Exercise 4 (Due July 18)

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(α, β) **shortcuts.** Given is an graph $G = (V, E)$ and a collection of vertex-disjoint subsets $S_1, \ldots, S_N$ where $G[S_i]$ (i.e., the induced subgraph) is connected for every S_i. An (α, β) shortcuts for these sets is collection of subgraphs $H_1, \ldots, H_N$ that satisfies the following: (1) the diameter of each subgraph $G[S_i] \cup H_i$ is at most α and (2) each edge $e \in G$ appears on at most β subgraphs, i.e., the number of H_i subgraphs containing a fixed edge e is at most β. (α, β) shortcuts provide a basic communication backbone that is useful for many distributed algorithms in the CONGEST model. The running time of these algorithms depend on the summation $\alpha + \beta$ of the shortcuts. It is therefore desirable to design (α, β) shortcuts with small value of $\alpha + \beta$. (i) Show that for every n-vertex graph $G = (V, E)$ with diameter D, and for every collection of connected vertex-disjoint subsets $S_1, \ldots, S_N$, there exist (α, β) shortcuts with $\alpha + \beta = O(D + \sqrt{n})$. (ii) Show that every graph with minimum degree k has (α, β) shortcuts with $\alpha = O(n/k)$ and $\beta = 2$.

MST on Clique Graphs. (iii) Show that one can compute an MST on the clique graph within $O(\log n)$ rounds w.h.p.

Connectivity Identification for a Subgraph. Let G be an n-vertex D-diameter graph and let H be a subgraph of G given in a distributed manner. I.e., each vertex v knows its incident edges in H. In the Connectivity Identification task it is required that each vertex v knows the ID of its connected component in H where the ID of a component is simply the largest vertex ID in the component. (iv) Show a distributed algorithm for this problem that runs in $\widetilde{O}(D + \sqrt{n})$ rounds w.h.p.