non-interacting gas of quasiparticles with a fractional charge $e^* = e/q$.

In a beautiful experiment, Goldman and Su [44] showed that the variation of a polarization charge on a capacitor plate, associated with a periodic conductance oscillations through an anti-dot (weakly coupled to leads), is accurately quantized in unit of e/q. Goldman's experiment showed that two consecutive ground states are related by a deficiency of a fractional charge at the anti-dot (this does not violate the fact that the total charge of the whole circuit remains a multiple of e). One may argue that such an equilibrium experiment does not probe the quasiparticles which, by definition, are the excitations above the ground state. In contrast, shot noise, which is a non-equilibrium property, probes the charge of these quasiparticle which carry the current. As long ago as 1987, Tsui suggested that the quasiparticle charge could, in principle, be determined by measuring shot noise in the FQH regime. However, no theory was available until Wen [45] recognized that transport in the FQH regime could be treated within a framework of 1D interacting electrons, propagating along the edge of the 2D plane, making use of the so called (chiral) Luttinger liquid model. Based on this model, subsequent theoretical works [46, 47, 48] predicted that shot noise, generated due to weak backscattering of the current, at fractional filling factors $\nu = 1/q$ and at zero temperature, should be simply related to the quasiparticle charge $e^* = e/q$ and to the *backscattered* current I_B [46]:

$$S_i = 2e^* I_B. \tag{11}$$

In the opposite limit of strong backscattering, the prediction is that the noise should be proportional to the transmitted current, I_t, and to the full charge of an electron [46]:

$$S_i = 2eI_t. \tag{12}$$

This seemingly strange result is a manifestation of the fact that fractionally charged quasiparticles exist only within the interacting 2DEG and not in regions where electrons do not exist. For weak backscattering, quasiparticles with a fractional charge make up the backscattered current in the 2DEG while it is the electrons that tunnel through the (empty) barrier and make up the transmitted current when the transmission is small. The noise, accordingly, is proportional to the corresponding charge.

Calculations of the zero-frequency spectral density of shot noise in the more general case of an arbitrary transmission probability, t, were performed analytically [47] and for finite temperature numerically [49]. Without an applied voltage, the results reduce to the expected thermal noise, $4k_B TG$, where the conductance, G, is temperature dependent as a result of the non-fermi liquid properties of the fractional edge channels. In addition, the spectral density of the noise at large DC currents (for applied voltages much larger than temperature) has a non-trivial variation, reflecting the stochastic transmission process and the non-trivial statistics of the quasiparticles. In the two limiting cases of very weak and very strong backscattering, keeping only the first-order term of the reflection or transmission probability, respectively, and at zero temperature, the numerical calculations reproduce the results given by eqns (11) and (12).

Unfortunately, in the FQH regime the shot noise signal is very small compared to the background noise, contributed mostly by the amplifiers. The shot noise is small due to the smaller charge and the small allowed DC current. The latter is restricted by the fact that the FQH effect 'breaks down' when the applied voltage is larger than the excitation gap. This excitation gap, in turn, depends crucially on the quality of the material in which the 2DEG resides. State-of-the-art technology currently yields samples with an excitation gap of the of the order of 100 μeV, leading to shot noise levels in the 10^{-29} A^2 Hz^{-1} range.

Two groups, one in the Saclay Research Center in France [12] and the other in the Weizmann Institute of Science in Israel [13], tackled these difficult measurements. A QPC was used by both groups in order to partially reflect the incoming current. The French group utilized a cross correlation technique [9] described in Section 2.3. They utilized a magnetic field corresponding to a filling factor, $\upsilon_{\text{bulk}} = \frac{2}{3}$, in the bulk of the sample and a small region of filling factor, $\upsilon = \frac{1}{3}$, within the QPC opening. Careful measurements of a resonant tunneling type behavior of the QPC allowed for a confirmation that tunneling was coherent and occurred in a single step, thus ruling out the possibility of noise suppression due to multiple uncorrelated steps. The Israeli group used a quieter, home-made, amplifier which was cooled down to liquid He temperature; thus a shorter integration time was needed in order to extract shot noise from the smaller background. Here the magnetic field corresponded to a filling factor $\upsilon = \frac{1}{3}$ throughout the sample's bulk and the QPC was used to locally reduce it to below $\frac{1}{3}$, again in order to slightly reflect the impinging current.

Both groups arrived at the same conclusion (see Figs 8, 9) that in the weak backscattering limit, near filling factor $\frac{1}{3}$, shot noise is threefold suppressed, proving that the charge of quasiparticles is $e^* = e/3$. In addition, the data in both experiments seem to deviate significantly from the thermal noise when the applied voltage satisfies the inequality $eV_{DS}/3 > 2k_B T$ (rather than $eV_{DS} > 2k_B T$), indicating that the potential energy of the quasiparticles is also threefold smaller.

De Picciotto *et al.* [13] compared their results, presented in Fig. 9, with the analytical expression in eqn (8)

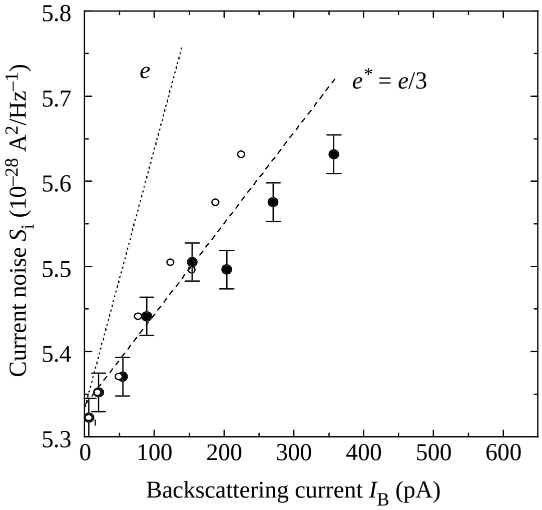

Fig. 8. Current noise as a function of $I_B = (e^2/3h)V - I$ (filled circles). The temperature is 25 mK. The slopes corresponding to the poissonian noise of backscattered charges $e/3$ (dashed line) or e (dotted lines) are shown for comparison. The open circles correspond to the data plotted against $I_B(1 - r(I_B))$ assuming a fermionic statistics. The reflection coefficient, r, is defined here as $r(I_B) = I_B/I_i$ with I_i the impinging current (the current which was to flow without reflection). The reflection coefficient increases linearly with I_B from 0.05 to 0.35 (Saminadayar *et al.* [12]).

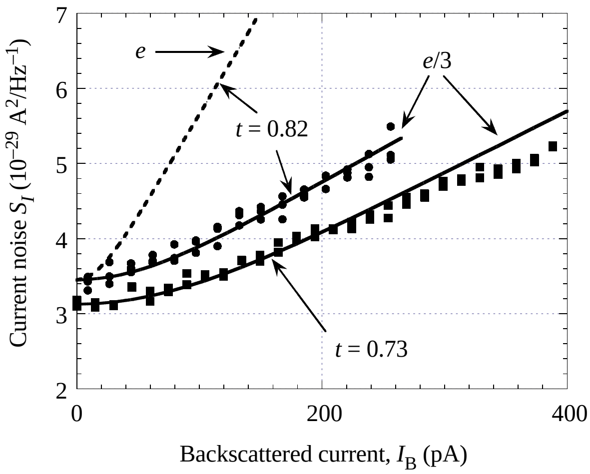

Fig. 9. Current noise as a function of the backscattered current, I_B, in the FQH regime at $\nu = \frac{1}{3}$ for two different transmission coefficients through the QPC (full circles and squares). The solid lines correspond to eqn (8) with a charge $e^* = e/3$ replacing e and the appropriate t. For comparison the expected behavior of the noise for $e^* = e$ and $t = 0.82$ is shown by the dotted line (de Picciotto *et al.* [13]).

rewritten for a single channel with electron charge e substituted by quasiparticle charge e^* :

$$S_I = 4k_B T G + 2e^* I_i t_n (1 - t_n) \left(\coth \left(\frac{e^* V_{DS}}{2k_B T} \right) - \frac{2k_B T}{e^* V_{DS}} \right). \tag{13}$$

Note that this expression extrapolates between thermal noise without an applied voltage and shot noise, corresponding to eqn (11) for $e^* V \gg 2k_B T$ (up to first order of reflection probability). In eqn (13) $I_i = V_{DS} \frac{\nu e^2}{h}$

is the impinging current (the current which was to flow without reflection) and the transmission coefficient is defined as the ratio between the transmitted current and I_i. However, as far as higher orders of $(1 - t)$ are concerned the situation is complicated. The fractional statistics of quasiparticles actually suggests larger noise (rather than the reduced fermionic one suggested by eqn (13)).

These two experiments are a fine example of the utilization of shot noise measurements in order to extract information not easily accessible via other measurements.

5. Summary

Shot noise measurements provide a powerful tool with which to extract information not given by DC conductance measurements. While the manifestation of correlation between electrons can be also seen in the conductance, shot noise measurements, depending directly on the temporal behavior of the current, provide the most direct and unambiguous insight to such effects. Such correlation results from the fermi statistics as well as from coulombic interaction between the electrons. Examples of such effects have been demonstrated. The fermi statistics, at zero temperature, leads to perfect ordering of the electrons in such a way that shot noise vanishes all together. This was clearly demonstrated in the example of transport through a QPC. The Coulomb interaction might introduce a spectral peak in the otherwise white nature of the noise and/or lead to suppression in the zero-frequency component. Such suppression at zero frequency was observed in the FQH regime. Measurements of the spectral dependence over a wide frequency range still need to be done. Measuring cut-off frequencies, spectral peaks, non-white behavior and the like will provide more information and make such measurements even more powerful.

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