Fine-Grained Complexity 20224191 List of Open Questions
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This document summarizes some open questions that we saw in class. You are welcome to send me an e-mail and ask for the relevant references (or recent developments) for any of them. You are also welcome to do a research project with me on any of them. The number of stars indicates how hard I expect them to be.

1 From Lecture 2

- Either (**) solve 3-SUM with $U = n^2$ in $O(n^{2-\varepsilon})$ time (refuting the Strong 3-SUM Conjecture), or (*) reduce the general case (with $U = n^3$ as we saw) to the $U = n^2$ case (showing an equivalence between the 3SUM and Strong-3SUM conjectures).
- (***) Prove that k-SUM cannot be solved in $O(n^{\lceil k/2 \rceil - \varepsilon})$ time, for any $\varepsilon > 0$, assuming SETH.
- (**) Prove that k-SUM cannot be solved in $O(n^{k/2-\varepsilon})$ time, for any $\varepsilon > 0$, assuming SETH.
- (*) Prove that k-SUM cannot be solved in $O(n^{k/1000})$ time, for any $\varepsilon > 0$, assuming SETH.

2 From Lecture 3

- (*) Prove that deciding if there is a line that passes through *four* among a set of n points in the plane is 3SUM-hard.
- Hardness of approximation: (**) Prove 3SUM hardness for distinguishing between the *no-case* where no line passes through > 2 points among a set of n points in the plane, and the *yes-case* where there is a line that passes through ≥ 100 points.