Concepts of condensed matter physics

Spring 2013

Exercise #1

Due date: 03/04/2013

1. **Symmetry and Goldstone.** Consider the following Ginzburg-Landau theory (please consult “Quantum Field Theory of Many-Body Systems” by Xiao-Gang Wen; page 82-83)

$$S_{GL} = \int_0^\beta d\tau \int d^d x\, \psi^* \left(\partial_\tau - \frac{\nabla^2}{2m} + \alpha + \beta |\psi|^2 \right) \psi$$

   a. Write the complex field ϕ in a polar representation, i.e. $\psi = \sqrt{\rho}\, e^{i\theta}$. What is the mean-field value of ρ?

   b. Show the GL action has global gauge symmetry, i.e. $\theta \to \theta + const.$

   c. Since this is a continues symmetry it implies that there is a Goldstone mode. Show that the action of this Goldstone mode has the form

$$S_{XY} = -\int_0^\beta d\tau \int d^d x\, \theta \left(\chi\, \partial_\tau^2 + r_0 \nabla^2 \right) \theta$$

   This action is known as the XY-model, and it describes the low energy excitations of a superconductor/superfluid. Note that here r_0 is the superfluid stiffness and χ is the compressibility.

   d. We know that S_{GL} has vortex excitations. Explain how to obtain them from S_{XY}. For example, what is the energy of a vortex? (Hint: use the static limit in-which there is no compressibility. Also, remember that the GL theory is an effective theory, which is correct at long-distances, such that a short-range cutoff, a_0, may be introduced.)

2. **“More is different”**

   In this question you are asked to read the article “More is different” (Phillip W. Anderson, Science, **177**, 393 – 396 (1972)) and answer the questions below. The goal is to make you think about the complexity of many-body phenomena, and therefore the questions do not have well defined answers, nonetheless they will be graded. (The length of the answer for each item should be no longer than 200 words (about half a page).

- **a.** Explain the viewpoint of Anderson regarding the hierarchical relation between different fields of science, for example condensed matter physics biology and nuclear physics?
- **b.** A single magnet (spin) will have on average a zero magnetic moment. Two spins with ferromagnetic interaction between them still have a zero magnetic moment on average. A macroscopic number of spins will break the symmetry at low enough temperature. What determines the transition between the two behaviors?
- **c.** Anderson thinks that an "additional" symmetry breaking occurs in living things. What is this additional symmetry breaking? Why do you think that this breaking is so necessary for life?