Evolution of Quasiparticle Charge in the Fractional Quantum Hall Regime

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The charge of quasiparticles in a fractional quantum Hall (FQH) liquid, tunneling through a partly reflecting constriction with transmission t, was determined via shot noise measurements. In the $\nu = 1/3$ FQH state, a charge smoothly evolving from $e^* = e/3$ for $t_{1/3} \cong 1$ to $e^* = e$ for $t_{1/3} \ll 1$ was determined, agreeing with chiral Luttinger liquid theory. In the $\nu = 2/5$ FQH state the quasiparticle charge evolves smoothly from $e^* = e/5$ at $t_{2/5} \cong 1$ to a maximum charge less than $e^* = e/3$ at $t_{2/5} \ll 1$. Thus it appears that quasiparticles with an approximate charge $e/5$ pass a barrier they see as almost opaque.

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The fractional quantum Hall (FQH) effect is a manifestation of the prominent and unique effects resulting from the Coulomb interactions between electrons in a two-dimensional electron gas (2DEG) under the influence of a strong magnetic field [1]. In this regime the lowest Landau level is partially populated. Laughlin's seminal explanation of the FQH effect [2] involved the emergence of intriguing fractionally charged quasiparticles. Recently, shot noise measurements confirmed the existence of such quasiparticles with charge $e/3$ and $e/5$ at filling factors $\nu = 1/3$ [3] and $\nu = 2/5$ [4], respectively. These experiments relied on the fact that shot noise, resulting from the granular nature of the quasiparticles, is proportional to their charge. Since current flowing in an ideal Hall state is noiseless [4], a quantum point contact (QPC) constriction was used to weakly reflect the incoming current, leading to partitioning of the incoming carriers and hence to shot noise. A charge e^* was then deduced from the shot noise expression derived for noninteracting particles [5]. In this paper, we extend the range of QPC reflection to the strong backscattering limit, where the apparent noise-producing quasiparticle charge is expected to be different. Specifically, an opaque barrier is expected to allow only the tunneling of electrons, as both sides of the barrier should be quantized in units of the electronic charge. How this charge evolves is an important question in the understanding of the behavior of quasiparticles, and here we explore the evolution of the charge of the $e/3$ and $e/5$ quasiparticles. We first briefly describe the expected dependence of shot noise on charge and transmission.

At zero temperature ($T = 0$), the shot noise contribution of the pth channel is [5,6]

$$S_{T=0} = 2e^* V g_p t_p (1 - t_p), \tag{1}$$

where S is the low frequency ($f \ll eV/h$) spectral density of current fluctuations ($S\Delta f = \langle i^2 \rangle$), V is the applied source-drain voltage, g_p is the conductance of the fully transmitted pth channel in the QPC, and t_p is its transmission coefficient. This reduces to the well known *classical* Poissonian expression for shot noise when $t_p \ll 1$ (the "Schottky equation"), $S_{T=0} = 2eI$, with $I = V g_p t_p$ the dc current in the QPC.

The justification for the use of Eq. (1) comes from current theoretical studies of shot noise in the FQH regime, based on the chiral Luttinger liquid model. They are applicable only for Laughlin's fractional states, $\nu = 1/3, 1/5$, etc. [7–9] (where the edge is composed of one channel only) and not for more general filling factors. They predict the following:

$$\begin{aligned} S_{T=0} &= 2e^* V g_p (1 - t_p) = 2e^* I_r, \qquad t_p \approx 1, \\ S_{T=0} &= 2eV g_p t_p = 2e I_t, \qquad t_p \approx 0, \end{aligned} \tag{2}$$

where I_r and I_t are the reflected and transmitted dc currents, respectively. The most important result of Eq. (2) is that the tunneling of quasiparticles with charge $e/3$, $e/5$, etc., in Laughlin states, at weak reflection ($t_p \approx 1$), changes to that of electrons at strong reflection ($t_p \approx 0$).

One can gain insight into the characteristics of the expected shot noise in the FQH regime [4], and some insight into Eq. (1), by considering the composite fermion (CF) model [10]. In the simplest approximation for the CF model the fractionally filled electronic Landau level with $\nu = p/(2p + 1)$ is identified as p filled Landau levels of CFs, $\nu_{\mathrm{CF}} = p$, with each CF consisting of an electron with *two* attached magnetic flux quanta $\phi_0 = h/e$. The effective magnetic field sensed by the CFs is $B - 2n_s h/e$, with n_s the density of the 2DEG. Under this weaker effective magnetic field the CFs are approximated as weakly interacting quasiparticles, flowing in separate and noninteracting edge channels, hence justifying the application of the above-mentioned formulas for the noise. When the QPC constriction is reduced in width and the conductance is in a transition between two different FQH plateaus of the series $p/(2p + 1)$ only one edge channel is partitioned. The others can be approximated as being perfectly transmitted. Consequently, in Eqs. (1) and (2), p designates the CF edge channel that is being partitioned. As examples, for the transition between $\nu = 1/3$ and the insulator, $p = 1$, $g_1 = g_0/3$, and $t_1 = 3g/g_0$; while for the transition between $\nu = 2/5$ and $\nu = 1/3$, $p = 2$,

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$g_2 = (2/5 - 1/3)g_0$, and $t_2 = \frac{g/g_0 - 1/3}{2/5 - 1/3}$, with g being the total conductance and $g_0 = e^2/h$ the quantum conductance. The dependence of the charge on transmission, in the simplest model, can be evaluated by considering the added current due to the two flux quanta attached to the electron. Doing this, de Picciotto predicted [11] the quasiparticle charge to vary from $e^* = e/(2p + 1)$ at $t_p \approx 1$ to $e^* = e/(2p - 1)$ at $t_p \approx 0$ as a linear function of t_p, namely, for $p = 1$, $e/3 \to e$, and for $p = 2$, $e/5 \to e/3$.

In order to apply the above principles in a realistic experiment a more general expression for the shot noise [12] applicable at finite temperatures has to be used [3,4]:

$$S_T = 2e^* V g_p t_p (1 - t_p) \times \left[\coth\left(\frac{e^* V}{2k_B T}\right) - \frac{2k_B T}{e^* V}\right] + 4k_B T g\,. \quad (3)$$

This equation leads to a finite noise at zero applied voltage, $S = 4k_B T g$—the Johnson-Nyquist formula. When $V > V_T \sim 2k_B T/e^*$ the noise approaches the linear behavior predicted by Eqs. (1) and (2).

Measuring quasiparticle charge in the strong backscattering limit is difficult, and results so far were inconclusive [13]. As the QPC constriction is closed to reflect a larger portion of the incident current, the conductance exhibits the familiar *impurity resonances* as a function of constriction width ([14], and see also Fig. 1). Moreover, the I-V characteristic becomes highly nonlinear (g and t depend on current), making the analysis difficult. Measuring a large number of samples across the full range of the transmission coefficient in the first two CF channels, $\nu = 1/3$ and $\nu = 2/5$, we found relatively resonant-free samples. Moreover, we extended Eq. (3) to cases of nonlinear I-V characteristics allowing also the charge to change with the transmission coefficient. Consequently, we have found a universal behavior of the charge as a function of transmission in the $\nu = 1/3$ channel, and qualitatively quite different behavior for the charge in the $\nu = 2/5$ channel.

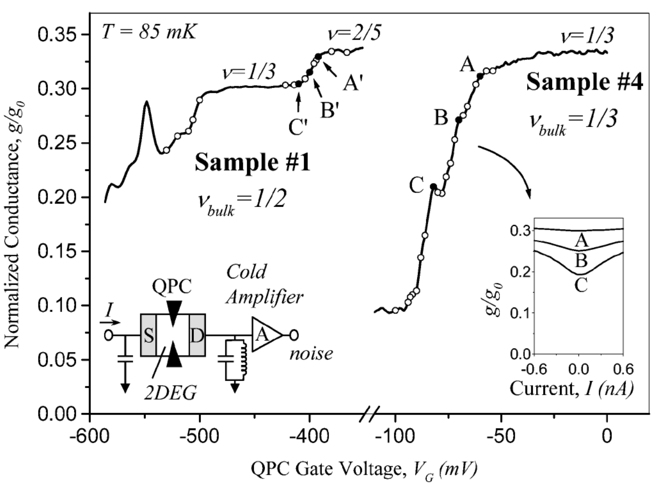

FIG. 1. Two-terminal conductance as a function of QPC gates voltage for samples #1 and #4. The deviations from the quantized values of the conductance are due to the bulk longitudinal resistance. The markers show the conductance values at which conductance and noise measurements were made. Right inset: Conductance as a function of applied dc current at the points shown. Left inset: Schematic of sample and measurement system.

Our samples were 2DEG's embedded in GaAs-AlGaAs heterostructures with a low-temperature concentration of 9.8×10^{10} cm^{-2} and a mobility of 4×10^6 cm^2/V s. A perpendicular magnetic field of 12.15 T is needed to reach the center of the $\nu = 1/3$ plateau. The left-hand inset in Fig. 1 shows the schematic of the two-terminal Hall samples with source (S), drain (D), and a QPC. The Hall sample's width was 100 μm and the QPC opening width was 300 nm. The QPC gate's potential was used to control the partitioning of the incoming current. Measurements were made in a dilution refrigerator at a lattice temperature of 55 mK and a measured electron temperature of 85 mK (see [3] for details). Noise was measured within a bandwidth of 30 kHz around a frequency of 1.6 MHz, chosen to be above the $1/f$-noise knee and much lower than eV/h. An LRC circuit determined the central frequency and bandwidth, with R dominated by the resistance of the QPC and C by the capacitance of the coaxial lines. A cold preamplifier, with a current noise of $\sim 3 \times 10^{-29}$ A^2/Hz, amplified the noise signal.

We present here results from four samples (#1–#4): three measured in the $\nu = 1/3$ FQH state and two in the $\nu = 2/5$ FQH state. The bare samples (without applied gate voltage) exhibit, as a function of magnetic field, an accurate $\nu = 1/3$ quantization of the resistance but deviate at the $\nu = 2/5$ plateau due to finite bulk longitudinal resistance. The measurements in the $\nu = 2/5$ state were conducted at two different bulk filling factors: $\nu_{\text{bulk}} = 2/5$ and $\nu_{\text{bulk}} = 1/2$ (see sample #1 in Fig. 1), while for the measurements in the $\nu = 1/3$ state the bulk filling factors were $\nu_{\text{bulk}} = 1/3$ and $\nu_{\text{bulk}} = 1/2$ (see sample #4 in Fig. 1). Typical problems are seen in Fig. 1: sample #1 shows a single large "resonance"—the large spike on the left-hand side of the graph—which prohibits further measurement into the 1/3 state; and the reduction of the transmission of the 1/3 state in sample #4, although much smoother, saturates at about $0.1e^2/h$, presumably due to leakage across the QPC. The open circles on the graphs show where noise and I-V measurements were made.

In our experiment we measured two quantities: the differential conductance g and the shot noise. Using $g \propto e^* t$ and S_T from Eq. (3) we extracted the transmission probability t and the quasiparticle's charge e^*. However, the analysis is complicated by the strong dependence of the conductance on the current—see the right-hand inset in Fig. 1. This inset shows the differential conductance of the QPC as a function of dc current for three different conductances indicated by points A, B, and C. While at point A, where t is relatively large, the conductance is almost constant with current ($\Delta g/g_{I=0} = 0.05$), at point C, where t is very small, there is a significant change in the differential conductance at large currents ($\Delta g/g_{I=0} = 0.3$). To account for this nonlinearity, the energy independent Eq. (3) was modified by resorting to the integral over energy used in its derivation [12]. However, the dependence of conductance on the current (in a small range), for a fixed QPC width, was all attributed to a changing t; i.e., the charge e^* was approximated not to vary with current. Transforming from the integration over energy to a sum over discreet current points, and substituting t in terms of g and e^* in Eq. (3), $t_{p=1} = \frac{(g_i/g_0)}{e^*/e}$, we get for $\nu = 1/3$

$$S_T(I) = 2e^*I \frac{1}{N} \sum_{i=1}^{N} \left(1 - \frac{g_i/g_0}{e^*/e}\right) \times \left[\coth\left(\frac{e^*V}{2k_BT}\right) - \frac{2k_BT}{e^*V}\right] + 4k_BTg. \quad (4)$$

Here i runs over the measured points (N) up to current I and g_i is the differential conductance at each point. In the $\nu = 2/5$ state we substitute for the total current I_T only that fraction which flows through the second edge channel (using the CF model), $I_{p=2} = \frac{(g/g_0)-1/3}{g/g_0} I_T$, and for the transmission $t_{p=2} = \frac{(g/g_0)-1/3}{(2/5-1/3)5e^*/e}$. Indeed, if $e^* = e/5$, $t_{p=2}$ is the expected bare transmission of the second CF channel given above. The noise expression now contains a single fitting parameter e^*.

Figure 2 shows noise results for a partitioned $\nu = 1/3$ channel in sample #4. There is no noise on the $\nu = 1/3$ plateau. The top part of the graph shows the differential conductance of the QPC against dc current I at points A, B, and C shown in Fig. 1. The current range we used for the extraction of the charge is $\Delta g/g_{I=0} = 0$–0.2 in order to reduce the effect of the charge variation with current while still being able to fit the curves to Eq. (4). The measured noise, with the background thermal noise subtracted, is shown in the lower part of Fig. 2. The curves are offset for clarity. Also shown is the behavior of Eq. (4) with $e^* = e/3$ (solid lines) and $e^* = e$ (dashed lines). For each width of the QPC constriction we find the best fitting quasiparticle charge e^* and consequently the channel transmission t near $I = 0$. In previously published high-t data the noise is that of $e/3$ charges [3]. As the transmission is reduced, the apparent charge increases to a maximum around charge e. Consistent results were obtained for the two other samples (as seen in Fig. 4). Similarly, Fig. 3 shows similar graphs for the measurements in the $\nu = 2/5$ state in sample #1 (points A', B', and C' in Fig. 1). Again, no noise is measured on the $\nu = 2/5$ plateau. The theoretical lines correspond to charges $e^* = e/5$ (solid lines) and $e^* = e/3$ (dashed lines). The other sample provided similar results.

The dependences of the quasiparticle charge on the transmission coefficient for all four samples are summarized in Fig. 4. All results approximately collapse onto two separate curves. While in the $\nu = 1/3$ case the deduced charge changes smoothly from $e/3$ at weak reflection (large t) to around e at strong reflection ($t \cong 0.1$), the deduced charge in the $\nu = 2/5$ case stays near $e/5$ over almost the full range of transmission. There is an apparent slight increase of e^* at lower transmissions. Although scattering of the data due to the small signal prevents a more accurate determination of the charge for $t < 0.3$, it clearly does not show the steep rise to $e^* = e$ observed at $\nu = 1/3$.

Adopting the CF picture in accordance with Ref. [11], the difference between the two channels can be understood by considering how much charge crosses the constriction when a composite fermion, composed of an electron and

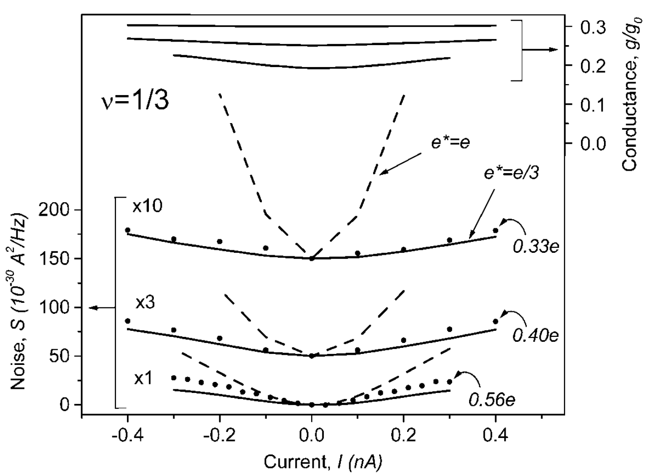

FIG. 2. Top: Differential conductance as a function of dc current for different transmissions in the $\nu = 1/3$ channel for sample #4. Bottom: Measured excess noise as a function of dc current for the same transmission points. The solid lines are the result of Eq. (4) with a charge $e^* = e/3$; the dashed lines are the result with charge e. The numbers near the data points give the best-fit value to e^* from Eq. (4).

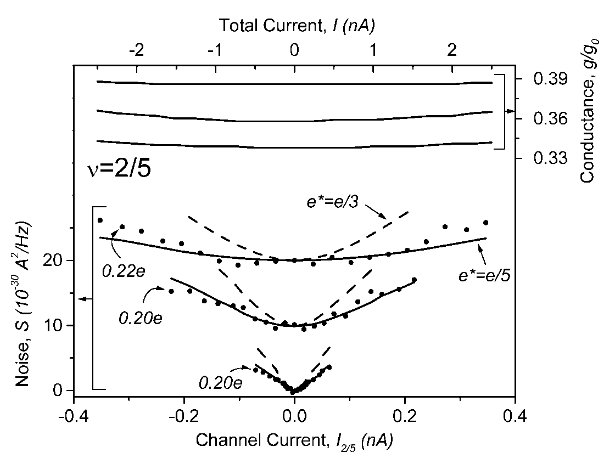

FIG. 3. Top: Differential conductance as a function of dc current for different transmissions in the $\nu = 2/5$ channel for sample #1. Bottom: Measured excess noise as a function of dc current for the same transmission points. The solid lines show the result of Eq. (4) with a charge $e^* = e/5$; the dashed lines are the result with charge $e^* = e/3$. The expected noise with charge e lies much above that of $e/3$.

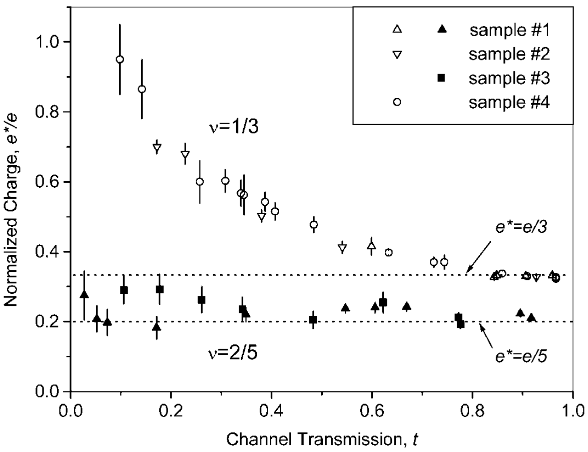

FIG. 4. Summary of the results of the determined evolution of the charge of the quasiparticles as a function of transmission, for all four samples, for the $\nu = 1/3$ and $\nu = 2/5$ channels.

two flux quanta, traverses it. In the $\nu = 1/3$ case, a strongly closed constriction, reflecting almost all the incident current, is almost an insulator and the extra charge induced by the fluxes is negligible, leading to a quasiparticle charge approximately e. In contrast, in the $\nu = 2/5$ case only one of the edge channels is strongly reflected, and consequently the constriction is not an insulator. Thus the extra transferred charge is finite and the quasiparticle's charge is not e. Equations (1)–(4) are based on a picture in which the noise is produced by independent quasiparticles whose partitioning obeys binomial statistics. In fact, the noise can be interpreted also as being generated by quasiparticles of fixed charge whose partitioning statistics are not binomial. For example, the measured charge of $e^* = e$ could be interpreted as a quasiparticle of charge e (a single electron) or as three quasiparticles of charge $e^* = e/3$ bunched together. For the $\nu = 2/5$ channel, we may conclude that the $e^* = e/5$ quasiparticles traverse an opaque barrier without fully bunching, which would produce a charge $e^* = e$. However, these are qualitative arguments, and as yet there is no rigorous theory for the $\nu = 2/5$ case.

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