Volumes in High Dimensions - Home Work 9

It is highly recommended to do all the questions. You need to submit the solution to question 2.

Question 1 *Let $L_2(\gamma_k, X) = L_2(\gamma_k) \otimes X$ as defined in class. Prove that,*

$$\left(L_2\left(\gamma_k, X\right)\right)^* = L_2\left(\gamma_k, X^*\right).$$

(Hint: for every norm $\|\cdot\|_K$ we have $\langle x, y\rangle \leq \|x\|_K \|y\|_{K^\circ}$. Find the equality case in the Cauchy-Schwarz inequality.)

Question 2 *We defined the K convexity parameter of a finite dimensional normed space X by*

$$K(X) = \left\|\vec{Q}_1\right\|_{L_2(\gamma_k, X) \to L_2(\gamma_k, X)}.$$

Show that

$$K(X) = K(X^*).$$

(Hint: use Question 1 to show $\|Q_1 \otimes I_{X^*} G\| \leq K(X)\|G\|$, and that every finite dimensional normed space is reflexive.)

Question 3 *Prove that $K(\ell_p^n) \leq C(p)$ for all $1 < p < \infty$.*

Question 4 *For a bounded holomorphic function on the strip $[0,1] \times i\mathbb{R}$, prove that the function $M(x) = \log \sup_y |f(x+iy)|$ is convex.*

(Hint: First prove the maximum principle for bounded functions on the strip, then use it with $f(z) = e^{\alpha z}$ for an appropriate α.)