Fine-Grained Complexity - Lecture 8:

SETH-Hardness for Longest Common Subsequence

Sequences (a.k.a. strings):

An ordered set of n symbols over some alphabet Σ.

Examples:

DNA, text, programs, machine code, curves (e.g. GPS trajectories), time-series (e.g. audio), …

20.12.21

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(and other similarity measures)

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(and "better" SETH)

(and other similarity measures)

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Longest Common Subsequence (**LCS**)

Input: two sequences of length n

S = cddcabbabcbaa

T = adbdbbcabacdd

Output: the length of the longest common subsequence

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Classic Dynamic Programming: $O(n^2)$ [Wagner-Fisher '74]

$$M[i,j] = \max \begin{cases} M[i-1, j-1] + (S[i] == T[j]), \\ M[i-1, j], \\ M[i, j-1] \end{cases}$$

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[Masek - Paterson '80]

$$O\left(\frac{n^2}{\log n}\right)$$

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Classic Dynamic Programming: *O(n²)* → $O(n^2)$ [Wagner-Fisher ’74]

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[Masek - Paterson ’80]

$$O\left(\frac{n^2}{\log n}\right)$$

Can we do better?

How similar are two sequences?

The following popular similarity measures can be computed in $O(n^2)$ time with dynamic programming:

- **LCS**
- **Edit-Distance**
- **Local Alignment (BLAST)**
- **Frechet**
- **Dynamic Time Warping**

And cannot be computed in $O(n^{1.99})$ time assuming SETH.

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A celebrated result!

[Backurs-Indyk STOC'15]:

"$\forall \varepsilon > 0 : O(n^{2-\varepsilon})$ for **Edit Distance** refutes SETH"

The Boston Globe

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BRAINIAC

For 40 years, computer scientists looked for a solution that doesn't exist

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- Paris attacks: Baker, others wrong to reject Syrian refugees
- Paris attacks | Andrew J. Bacevich: A war the West cannot win
- A Patriots-Giants rematch in February: You can't beat that
- How will Patriots fill void while Julian Edelman is out?

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[A. - Backurs - Vassilevska W. FOCS'15] and [Bringmann - Kunnemann FOCS'15]:

"$\forall \varepsilon > 0 : O(n^{2-\varepsilon})$ for **LCS** refutes SETH"

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The Reduction:

OV → **LCS**

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The Reduction:

OV $\longrightarrow$ **LCS**

OV: Given $A, B \subseteq \{0,1\}^d$, $|A| = |B| = n$,
is there $a \in A$, $b \in B$ s.t. $\langle a, b \rangle = 0$?

Last time: $\forall \varepsilon > 0 : O(n^{2-\varepsilon} \cdot \text{poly}(d))$
for OV refutes SETH.

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The Reduction:

OV → **LCS**

An instance of OV on two sets of n vectors of dimension d

$O(n')$ time

An instance of LCS on two strings of length $n' = O(n \cdot \text{poly}(d))$

OV: Given $A, B \subseteq \{0,1\}^d$, $|A| = |B| = n$, is there $a \in A$, $b \in B$ s.t. $\langle a, b \rangle = 0$?

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Reduction Plan:

OV: Given $A, B \subseteq \{0,1\}^d$, $|A| = |B| = n$, is there $a \in A$, $b \in B$ s.t. $\langle a, b \rangle = 0$?

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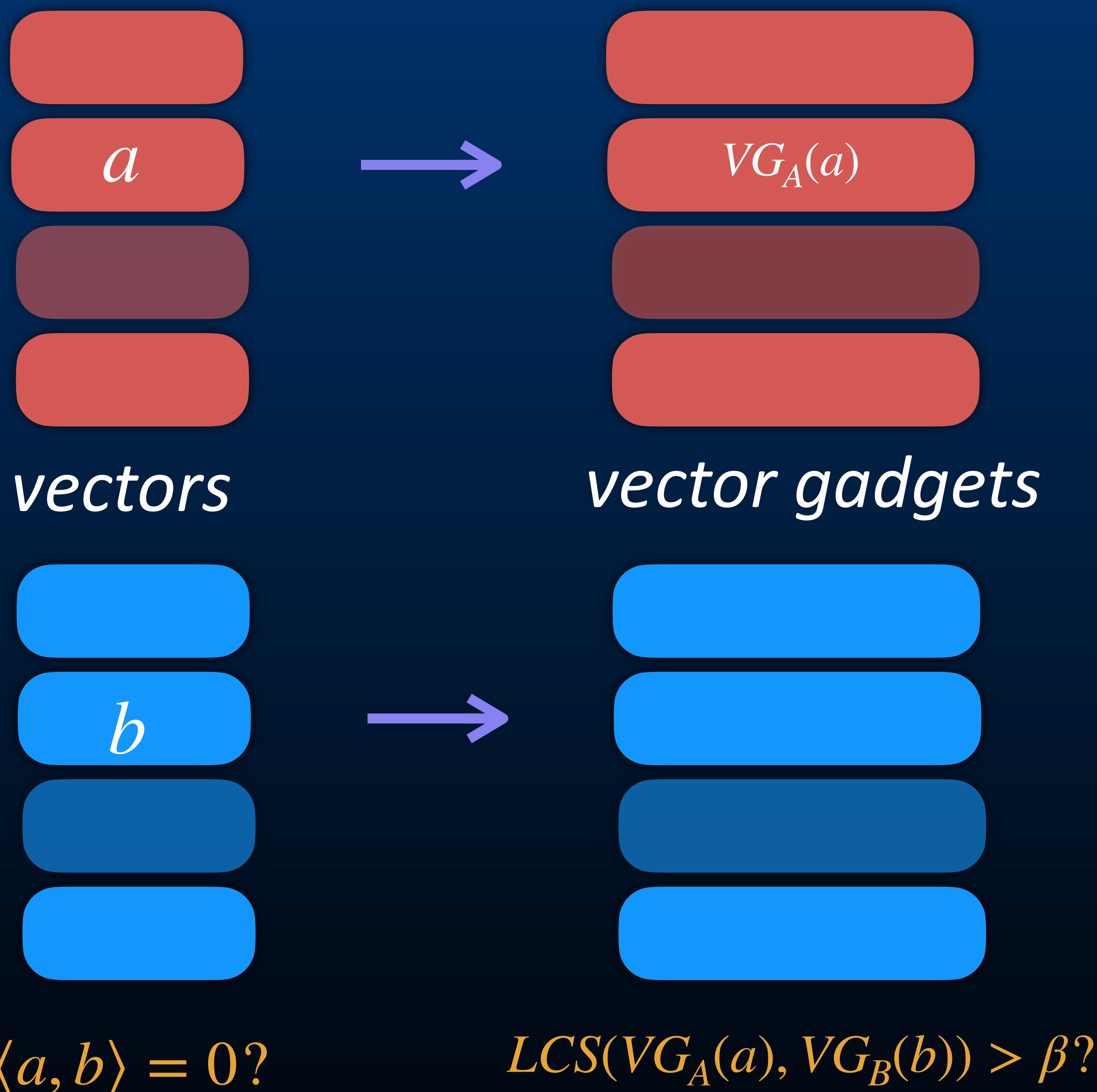

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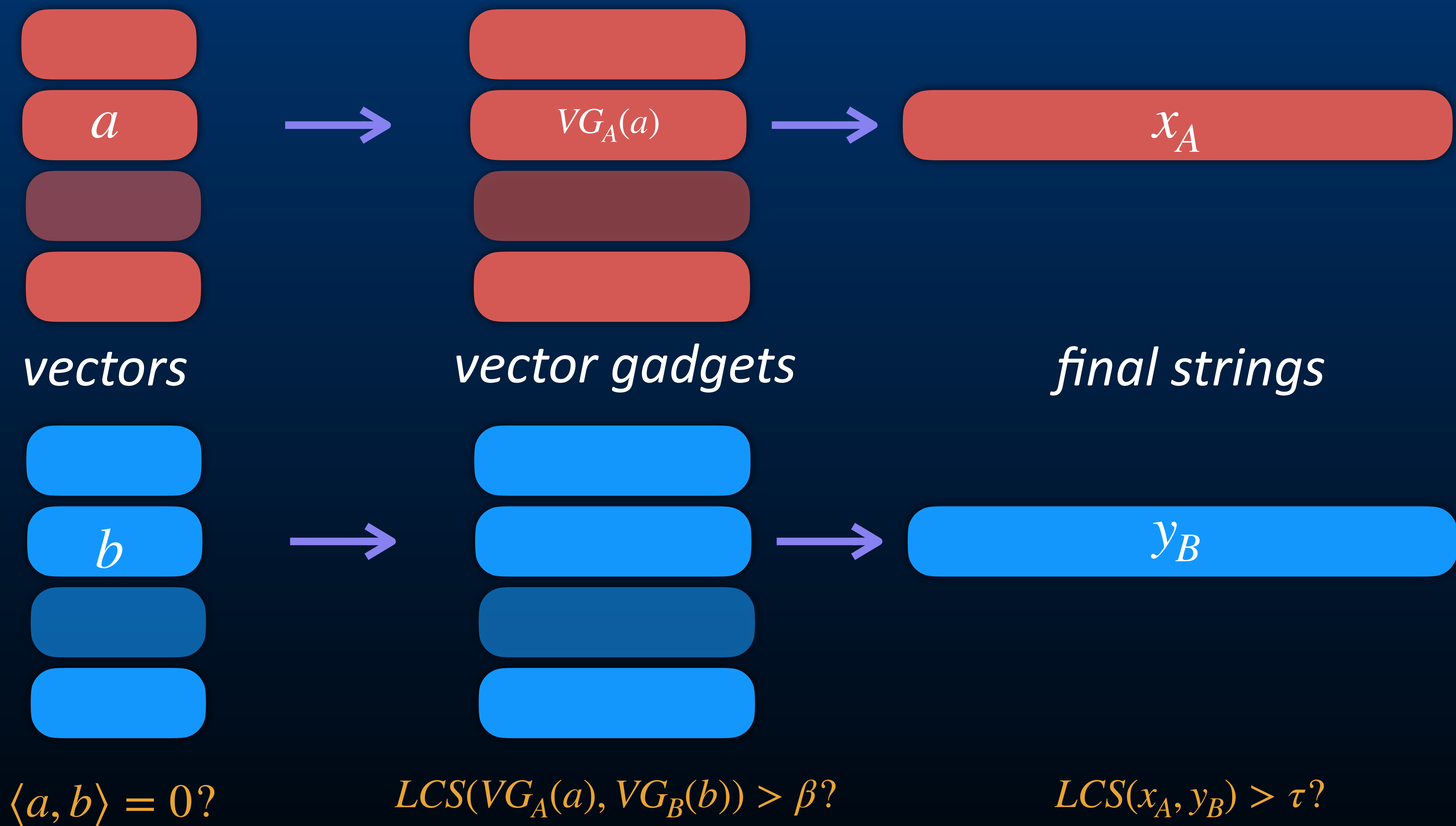

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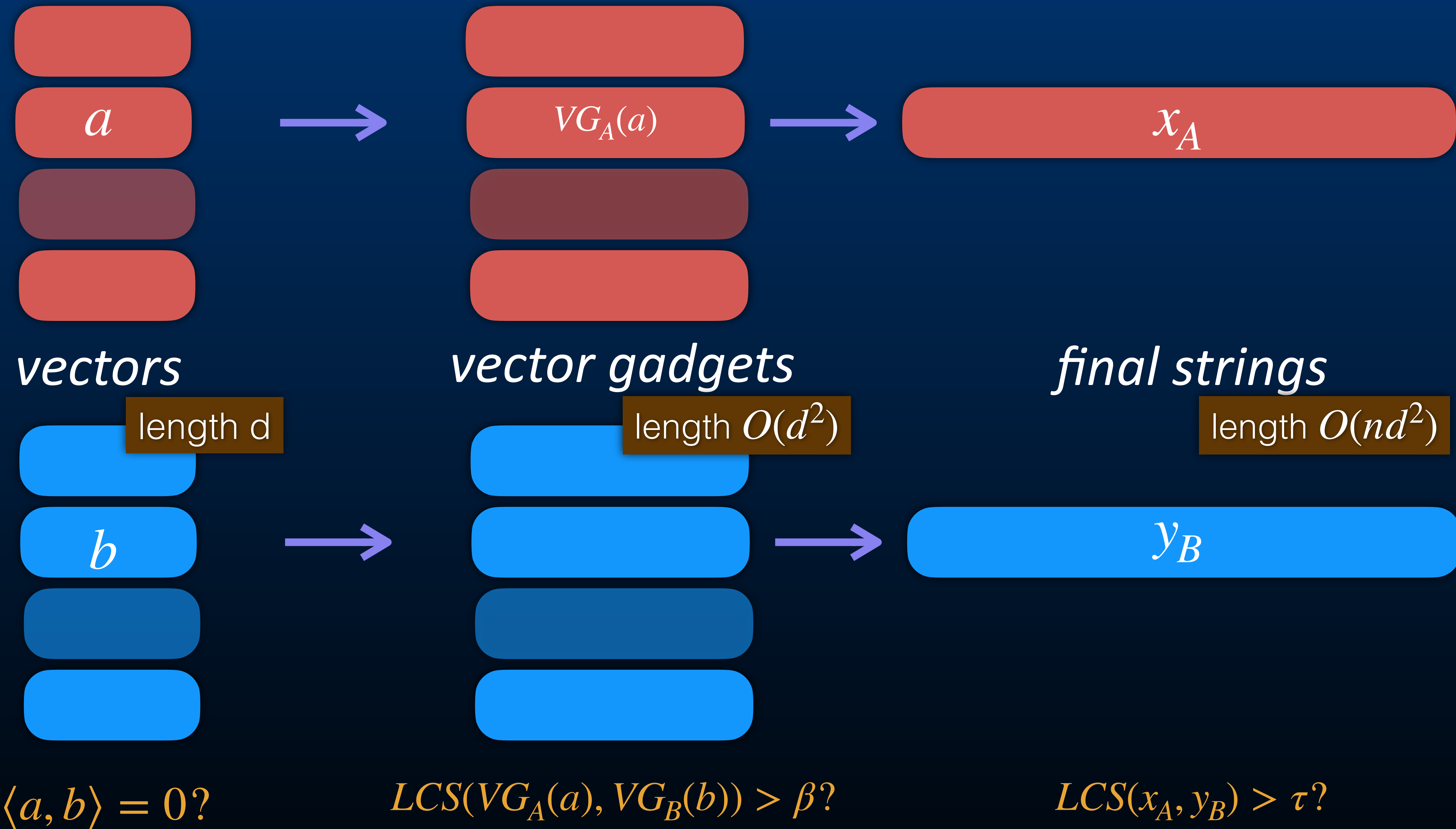

Step 0: **Coordinate Gadgets**

Lemma 0: There are atomic strings $CG_A(0), CG_A(1)$ and $CG_B(0), CG_B(1)$ of length 3 s.t. for any $s, t \in \{0,1\}$:

$s \cdot t = 0$	$\Longrightarrow$	$LCS(CG_A(s), CG_B(t)) = 2$	(Large LCS)
$s \cdot t \neq 0$	$\Longrightarrow$	$LCS(CG_A(s), CG_B(t)) = 0$	(Small LCS)

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Proof:

$CG^A(0) :=$

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Proof:

$CG^A(0)$:= 001

$CG^A(1)$:= 111

$CG^B(0)$:= 011

$CG^B(1)$:= 000

Step 1: **Vector Gadgets**

Lemma 1: There are mappings $VG_A, VG_B : \{0,1\}^d \rightarrow \{0,1,2,3\}^{6d^2-d-2}$ computable in $O(d^2)$ time s.t. for any vectors $a, b \in \{0,1\}^d$:

$\langle a,b \rangle = 0 \implies LCS(VG_A(a), VG_B(b)) = \beta_d + 2$ (Large LCS)

$\langle a,b \rangle \neq 0 \implies LCS(VG_A(a), VG_B(b)) = \beta_d$ (Small LCS)

where $\beta_d := 3d^2 - d - 2$ is a fixed value.

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Attempts:

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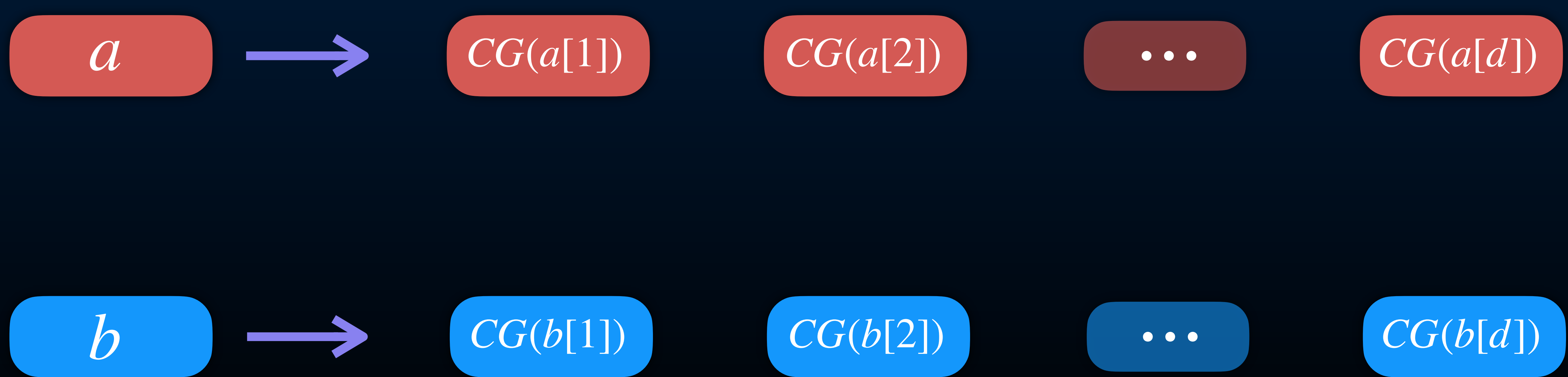

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Attempts:

$a \longrightarrow CG(a[1])\ 2^{3d}\ CG(a[2])\ 2^{3d}\ \cdots\ 2^{3d}\ CG(a[d])$

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a → $CG(a[1])$ 2^{3d} $CG(a[2])$ 2^{3d} $\cdots$ 2^{3d} $CG(a[d])$

Length d | Length $O(d^2)$

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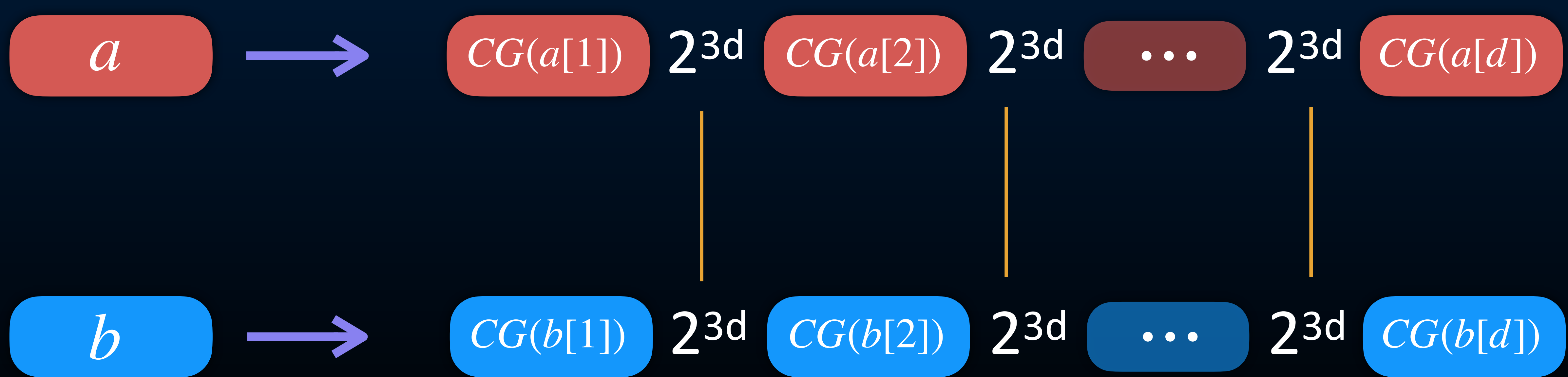
Claim: LCS = $(d-1)\cdot 3d + 2\cdot(d - \langle a,b\rangle)$



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$$\langle a,b \rangle = 0 \implies LCS(VG_A(a), VG_B(b)) = \beta_d + 2$$ Done! (Large LCS)

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$a \to CG(a[1])\ 2^{3d}\ CG(a[2])\ 2^{3d} \cdots 2^{3d}\ CG(a[d])$

Loses $3d > |CG|$ ... never worth it.

$b \to CG(b[1])\ 2^{3d}\ CG(b[2])\ 2^{3d} \cdots 2^{3d}\ CG(b[d])$

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Almost done: $\leq \beta_d$)

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If we match any 3, we cannot match any non-3's.

$b \longrightarrow$ $CG(b[1])$ 2^{3d} $CG(b[2])$ 2^{3d} $\cdots$ 2^{3d} $CG(b[d])$ 3^{β_d}

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Proof:

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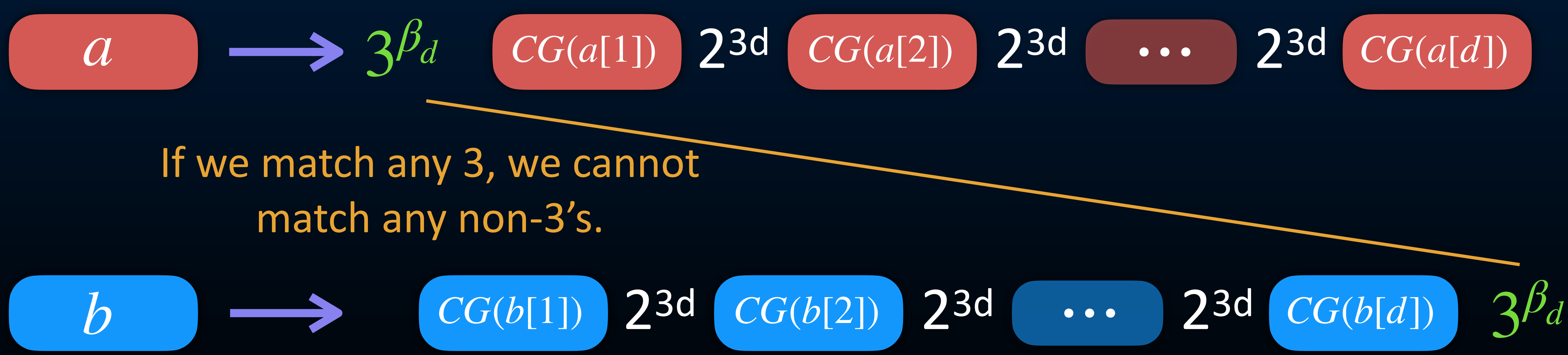
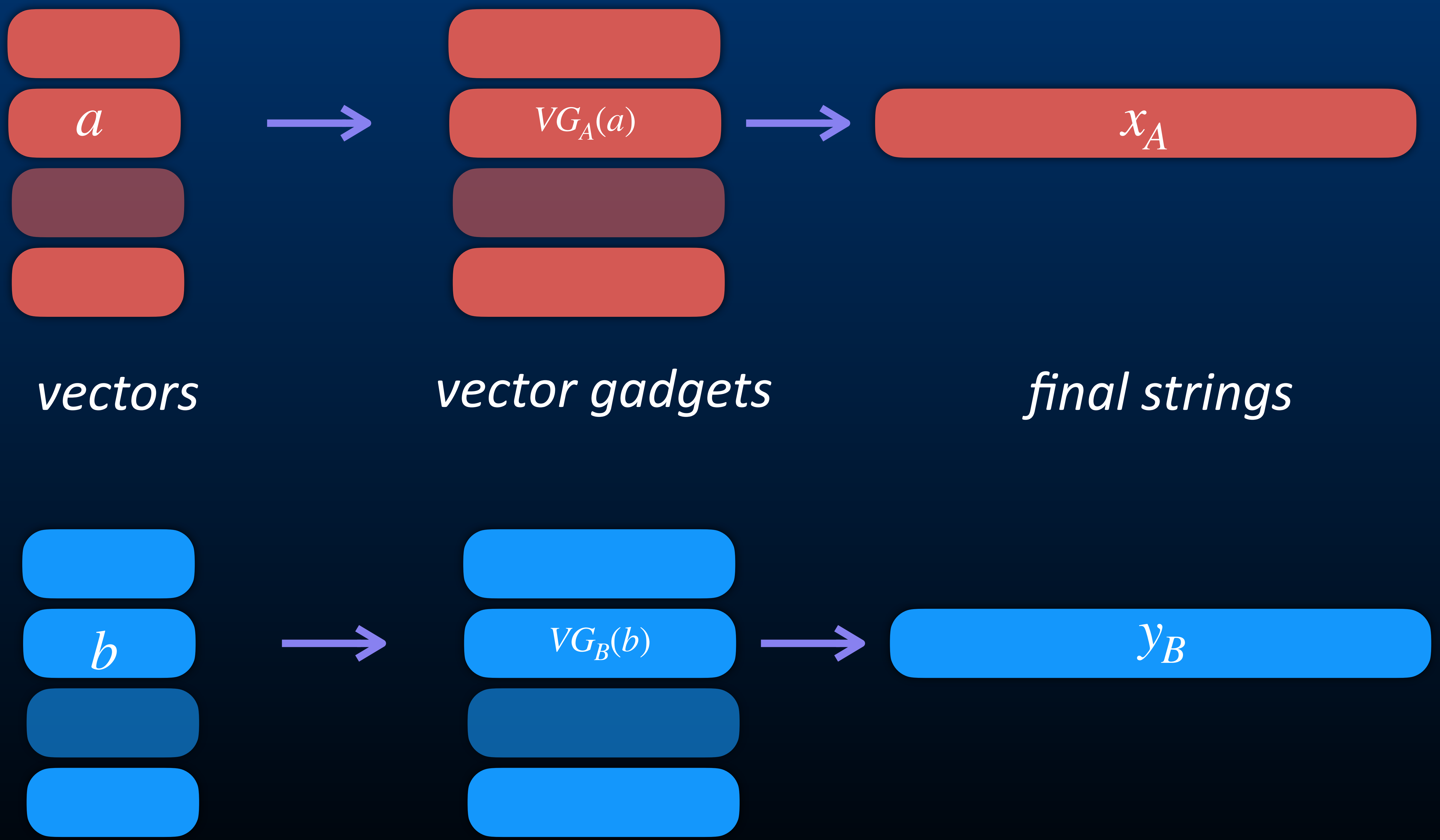


Step 2: **Outer-OR Gadget**



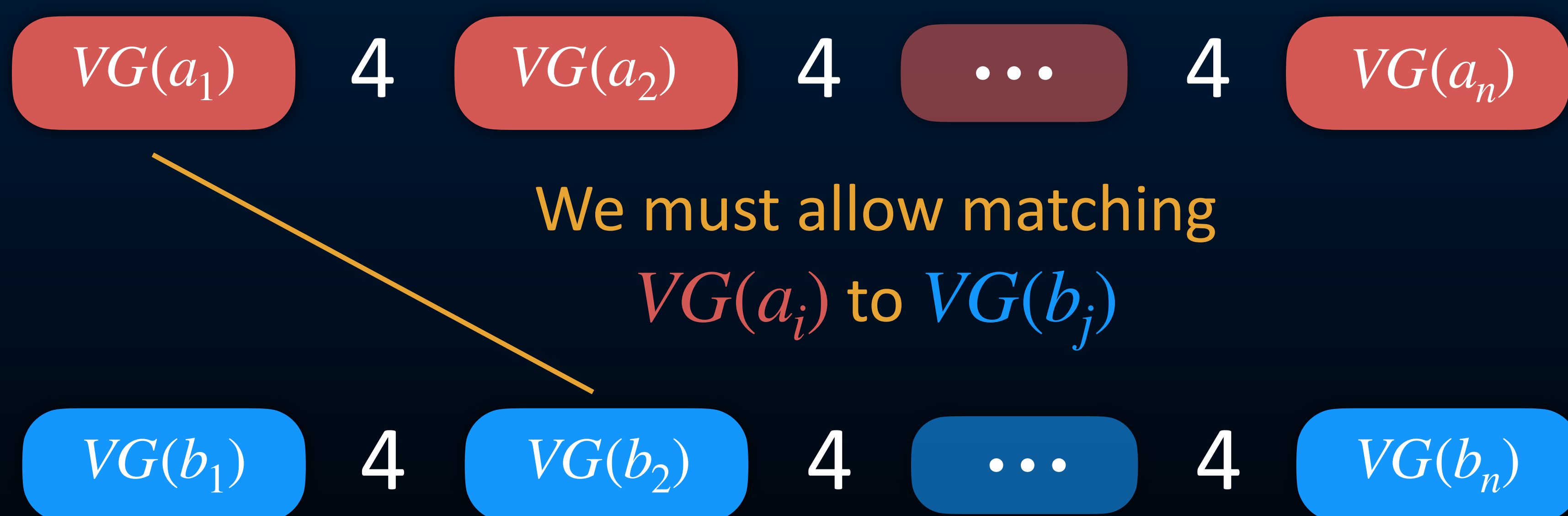
Step 2: **Outer-OR Gadget**

Thm: Given $A, B \subseteq \{0,1\}^d$ can construct x_A , y_B $\in \{0,1,2,3,4\}^{O(nd^2)}$ and an integer τ, in $O(nd^2)$ time s.t.

$\exists a \in A, b \in B : \langle a, b \rangle = 0 \implies LCS(x_A, y_B) \geq \tau$ (Large LCS)

$\forall a \in A, b \in B : \langle a, b \rangle \neq 0 \implies LCS(x_A, y_B) < \tau$ (Small LCS)

Attempts:



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Attempts:

$VG(a_1)$ 4 $\cdots$ 4 $VG(a_n)$ 4 $VG(a_1)$ 4 $\cdots$ 4 $VG(a_n)$

$VG(b_1)$ 4 $\cdots$ 4 $VG(b_n)$

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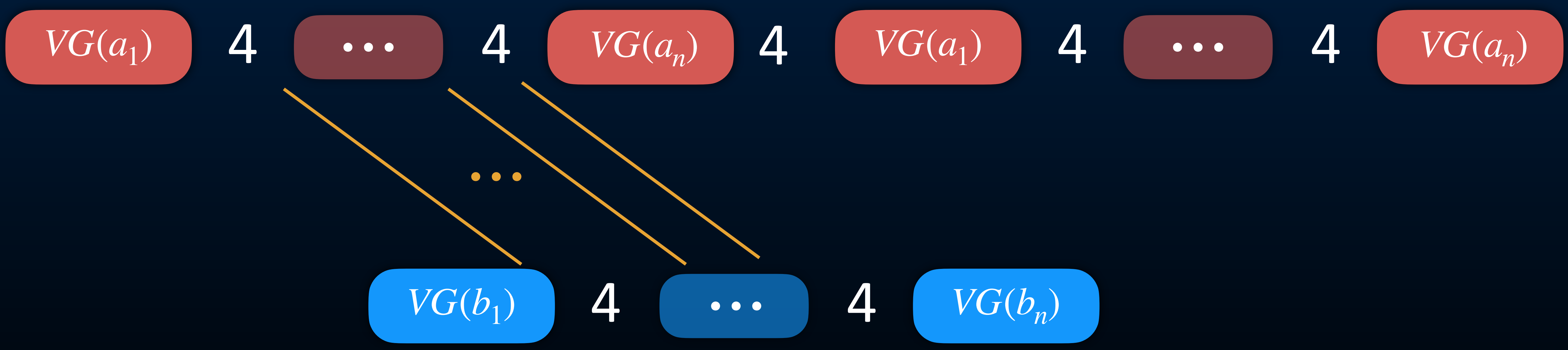

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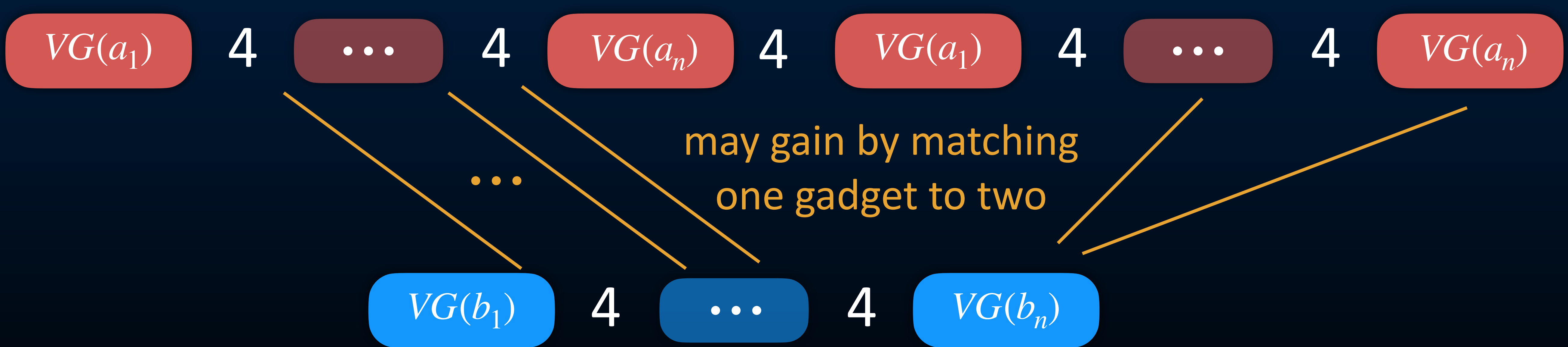

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Proof:

$VG(a_1)$ 4^γ $\cdots$ 4^γ $VG(a_n)$ 4^γ $VG(a_1)$ 4^γ $\cdots$ 4^γ $VG(a_n)$

$(4^\gamma)^n$ $VG(b_1)$ 4^γ $\cdots$ 4^γ $VG(b_n)$ $(4^\gamma)^n$

Set: $\gamma := 6d^2 - d - 2 = |VG_B(b)|$, and $\tau := (2n-1) \cdot \gamma + n\beta_d + 2$.

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All 4's in x get matched: $+(2n-1)\cdot\gamma$

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All 4's in x get matched: $+(2n-1)\cdot\gamma$

All VG(b) gadgets get matched: $+n \cdot \beta$ at least one of them gives +2

$(4^\gamma)^n$ $VG(b_1)$ 4^γ $\cdots$ 4^γ $VG(b_n)$ $(4^\gamma)^n$

Set: $\gamma := 6d^2 - d - 2 = |VG_B(b)|$, and $\tau := (2n-1)\cdot\gamma + n\beta_d + 2$.

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Contribution from 4's:

$\leq (2n-1) \cdot \gamma$

$(4^\gamma)^n$ $VG(b_1)$ 4^γ $\cdots$ 4^γ $VG(b_n)$ $(4^\gamma)^n$

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Contribution of each $VG_B(b)$:

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Contribution of each $VG_B(b)$:

$= 0$

unmatched

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Contribution of each $VG_B(b)$:

- $= 0$ *unmatched*
- $\leq \beta_d$ *matched to one* $VG_A(a)$
- $\leq |VG_B(b)| - \gamma = 0$ *matched to >1* $VG_A(a)$

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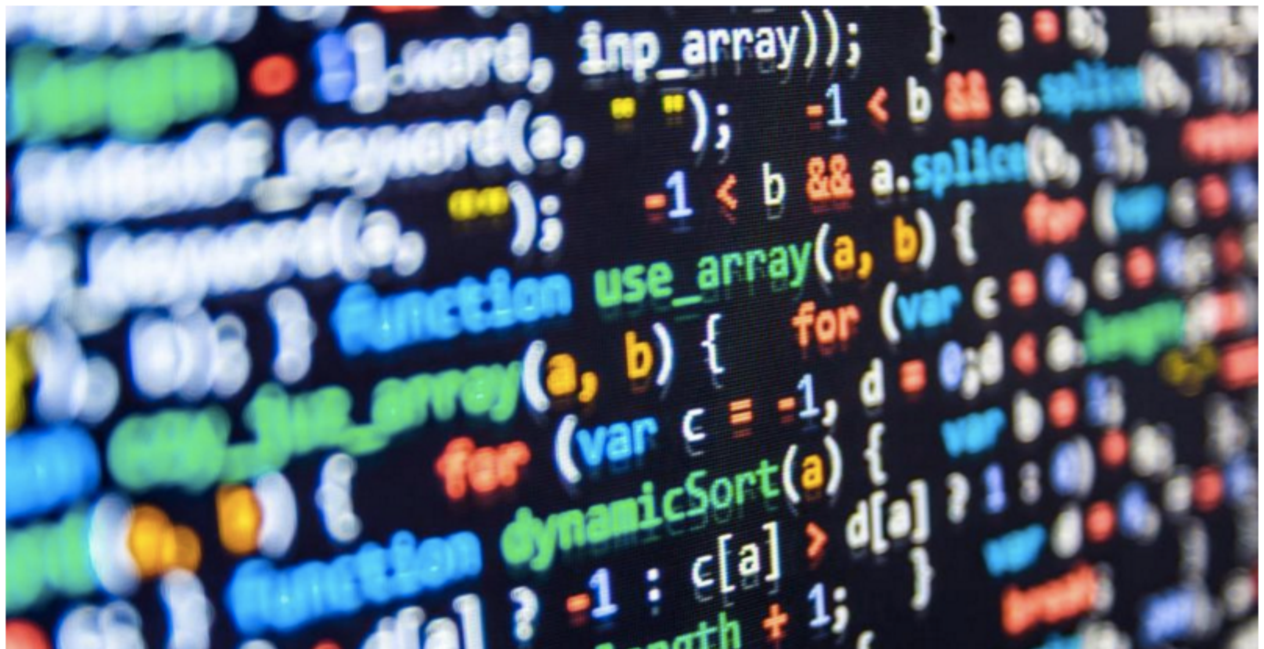
BRAINIAC

For 40 years, computer scientists looked for a solution that doesn't exist



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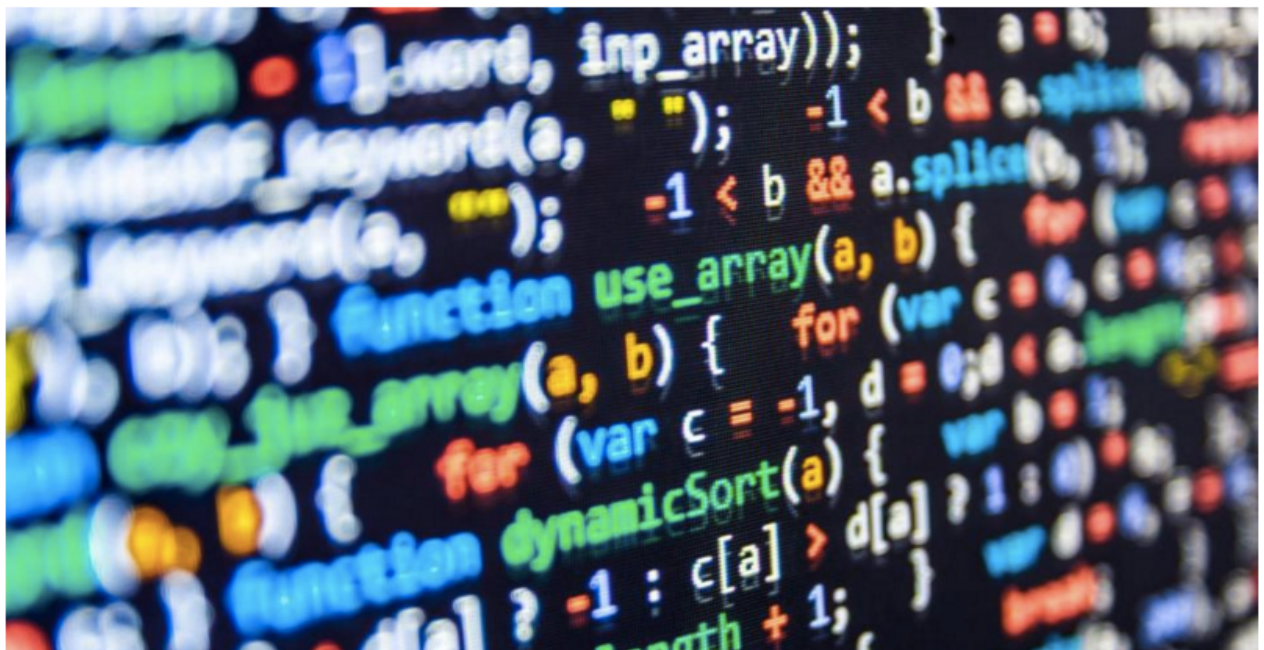

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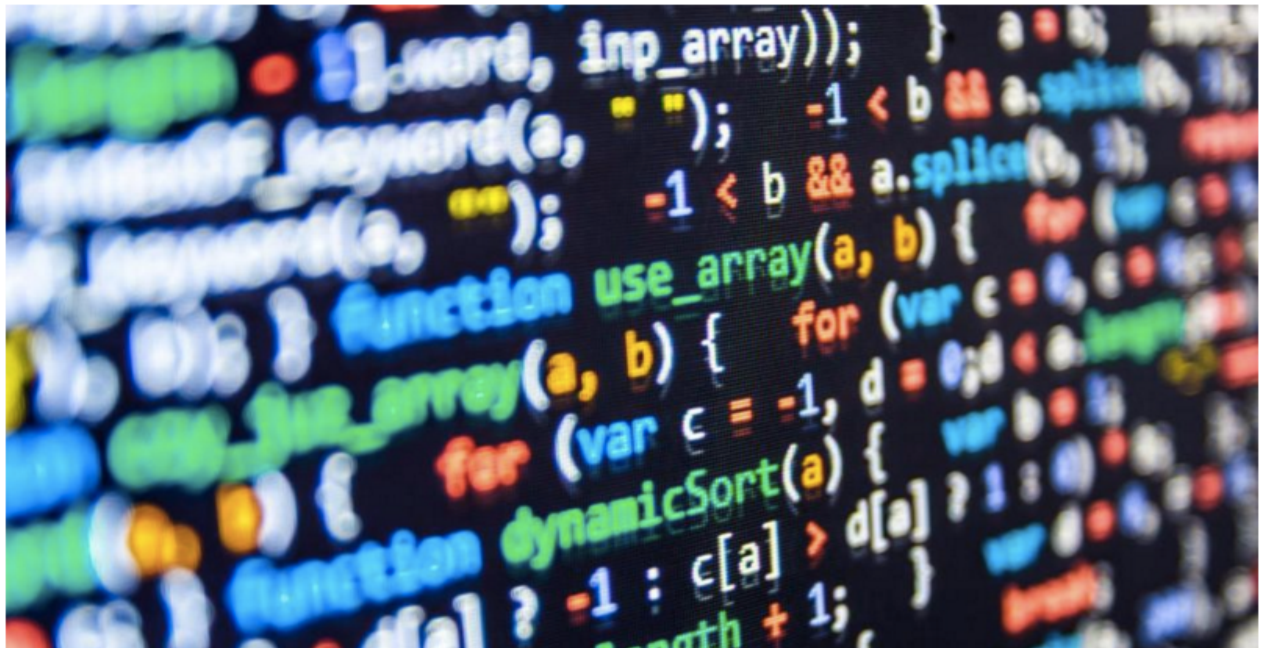

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Next: How Strong is SETH?