Advanced Distributed Algorithms	**Spring 2019**
Exercise 2 (May 3)	
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Randomized Symmetry Breaking. (1) Consider the following 1-round randomized algorithm: each vertex v picks a number r_v u.a.r in $[0,1]$, and joins a set R if it is a local minima, that is if $r_v < r_u$ for any neighbor u of v. We will now show two (unrelated) properties of this algorithm. (1a) Show that for each v it holds that $R \cap N^+(v) \neq \emptyset$ with probability at least $(deg(v)+1)/(deg(v)+deg_{max})$, where $N+(v) = N(v) \cup \{v\}$ and deg_{max} is the maximum degree among nodes in $\{u\} \cup N(u)$. (2b) Show that w.h.p. R is a $(2, O(\log n))$ ruling set.

Randomized Coloring. We discussed in class an algorithm for computing a $(\Delta + 1)$ coloring in $O(\log \Delta) + 2^{O(\sqrt{\log \log n})}$. Show that this algorithm implies an $O(\sqrt{\log n})$-round algorithm for a $G(n,p)$ graph[1].

From Local Complexity to Global Complexity. By now, we saw several algorithms that have a two-phase structure: randomized part followed by a deterministic part. Assume that we are given a pre-shattering randomized algorithm $\mathcal{A}$ (for either MIS or $\Delta + 1$ coloring) that has the following guarantee: every vertex v is *undecided*[2] at the end of $\mathcal{A}$ with probability at most $\Delta^{-\alpha}$; Furthermore, this holds even if the outcome of the coin tosses outside $N^+_\beta(v) = \{v\} \cup \{u \mid dist(u,v,G) \leq \beta\}$ are determined adversarially. Bound the round complexity of the second deterministic phase that solves the remaining undecided subgraph as a function of α, β, n and Δ. The final algorithm should succeed with high probability of $1 - 1/n^c$. (

Bonus Question: This question is about the MIS algorithm by [Ghaffari, SODA'16] that we presented in the last class. Recall that a round t is a golden-out round for v if $d_t(v) \geq 1$, and at least $d_t(v)/10$ is due to low-deg nodes. Explain why it is important to set $d_t(v) \geq 1$ in this definition. That is, what might go wrong in the analysis if we change the definition by replacing the requirement of $d_t(v) \geq 1$ with $d_t(v) \geq 2$.

[1]The Erdős and Rényi random graph $G(n,p)$ is a graph on n vertices in which each edge $e = (u,v) \in V \times V$ is taken into the graph with probability of p.

[2]For MIS: undecided means that v is not yet in MIS and that none of its neighbors are in MIS. For coloring, undecided means uncolored.