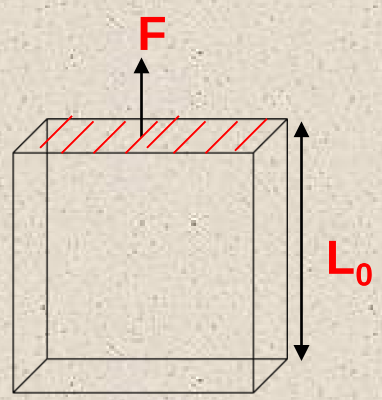
Mechanical Properties

Primer Materials
Spring 2021

Definitions (engineering)

Stress (σ): force divided by cross sectional area $\frac{N}{mm^2} = MPa$

$$\sigma = \frac{F}{A_0}$$



Strain (ε): elongation divided by the original length

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0}$$

Elasticity

Elastic strain (ε_e) is reversible – when the stress goes to 0 the strain goes to 0

Plastic strain (ε_p) is irreversible – when the stress goes to 0 the strain remain

Hooke's Law

Stiffness

$$\sigma_i = C_{ij} \cdot \varepsilon_j$$

$$\varepsilon_i = S_{ij} \cdot \sigma_j$$

Compliance

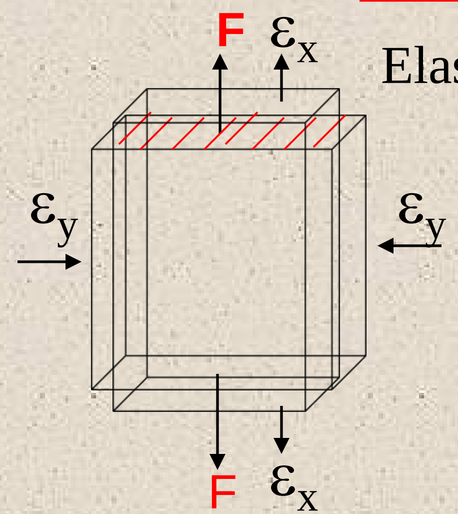
$$A = \frac{2C_{44}}{C_{11} - C_{12}}$$

Poisson's ratio

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

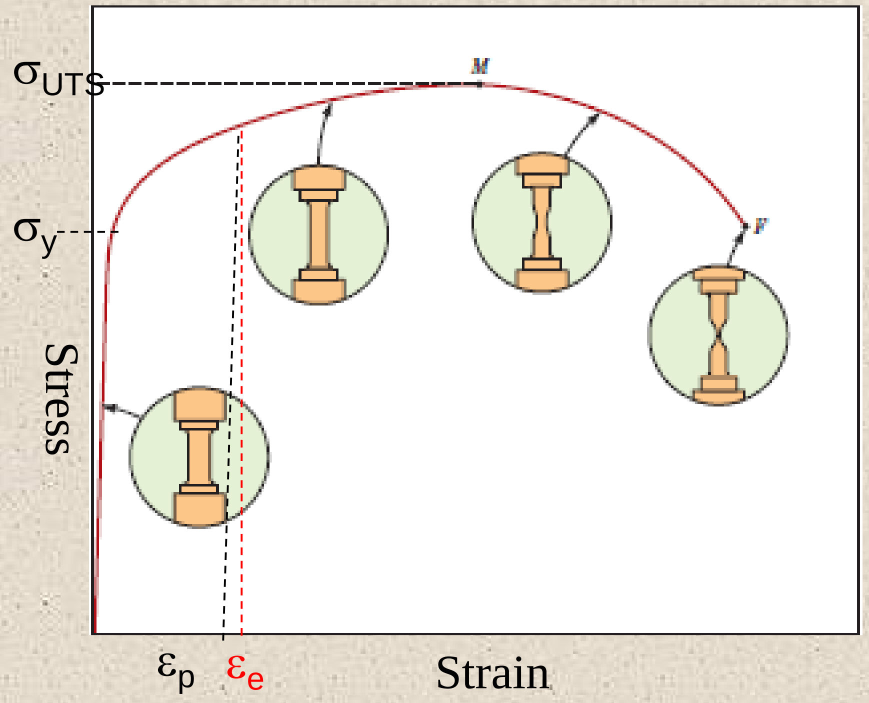
Young's modulus

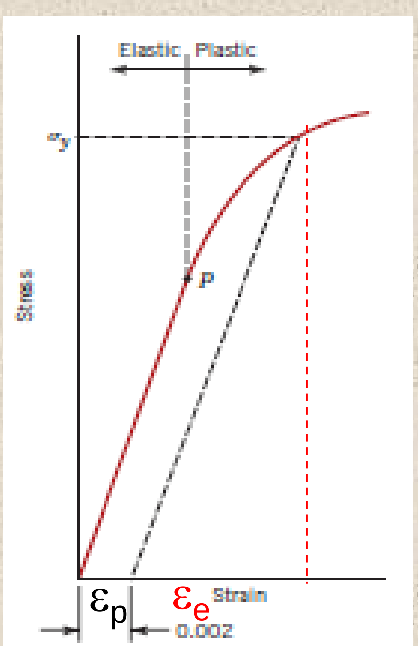
For uniaxial loading: $\sigma = E \cdot \varepsilon_e$



Elastic Energy: $\frac{J}{cm^3} = MPa$

$$W = \frac{1}{2} \cdot \sigma \cdot \varepsilon_e = \frac{1}{2} \cdot E \cdot \varepsilon_e^2 = \frac{1}{2} \cdot \frac{\sigma^2}{E}$$





Yield Stress (σ_y): the stress in which the plastic deformation begins. Engineering yield Stress is the stress in which the plastic deformation equal to 0.002.

Ultimate Tensile Strength (σ_{UTS}) or (σ_M): The maximum engineering tensile stress. An additional deformation will cause the formation of “neck” in the sample.

Fracture Energy (W_f): equal to the area below the Strain-Stress curve.

Total Strain (ε_{tot}): $\varepsilon_{total} = \varepsilon_p + \varepsilon_e$

There is no plastic stress

For shear stress:

$$\sigma_j = G \cdot \varepsilon_i$$

shear modulus

For isotropic material:

$$G = \frac{E}{2(\nu + 1)}$$

For Cubic:

$$E = \frac{1}{S_{11}} = \frac{(C_{11} - C_{12})(C_{11} + 2C_{12})}{(C_{11} + C_{12})}$$

$$G = \frac{1}{2(S_{11} - S_{12})} = \frac{(C_{11} - C_{12})}{2} \approx C_{44}$$

$$\nu = -\frac{S_{12}}{S_{11}}$$

$(in\ isotropic\ material\ A = \frac{2C_{44}}{(C_{11} - C_{12})}$=1)

Example 7.1

- Calculate the volume change for small strains.

$$\frac{\Delta V}{V_0}=\frac{V-V_0}{V_0}=\frac{(L_{x0}+\Delta L_x)\cdot(L_{y0}+\Delta L_y)\cdot(L_{z0}+\Delta L_z)-L_{x0}\cdot L_{y0}\cdot L_{z0}}{L_{x0}\cdot L_{y0}\cdot L_{z0}}\approx$$

$$\approx\frac{L_{x0}\cdot L_{y0}\cdot L_{z0}+\Delta L_x\cdot L_{y0}\cdot L_{z0}+\Delta L_y\cdot L_{z0}\cdot L_{x0}+\Delta L_z\cdot L_{x0}\cdot L_{y0}-L_{x0}\cdot L_{y0}\cdot L_{z0}}{L_{x0}\cdot L_{y0}\cdot L_{z0}}=$$

$$\varepsilon_x+\varepsilon_y+\varepsilon_z$$

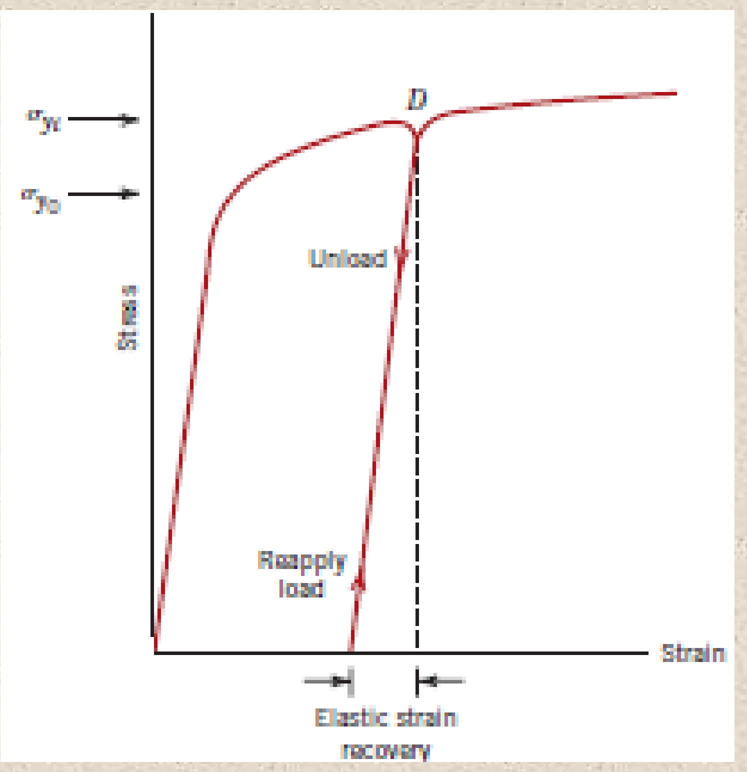
- Show that if the Poisson ratio is 1/2 the volume do not change under elastic deformation.

$$\frac{\Delta V}{V_0}=\varepsilon_x-\nu\cdot\varepsilon_x-\nu\cdot\varepsilon_x=\varepsilon_x\cdot(1-2\cdot\nu)=\frac{\sigma}{E}\cdot(1-2\cdot\nu)$$

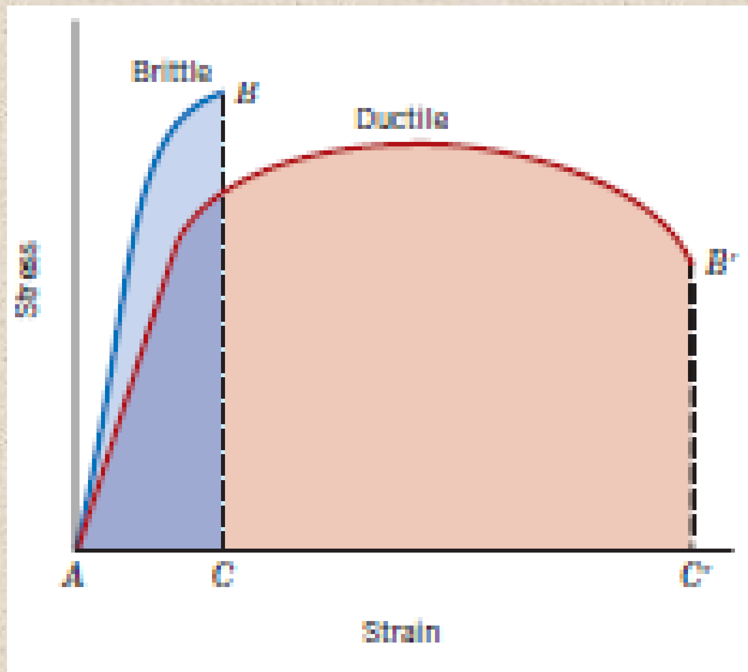
- **Hard- resistance to plastic deformation** (σ_y)
- **Stiff - resistance to elastic deformation** (E)
- **Tough- resistance to fracture** (W_f)

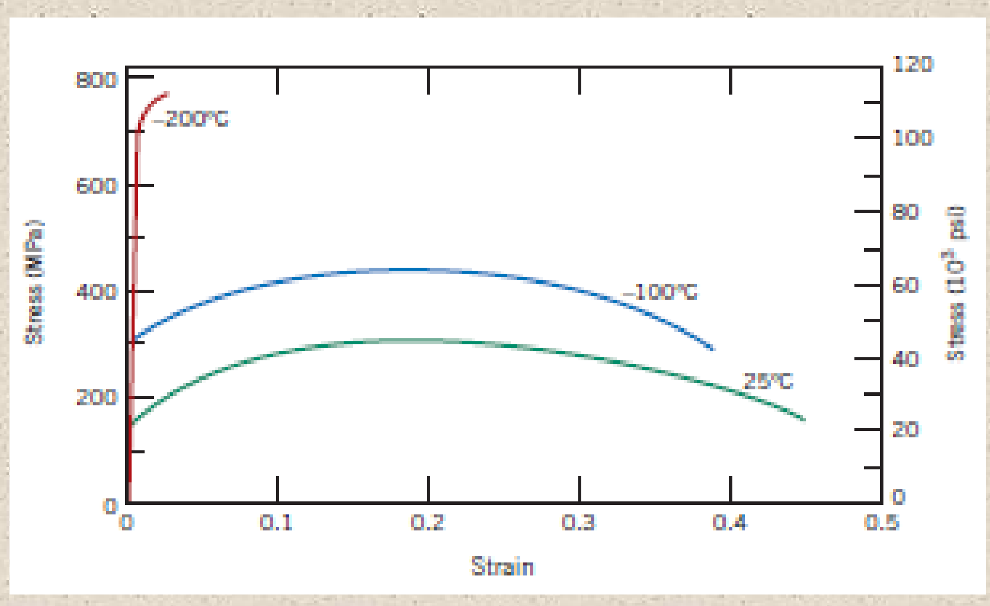
Example 7.2

- Show by using strain-stress curve that plastic deformation make the materials harder.



Ductile and Brittle Materials

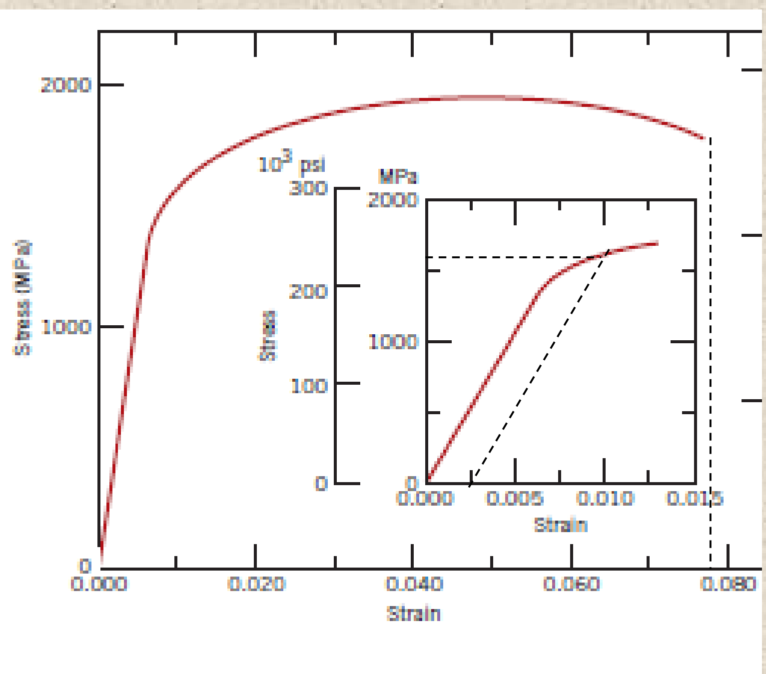




- **Brittle material** is a material that can not stand even small plastic deformation.
- **Ductile material** is a material that can undergo large plastic deformation.

Example 7.3

- Using the following stress-strain curve of 8mm diameter 150mm length steel bar calculate the yield force and the maximum elongation of the sample.



From the stress-strain curve the yield stress is 1600 MPa

$$F_y = \sigma_y \cdot A_0 = 1600 \cdot \pi \cdot 4^2 = 25{,}600\, N$$

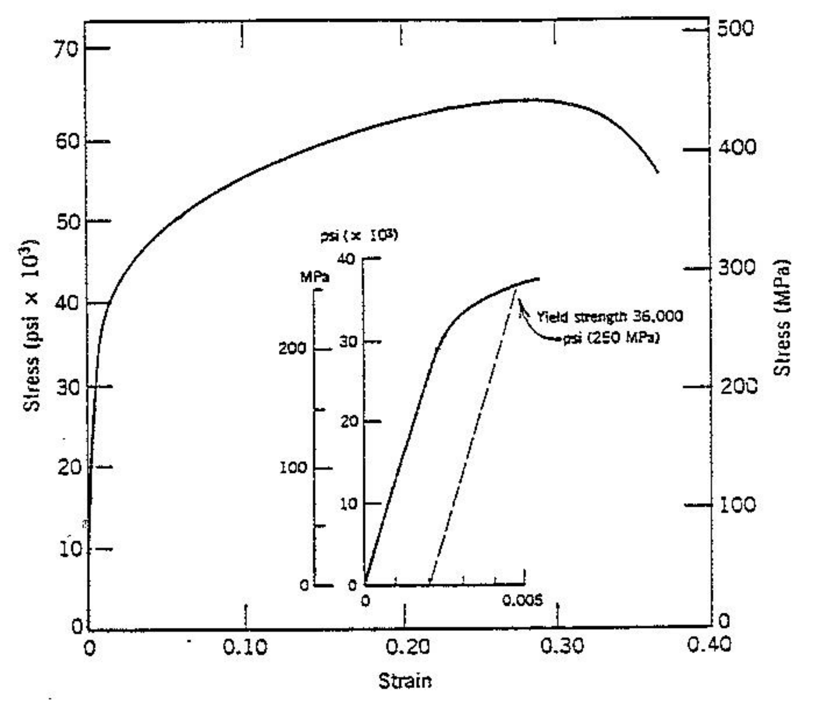
From the stress-strain curve the strain for break is 0.077

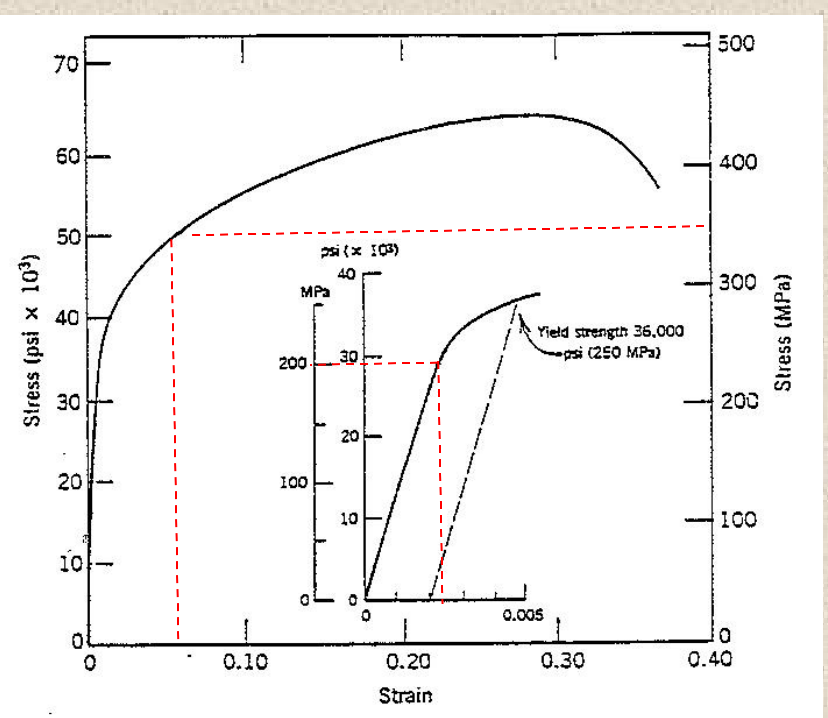
$$\Delta L = \varepsilon_f \cdot L = 0.077 \cdot 150 = 11.55 mm$$

Example 7.3

Using the following stress-strain curve of 10mm diameter 26mm length brass bar calculate:

1) The Young's modulus
2) The total elongation and the volume strain of the sample at load of 27000N, if the Poisson ratio of brass is 0.33





$r=5mm$; $L_0=26mm$; $\nu=0.33$

1.

$E=\sigma/\varepsilon_{el}=200/0.0022=91,000$ MPa=91GPa

2. a)

$\sigma=F/A=27000/(\pi r^2)=27000/(\pi\cdot 25)=343.77MPa$

$\varepsilon_{tot}=0.055$, $\Delta L=L_0\cdot\varepsilon$; $\Delta L_{tot}=26\cdot 0.055=1.43$ mm

b)

$\Delta V/V_0=\sigma/E\cdot(1-2\nu)=343.77/91000\cdot 0.34=0.0013$