Fine-Grained Complexity 20224191, Final Exam **Due on:** March 1st, 22:00, 2022
Please write solutions in *English*, ideally in LaTeX, and submit to: fgcweizmann@gmail.com

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Guidelines: You are expected to solve the exam on your own, and without looking for answers in research papers or online forums. If you do get ideas from others or online, please write this in your solution.

The task: For each of the following problems, determine the time complexity up to $n^{0.001}$ factors, assuming the standard conjectures we saw in class. That is, for each problem:

1. Design an algorithm proving that the problem can be solved in time $O(n^c)$.
2. Prove a matching lower bound of $\Omega(n^{c-\varepsilon})$ conditioned on one of 3SUM, APSP, or the SETH conjectures.

Please provide complete proofs of the lower bounds (you may use any of the results we saw in the course). For the upper bounds, it is enough to sketch the ideas in the algorithm without formal correctness proofs.

1 Importance of a Node

The input is a weighted undirected graph $G = (V, E, w), w : E \to \{1, \ldots, U\}$ with $|V| = n$ and $U = n^{O(1)}$ and a special node $x \in V$. The goal is to compute the *importance* of x defined as the number of pairs with a shortest path going through x:

$$I(x) = |\{s, t \in V \ \mid \ \text{there exists a shortest } s,t\text{-path that contains } x\}|.$$

2 Three-Hop-Sum

The input is a weighted undirected graph $G = (V, E, w), w : E \to \{-U, \ldots, U\}$ with $|V| = n$ and $U = n^{O(1)}$. A *three-hop* is a set of four nodes $a, b, c, d \in V$ such that $\{a, b\}, \{b, c\}, \{c, d\} \in E$ are edges, and the weight of the three-hop is the sum of its three edges. The goal is to determine if G contains a three-hop of weight zero.

3 Best Match

The input is two strings S, T of length n over an alphabet Σ of size 10. For a substring[1] x of S and a substring y of T, such that $|x| = |y|$, we define the *match-score* α to be the number of indices on which they agree minus the number of indices on which they disagree, i.e. $\alpha(x, y) = |\{i \mid x[i] = y[i]\}| - |\{i \mid x[i] \neq y[i]\}|$. The goal is to compute the maximal match-score of any two same-length substrings x, y of S, T, i.e. the maximum $\alpha(x, y)$ over all pairs of substrings x, y (that could have any length from 0 to n).

[1] Unlike a subsequence, a substring must be contiguous (i.e. no skipping letters).