Primer Materials
Spring 2021

Example 8.1

Calculate the bond stiffness of high purity diamond from its dielectric constant at low frequencies, ε_r=5.5. Density of diamond is 3.515 gm/cm^3.

$$el := 1.6\cdot 10^{-19}\cdot C$$

$$\varepsilon 0 := 8.85\cdot 10^{-14}\cdot\frac{F}{cm} \qquad Nav := 6.02\cdot 10^{23}\frac{1}{mol}$$

$$\varepsilon := 5.5$$

Number of atoms per unit volume $Nat := \dfrac{3.515\cdot\dfrac{gm}{cm^3}}{12\cdot\dfrac{gm}{mole}}\cdot Nav \qquad \dfrac{Nat}{cm^{-3}} = 1.763\times 10^{23}$

For pure electronic polarization $\varepsilon r := \varepsilon 0 + \dfrac{el^2\cdot Nat}{\omega^2\cdot mat} \qquad \omega^2\cdot mat := K$

Therefore

$$K := \frac{el^2\cdot Nat}{\varepsilon 0\cdot(\varepsilon - 1)} \qquad \frac{K}{\frac{N}{m}} = 113.351$$

Example 8.2

Dense fog makes ships invisible to a short-wave radars (GHz range 0.5-15 cm).
1. Can you guess why? What conclusion can you draw about the imaginary part of the dielectric constant of water at this frequencies?
2. Home microwave oven works almost in the same frequency range (2.45 GHz, wavelength of 12.24 cm). Why?
3. Can you think of a practical application of this effect to protect ships?

a) water is strong absorber at this frequencies. Thus the imaginary part of the dielectric constant is large.
b) home microwave works at the absorption band of water to warm the food.
c) If you create a dense water spray around the ship, it will become invisible for radars.

Example 8.3

The time necessary to bring a small quantity of water (<50 gm) from room temperature (25^o C) to a boiling point in a home microwave oven (f=2.45 GHz) is practically independent on the quantity of water (**do not test this at home!!!**). The imaginary part of the dielectric constant of water at 2.45 GHz is 6.64. The time necessary to boil water is ≈35 sec. Calculate the strength of the electric field in the oven. What is the reason that such a small electric field can heat so efficiently?

thermal capacitance

$t := 35s$ $\quad \Delta T := 75 \cdot K$ $\quad \varepsilon := 76.92 + i \cdot 6.64$ $\quad f := 2.45 \cdot GHz$ $\quad Cv := 1 \cdot \frac{cal}{kg \cdot K} = 4.184\ J \cdot gm^{-1} \cdot K^{-1}$

$\omega := 2 \cdot \pi \cdot f$ $\quad \rho := 1 \cdot \frac{gm}{cm^3}$

$P = j \cdot E$

$P := Im(\varepsilon) \cdot \omega \cdot E^2 \cdot \varepsilon 0$ delivered power per unit volume

$Q := \Delta T \cdot Cv \cdot \rho$ required amount of heat per unit volume $\frac{Q}{\frac{J}{cm^3}} = 314$

time = Q/P

$$t := \frac{\Delta T \cdot Cv \cdot \rho}{Im(\varepsilon) \cdot \omega \cdot E^2 \cdot \varepsilon 0} \qquad E := \sqrt{\frac{\Delta T \cdot Cv \cdot \rho}{Im(\varepsilon) \cdot \omega \cdot \varepsilon 0 \cdot 35 \cdot s}} \qquad \frac{E}{\frac{V}{cm}} = 31.49$$

Although the electric field seems to be weak, the power is large because the conductivity of water at this frequency is quit high (at ω=0, resitiivty is 18 M Ω *cm)

$\sigma := Im(\varepsilon) \cdot \omega \cdot \varepsilon 0$

$\frac{\sigma}{\Omega^{-1} \cdot cm^{-1}} = 9.046 \times 10^{-3}$ so that the current density is $j := \sigma \cdot E$ $\quad \frac{j}{\frac{mA}{cm^2}} = 285$

$\frac{\frac{1}{\sigma}}{\Omega \cdot cm} = 110.546$ and the power is $P := Im(\varepsilon) \cdot \omega \cdot E^2 \cdot \varepsilon 0$ $\quad \frac{P}{\frac{W}{cm^3}} = 8.972$

Example 8.4

Ordinary glass (sodium glass) has a refractive index of 1.41. Bohemian glass has a refractive index of 1.67. Which of these glasses would you like to use as a window glass? Why? Give a numerically motivated answer. (Consider light passing through a window glass and neglect all inner reflection except for the first one)

Reflection increases with increasing n. Thus to let maximum light in n must be minimized.

Transmission through a thick parallel glass plate is close to be proportional to the square of the transmission coefficient at the surface (two reflections outer and inner)

$$n1 := 1.41 \qquad T1 := \frac{4n1}{(n1+1)^2} \qquad T1^2 = 0.943$$

$$n2 := 1.67 \qquad T2 := \frac{4n2}{(n2+1)^2} \qquad T2^2 = 0.878 \qquad \frac{T1^2}{T2^2} = 1.074$$

Example 8.5

UV photolithography uses mirrors instead of lenses. Why? Calculate the shortest wavelength at which Ag mirror can still be used. Use the data from the lecture notes

$$c := 3 \cdot 10^{8} \cdot \frac{m}{s} \qquad h := 6.62 \cdot 10^{-34} \cdot J \cdot s \qquad eV := el \cdot V$$

$$Emeas := 3.93 \cdot eV \qquad f := \frac{Emeas}{h} \qquad f = 9.498 \times 10^{14} Hz$$

$$\lambda := \frac{c}{f} \qquad \lambda = 3.158 \times 10^{-7} m$$

Example 8.6

Density of thin films deposited in vacuum by electron beam evaporation or sputtering is often determined by comparing their refractive index with the refractive index of the corresponding bulk materials.

1. Why does this method is only applicable for comparison the films of the same (close) composition?
2. Derive the formula relating the densities and the refractive indices.
3. The method best works for fluorides (MgF_2, CaF_2) but it is significantly less accurate for the case of oxides (SiO_2, TiO_2 and Al_2O_3).

a) the ionic polarizability should be the same.

b) $\dfrac{\varepsilon_r - 1}{\varepsilon_r + 2} = \dfrac{1}{3\varepsilon_0}\sum_i N_i \alpha_i \Rightarrow \dfrac{\varepsilon_r - 1}{\varepsilon_r + 2} \propto \rho$; ρ is density

$$n = \sqrt{\varepsilon_r}; \qquad \frac{\rho_1}{\rho_2} = \left(\frac{n_1^2 - 1}{n_1^2 + 2}\right)\left(\frac{n_2^2 + 2}{n_2^2 - 1}\right)$$

c) Fluorides are not conductive even if they are non-stoichiometric. Non-stoichiometry in oxides causes them to be conductive. Therefore, the refractive index has a large imaginary part. So that the Clausius-Mossotti approach becomes inapplicable.