Weizmann PDEs Day

September 14, 2023

Venue: Weizmann Institute of Science, Ziskind Building Room 155 (in person). (Also, the lectures can be attended online via zoom)

Zoom Link:
https://weizmann.zoom.us/j/91658638395?pwd=d1l1TVlzYzAxTDE3NGpGakwzVUNKdz09

Titles and abstracts

Claude Bardos, Laboratoire J.-L. Lions, Paris, *"Boundary effects in the vanishing viscosity limit of solutions of Navier-Stokes equations with no slip boundary condition"*

Abstract: In this talk, I consider the zero viscosity ($\nu \to 0$) limit of solutions of the 2d Navier-Stokes equations, subject to no-slip boundary condition, and will elaborate on two complementary problems:

• The convergence to the solution of the Euler equations under strong analyticity hypothesis during a short time interval $0 < t < T$ to emphasize the role of the curvature of the boundary on this time T of validity in connection with the size of Gortler vortices.

• To prove that the Onsager's Hölder regularity exponent $\frac{1}{3}$ of the velocity field $u(x,t)$ of a weak solution of the Euler equations implies the same regularity for the pressure. Then to use this remark to prove that in the zero viscosity limit of u_ν bounded solutions of the Navier-Stokes equations, in $L^\infty((0,T); C^{0,\alpha})$ with $\alpha > \frac{1}{3}$, there is no anomalous energy dissipation.

These observations are part of a program initiated with E. Titi around 2007 and continuing with the contribution of other colleagues in particular presently Toan Nguyen, Trinh Nguyen and D. Boutros.

Matania Ben-Artzi, Hebrew University – Jerusalem, *"Non-Concentration for the acoustic operator in layered media"*

Abstract: Consider the operator $A = -\tilde{c}\Delta$ acting in bounded domains $\Omega \coloneqq \Omega' \times (0,H) \subset \mathbb{R}^d \times \mathbb{R}_+$. The diffusion coefficient $\tilde{c} > 0$ depends on one coordinate $y \in (0,H)$ and is bounded but may be discontinuous. This corresponds to the physical model of "layered media", appearing in acoustics, elasticity, optical fibers... Dirichlet boundary conditions are assumed. In general, for each $\varepsilon > 0$ the set of eigenfunctions is divided into a disjoint union of three subsets : $\mathfrak{F}_{NG}$ (non-guided), $\mathfrak{F}_G$ (guided) and $\mathfrak{F}_{res}$ (residual). The residual set shrinks as $\varepsilon \to 0$. The customary physical terminology of guided/non-guided is often replaced in the mathematical literature by concentrating/non-concentrating solutions, respectively.

For guided waves, the assumption of ``layered media'' enables us to obtain rigorous estimates of their exponential decay away from concentration zones. The case of non-guided waves has attracted less attention in the literature. It leads to some very interesting questions concerning oscillatory solutions and their asymptotic properties. Classical asymptotic methods are available for $c(y) \in C^2$ but a lesser degree of regularity excludes such methods. The associated eigenfunctions (in $\mathfrak{F}_{NG}$) are oscillatory. However, this fact by itself does not exclude the possibility of "flattening out" of the solution between two consecutive zeros, leading to concentration in the complementary segment. Non-concentration is established if $c(y)$ is of bounded variation, by proving a "*minimal amplitude hypothesis"*. However the validity of such results when $c(y)$ is not of bounded variation (even if it is continuous) remains an open problem.

(joint work with A. Benabdallah and Y. Dermenjian)

François Golse, École Polytechnique, Paris, *"The regularity problem for the Landau equation"*

Abstract: It is well known that the dynamics of particles interacting through the Coulomb potential cannot be described by the Boltzmann equation. In this case, the Boltzmann collision integral must be replaced with the Landau operator. In the late 1990's, Villani defined a notion of global, space-homogeneous solutions to the Landau equation, called H-solutions (in view of the importance of Boltzmann's H-Theorem in the definition of such solutions). This talk will review some recent progress on the regularity of Villani solutions of the Landau equation. (Based in particular on joint work with M.P. Gualdani, C. Imbert and A. Vasseur [1, 2].)

References:

[1] F. Golse, M.-P. Gualdani, C. Imbert, A. Vasseur, *Partial regularity in time for the space-homogeneous Landau equation with Coulomb potential*, Ann. Scient. Éc. Norm. Sup. 4e série, **55** (2022) 1575–1611.

[2] F. Golse, C. Imbert, A. Vasseur, *Local regularity for the space-homogeneous Landau equation with very soft potentials*, preprint arXiv:2206.05155 [math.AP].

Cy Maor, Hebrew University – Jerusalem, *"Non-Euclidean elastic ribbons: experiments, analysis and open problems"*

Abstract: Many bodies in nature that undergo inhomogeneous growth/shrinkage become "pre-strained"; that is, they are stressed even in the absence of external forces. They are typically modeled by having an intrinsic non-flat metric, giving rise to *non-Euclidean* elasticity.

Of particular interest are thin bodies: shell-like, rod-like or ribbon-like bodies (e.g., leaves or supramolecular assemblies), which exhibit interesting energy-driven patterns. In this talk I will describe the model of non-Euclidean elasticity, and then focus on thin bodies, the relations between their intrinsic curvature and their elastic behavior, and specifically on shape transitions in non-Euclidean ribbons.

Steve Schochet, Tel Aviv University, *“Sobolev estimates for nonlinear non-uniformly parabolic PDEs and applications to singular limits”*

Abstract: While there are many classical results for nonlinear parabolic equations and systems that are non-uniformly parabolic in specific ways, such as the porous media equation and the compressible Navier-Stokes system, the classical results of Oleinik and Kohn-Nirenberg for general non-uniformly parabolic equations inherently treat only the linear case.

Sobolev energy estimates are proven for solutions of initial-value-problems for general nonlinear non-uniformly parabolic second-order PDEs having symmetric coefficients depending on the independent and dependent variables and satisfying the non-strict Legendre condition. Local-in-time existence of solutions to initial-value problems for such systems are a consequence of those uniform bounds.

After mentioning applications of these results to eddy viscosity and nonlinear geometric optics, applications to a variety of singular limit problems will be described.

Marshall Slemrod, University of Wisconsin – Madison, *“Non-uniqueness on plane fluid flows”*

Abstract: Examples of dynamical systems proposed by Z. Artstein and C. M. Dafermos admit non-unique solutions that track a one parameter family of closed circular orbits contiguous at a single point. Switching between orbits at this single point produces an infinite number of solutions with the same initial data. Dafermos appeals to a maximal entropy rate criterion to recover uniqueness.

These results are here interpreted as non-unique Lagrange trajectories on a particular spatial region. The corresponding special velocity is proved consistent with plane steady compressible fluid flows that for specified pressure and mass density satisfy not only the Euler equations but also the Navier-Stokes equations for specially chosen volume and (positive) shear viscosities. The maximal entropy rate criterion recovers uniqueness.