Succinct Graph Structures and Their Applications **Spring 2018**

Exercise 3: May 17

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Routing Scheme

Exercise 1. Describe an efficient routing scheme for the unweighted $\sqrt{n} \times \sqrt{n}$ 2-dimensional grid. The labels and the routing tables should be of size $O(\log n)$ bits. Bonus: extend it to the d-dimensional n-vertex hypercube for $n = 2^d$.

Labeling Scheme

Exercise 2. Show that any labeling scheme for adjacency in general graphs has a label size of at least $\Omega(n)$ bits.

Exercise 3. We saw in class an $(2k-1)$ approximate labeling scheme for general graphs with $\widetilde{O}(n^{1/k})$ bits. In this exercise, we are required to design *exact* distance labels but only for nodes that are sufficiency far away from each other in G. Consider an unweighted graph G and define a δ-distance labeling scheme by $\langle L, D \rangle$ such that given $L(u)$ and $L(v)$, the decoding function should return the exact distance only for vertices u, v at distance at least δ in G. That is, it is required that $D(L(u), L(v)) = \texttt{dist}(u, v, G)$ for every $u, v \in G$ satisfying that $\texttt{dist}(u, v, G) \geq \delta$. Show an δ-distance labeling scheme with label size of $O(n \cdot \log^2 n/\delta)$. You are allowed to have a randomized construction and in such a case, guarantee that the decoding function returns the right answer with high probability.