Notation and preliminaries: The $\tilde{O}(\cdot)$ notation hides polylogarithmic (in n) factors.

1 SETH-Hardness

Assuming SETH, prove or disprove the following statements:

(a) Given two sets of n vectors $A, B \subseteq \{0,1\}^d$ where $d = \log^2 n$ we can find the pair $a \in A, b \in B$ with *maximum* inner product $\max_{a\in A, b\in B}\langle a, b\rangle = \max_{a\in A, b\in B}\sum_{i=1}^{d} a[i]\cdot b[i]$ in $\tilde{O}(n^{1.5})$ time.

(b) Given two rooted trees H and G of total size n we can decide if H is isomorphic to a subtree of G in $\tilde{O}(n^{1.5})$ time. The isomorphism must map the root of H to the root of G but is otherwise unrestricted: it is a mapping f from the nodes of H to the nodes of G such that if x is a child of y in H then $f(x)$ is a child of $f(y)$ in G.[1]

(c) OV on $n^{2/3}$ binary vectors of dimension $d = n^{1/3}$ can be solved in $\tilde{O}(n^{1.5})$ time.

2 SETH implies ETH

Prove that if ETH is false, then so is SETH. In other words, show a reduction from k-SAT to 3-SAT establishing that if 3-SAT can be solved in $O(2^{\delta n})$ time for all $\delta > 0$, then there is an $\varepsilon > 0$ such that for all $k \geq 3$ we can solve k-SAT in $O(2^{(1-\varepsilon)\cdot n})$ time.

[1]Note that the trees are *unordered*, i.e. there is no particular order on the children of a node that must be preserved by the mapping. By the way, in the ordered case the answer is positive: there are $\tilde{O}(n)$ time algorithms using Fast Fourier Transform.