Notation and preliminaries: The $\tilde{O}(\cdot)$ notation hides polylogarithmic (in $n$) factors.

1 SETH-Hardness

Assuming SETH, prove or disprove the following statements:

(a) Given two sets of $n$ vectors $A, B \subseteq \{0, 1\}^d$ where $d = \log^2 n$ we can find the pair $a \in A, b \in B$ with maximum inner product $\max_{a \in A, b \in B} \langle a, b \rangle = \max_{a \in A, b \in B} \sum_{i=1}^d a[i] \cdot b[i]$ in $\tilde{O}(n^{1.5})$ time.

(b) Given two rooted trees $H$ and $G$ of total size $n$ we can decide if $H$ is isomorphic to a subtree of $G$ in $\tilde{O}(n^{1.5})$ time. The isomorphism must map the root of $H$ to the root of $G$ but is otherwise unrestricted: it is a mapping $f$ from the nodes of $H$ to the nodes of $G$ such that if $x$ is a child of $y$ in $H$ then $f(x)$ is a child of $f(y)$ in $G$.

(c) OV on $n^{2/3}$ binary vectors of dimension $d = n^{1/3}$ can be solved in $\tilde{O}(n^{1.5})$ time.

2 SETH implies ETH

Prove that if ETH is false, then so is SETH. In other words, show a reduction from $k$-SAT to 3-SAT establishing that if 3-SAT can be solved in $O(2^{\delta n})$ time for all $\delta > 0$, then there is an $\varepsilon > 0$ such that for all $k \geq 3$ we can solve $k$-SAT in $O(2^{(1-\varepsilon)n})$ time.

\footnote{Note that the trees are unordered, i.e. there is no particular order on the children of a node that must be preserved by the mapping. By the way, in the ordered case the answer is positive: there are $\tilde{O}(n)$ time algorithms using Fast Fourier Transform.}