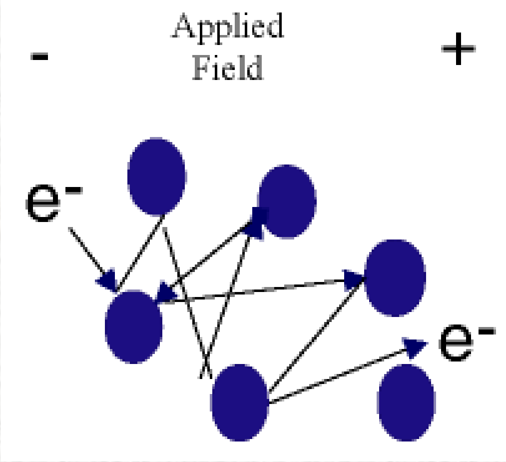
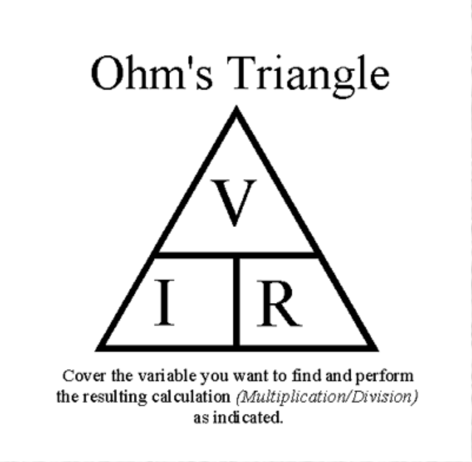
Electronic properties





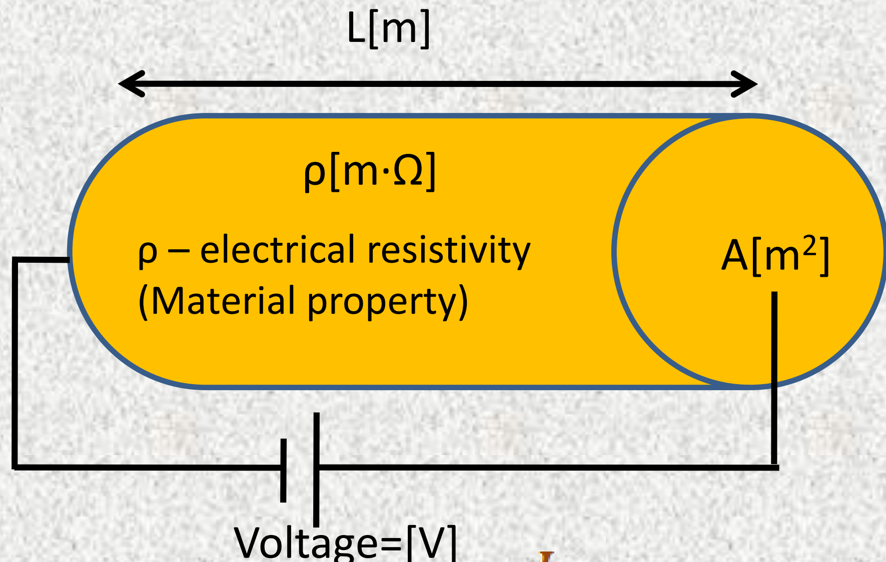
Primer in Materials

Spring 2021

Ohm's low

$$I = \frac{V}{R}$$

I – current [A]
V – Voltage [V]
R – Resistance [Ω]



Power dissipation:

$$P = I^2 R = V \cdot I = \frac{V^2}{R}$$

P – Watt [W]

$$R = \frac{L \cdot \rho}{A}$$

Resistance has geometrical dependence

$$\sigma = 1/\rho$$

σ – electrical conductivity [S/m]

$$J = I/A$$

J – Current density (flux, [A/m²])

$$J = \sigma \cdot E$$

$$E = V/L$$

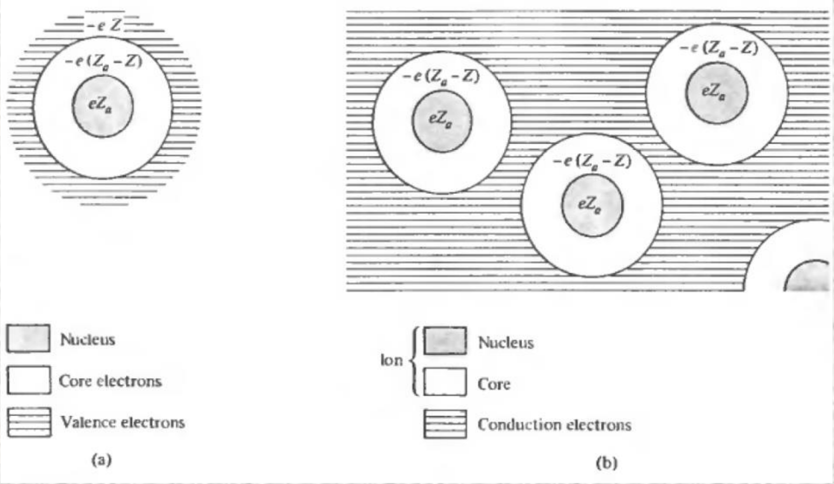
E – electric field [V/m]

$$p = J \cdot E = \sigma \cdot E^2$$

p – Watt [W/m³]

Drude model

n – electron density
n_i – metal ion density
e – electron charge
m_e- electron mass
μ- mobility
a- atomic (metallic) radius



Assumptions

- All atoms gives their valance electron to the ‘sea of electron’
- Ion are localized, Electron moves in straight lines between collisions
- Relaxation time: τ – time between collisions

$$\sigma = \mu n e \qquad \sigma = \frac{ne^2\tau}{m_e}$$

The electrical conductivity related to the relaxation time

$$\mu = \frac{e\tau}{m_e} \qquad \tau = \frac{m_e}{\rho n e^2} = \frac{1}{n_i \pi a^2} \cdot \sqrt{\frac{m_e}{3KT}}$$

Typical relaxation times

Table 1.2
ELECTRICAL RESISTIVITIES OF SELECTED ELEMENTS[a]

ELEMENT	77 K	273 K	373 K	$\frac{(\rho/T)_{373\ K}}{(\rho/T)_{273\ K}}$
Li	1.04	8.55	12.4	1.06
Na	0.8	4.2	Melted	
K	1.38	6.1	Melted	
Rb	2.2	11.0	Melted	
Cs	4.5	18.8	Melted	
Cu	0.2	1.56	2.24	1.05
Ag	0.3	1.51	2.13	1.03
Au	0.5	2.04	2.84	1.02
Be		2.8	5.3	1.39
Mg	0.62	3.9	5.6	1.05
Ca		3.43	5.0	1.07
Sr	7	23		
Ba	17	60		
Nb	3.0	15.2	19.2	0.92
Fe	0.66	8.9	14.7	1.21
Zn	1.1	5.5	7.8	1.04
Cd	1.6	6.8		
Hg	5.8	Melted	Melted	
Al	0.3	2.45	3.55	1.06
Ga	2.75	13.6	Melted	
In	1.8	8.0	12.1	1.11
Tl	3.7	15	22.8	1.11
Sn	2.1	10.6	15.8	1.09
Pb	4.7	19.0	27.0	1.04
Bi	35	107	156	1.07
Sb	8	39	59	1.11

[a] Resistivities in microhm centimeters are given at 77 K (the boiling point of liquid nitrogen at atmospheric pressure), 273 K, and 373 K. The last column gives the r… approximate linear temperature d… ture.
S… Physical and Chemical Constants, Longmans Green, London, 1966.

[μΩ·cm] = 1X10^{-8}[Ω ·m]

Table 1.3
DRUDE RELAXATION TIMES IN UNITS OF 10^{-14} SECOND[a]

ELEMENT	77 K	273 K	373 K
Li	7.3	0.88	0.61
Na	17	3.2	
K	18	4.1	
Rb	14	2.8	
Cs	8.6	2.1	
Cu	21	2.7	1.9
Ag	20	4.0	2.8
Au	12	3.0	2.1
Be		0.51	0.27
Mg	6.7	1.1	0.74
Ca		2.2	1.5
Sr	1.4	0.44	
Ba	0.66	0.19	
Nb	2.1	0.42	0.33
Fe	3.2	0.24	0.14
Zn	2.4	0.49	0.34
Cd	2.4	0.56	
Hg	0.71		
Al	6.5	0.80	0.55
Ga	0.84	0.17	
In	1.7	0.38	0.25
Tl	0.91	0.22	0.15
Sn	1.1	0.23	0.15
Pb	0.57	0.14	0.099
Bi	0.072	0.023	0.016
Sb	0.27	0.055	0.036

[a] Relaxation times are calculated from the data in Tables 1.1 and 1.2, and Eq. (1.8). The slight temperature dependence of n is ignored.

Drift velocity

V_{th} – thermal velocity
E – electric field
V_{drift} – drift velocity

$$V_{th} = \sqrt{\frac{3KT}{m_e}}$$

$$J = \sigma E = nev_{drift}$$

- When J = 0 the drift velocity is zero. Meaning that in average each electron has zero displacement
- When J ≠ electron moves with typical velocity of v_{drift}

Electron moves in metal

Example 9.1

Calculate DC conductivity of Na at room temperature using Drude model. What would you expect the conductivity of Li to be if 10% of the Li were replaced by He?

$density := 0.97 \cdot \frac{gm}{cm^3}$ number of ions per cm^3 $n := \frac{density}{23 \cdot \frac{gm}{mole}} \cdot Na$ $\frac{n}{cm^{-3}} = 2.539 \times 10^{22}$

$m_e = 9.10938215 \times 10^{-31}$ Kg KT= 4.11×10^{-21}J $e = 1.602 \times 10^{-19}$ C

covalent radius $a := 0.154 \cdot nm$

a) The relaxation time

$$\tau := \frac{\sqrt{me}}{n \cdot \pi \cdot a^2 \cdot \sqrt{3 \cdot k \cdot T}} \qquad \frac{\tau}{10^{-15}} = 4.525\,s$$

conductivity

$$\sigma := \frac{n \cdot e^2 \cdot \tau}{me} \qquad \sigma 1 := \frac{e^2}{\sqrt{me}} \cdot \frac{1}{\pi \cdot a^2 \cdot \sqrt{3\ TK}} \qquad \frac{\sigma 1}{S \cdot m^{-1} \cdot 10^6} = 3.232$$

Note: experimental value is 21.2

b) The relaxation time will not change significantly because the number of scattering centers will remain. The electron concentration and the condcutivity will drop accordingly by 10%

Example 9.2

The Hall constant of silver is 1.19 times larger than theoretical ($R_h \cdot e \cdot n = 1.19$) and the plasma frequency is only 3.93 eV instead of the theoretical 8.98 eV.

a) What can you conclude about the real number of free electrons contributed by one atom?

b) Does the plasma frequency agree with Hall data? Can you offer a reasonable explanation?

$$R_h = \frac{1}{e \cdot n_{real}} \qquad \omega_p = e \cdot \sqrt{\frac{n_{real}}{m_e \cdot \varepsilon_0}}; \mathrm{E} = \hbar \cdot \omega_p$$

1. Both number are hardly reconcilable.
According to Hall effect $n_{nominal}/ n_{real}=1.19$. n_{real} is smaller than $n_{nominal}$ by that factor of 1.19.
According to the plasma frequency n_{real} is smaller than $n_{nominal}$ by a factor of $(8.93/3.93)^2=5$.

2. It is clear from the above that the only way how the numbers can agree with each other, is that the effective mass of the electron in Ag differs from the effective mass of a free electron, *i.e.* the band structure of Ag is quite complicated.

Example 9.3

The resistivity of Cu is 1.7×10^{-8} Ωm at 300 K and the electron density is 8.5×10^{28} m^{-3}.

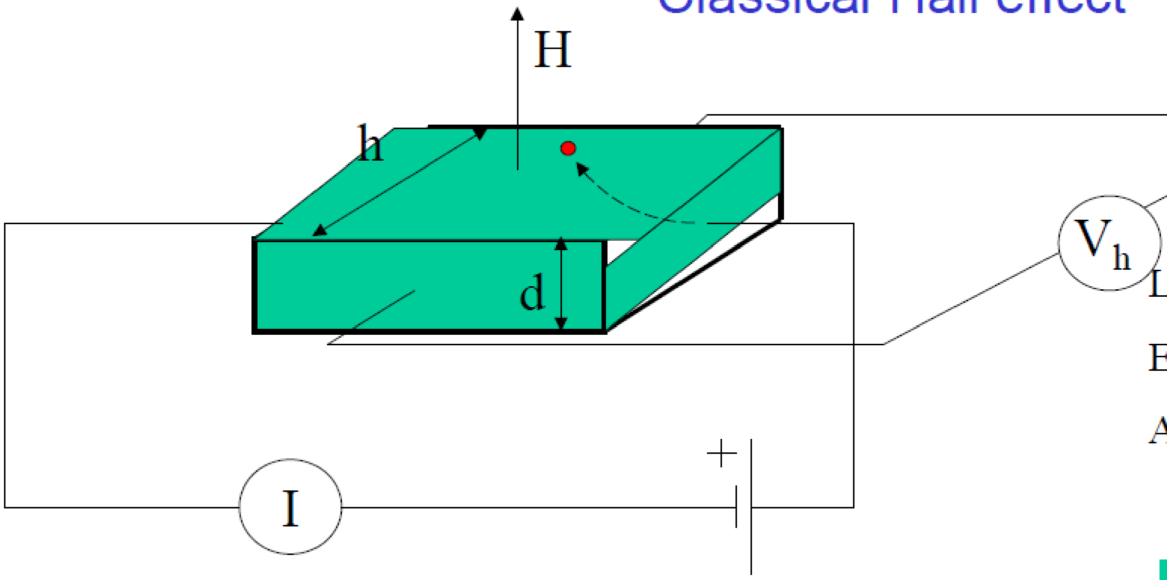
(a) Calculate the relaxation time of electrons in Cu at 300 K .

(b) Calculate the mean free path of the electrons using Drude approximation.

a) $\tau := \dfrac{me}{\rho \cdot n \cdot e^2}$ $\quad \tau = 2.46 \times 10^{-14} s$

b) $Vther := \left(\dfrac{3 \cdot k \cdot T}{me}\right)^{\frac{1}{2}}$ $\quad lther := Vther \cdot \tau$ $\quad \dfrac{lther}{10^{-9} \cdot m} = 2.874$ nm

Classical Hall effect



Lorentz force: $F_L = e\, v \times H$;

Electrostatic force: $F_E = Ee = V_h e / l$

At equilibrium $F_L = F_E$ and

$$v \times H = V_h / h$$

$$V_h = \frac{HI}{end} = R_h H \frac{I}{d}$$

Current density is related to the velocity as $ven = I/(dh)$ =>

$$R_h = \frac{1}{en}$$

R_h is Hall constant

Classical theory predicts that $-R_h en = 1$

Example 9.4

Prove that the combination of Hall effect measurements and resitivity measurements permits determination of the electron relaxation time

$$R_h = \frac{1}{\mathrm{e} \cdot \mathrm{n}}$$

$$\text{Hall voltage } \mathrm{V_h} = R_h \cdot \frac{IH}{d} = \frac{\mathrm{H} \cdot \mathrm{I}}{\mathrm{e} \cdot \mathrm{n} \cdot \mathrm{d}} \Rightarrow \mathrm{n} = \frac{\mathrm{H} \cdot \mathrm{I}}{\mathrm{e} \cdot \mathrm{d} \cdot \mathrm{V_h}}$$

$$\text{Conductivi ty } \sigma = \frac{\mathrm{ne}^2 \tau}{\mathrm{m_e}} \Rightarrow \tau = \frac{\mathrm{m_e} \sigma}{\mathrm{ne}^2} = \frac{\mathrm{m_e} \cdot \sigma \cdot \mathrm{d} \cdot \mathrm{V_h}}{\mathrm{H} \cdot \mathrm{I} \cdot \mathrm{e}}$$