Exercise 1 (List Coloring via Symmetric LLL). In the problem of list coloring, every vertex $v$ is associated with a list (or palette) of colors $\text{Pal}(v)$. It is required to compute a legal vertex coloring where the color of each vertex $v$ is taken from $\text{Pal}(v)$. (1a) Show that if for every $v$ it holds that: (i) $|\text{Pal}(v)| \geq \ell$ (ii) each color $c \in \text{Pal}(v)$ appears in at most $\ell/8$ of its neighbors, then there is a legal coloring (i.e., solution to the list coloring instance).

(1b) We consider a weighted variant of the list coloring problem. Given is a graph $G$ with maximum degree $\Delta$ where every vertex $v$ has a palette of colors $\text{Pal}(v)$. Each color $c \in \text{Pal}(v)$ has a weight $w_v(c)$ such that $\sum_{c \in \text{Pal}(v)} w_v(c) = 1$. Prove that if for every edge $(u, v)$ we have $\sum_{c \in \text{Pal}(u) \cap \text{Pal}(v)} w_v(c) \cdot w_u(c) \leq 1/(8\Delta)$ then $G$ has a legal coloring.

(1c) Use the LLL algorithms shown in class (as a black box) to devise distributed algorithms for computing the coloring in (1a) and (1b).

Exercise 2 (MST and Connectivity). (2a) Show that one can compute an MST on the clique graph within $O(\log n)$ rounds w.h.p.

(2b) Let $G$ be an $n$-vertex $D$-diameter graph and let $H$ be a subgraph of $G$ given in a distributed manner. I.e., each vertex $v$ knows its incident edges in $H$. In the Connectivity Identification task, it is required for each vertex $v$ to output the largest vertex identifier in its connected component in $H$. Show a distributed algorithm for this problem that runs in $\tilde{O}(D + \sqrt{n})$ rounds w.h.p.