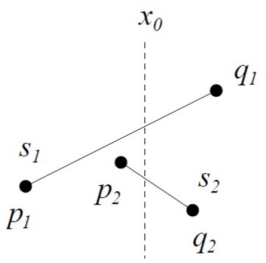
Questions from Past Exams
Computational Geometry

Question 1

We are given a collection $S = \{s_1, \ldots, s_n\}$, of $n \geq 3$ segments in the plane, with each segment $s_i = (p_i, q_i)$ given by the coordinates of its two endpoints. It is required to check if the segments form a closed polygon. Design an efficient algorithm for the problem and analyze its time complexity.

Question 2

We are given two nonintersecting segments $S_1 = (p_1, q_1)$ and $S_2 = (p_2, q_2)$ in the plane, represented by the coordinates of their endpoints, with $x_{p_i} \leq x_{q_i}$ for $i = 1, 2$, and an x-coordinate x_0 common to both segments, i.e., such that the line $\ell : x = x_0$ intersects both. Describe a procedure for deciding, in a constant number of operations, whether the intersection of ℓ with S_1 is higher or lower than its intersection with S_2. The procedure is not allowed to solve the equations and compute the coordinates of the intersection points. (The following figure illustrates one of the possible configurations for the problem.)



Question 3

Describe an efficient algorithm that given two sets A, B each containing n disjoint points in the plane, computes the shortest distance between a point in A and a point in B, i.e., $\min\{dist(p, q) \mid p \in A, q \in B\}$.

Question 4

Two sets of points S_1 and S_2 are known to have the same Voronoi diagram, i.e., $Vor(S_1) = Vor(S_2)$. Prove or disprove each of the following claims:

1. $S_1 = S_2$.
2. $|S_1| = |S_2|$.

Question 5

For a given collection of half-planes $H = \{h_1, \ldots, h_n\}$, we classified the half-plane intersection of H into the following types: (1) simple polygon, (2) generalized polygon, (3) half-plane,

(4) infinite strip, (5) single point, (6) empty region.

Let $H' = \{h'_1, \ldots, h'_n\}$ be the collection of complementary half-planes of H (namely, if h_i is defined by the inequality $a_i x + b_i y + c_i \leq 0$, then h'_i is defined by $a_i x + b_i y + c_i \geq 0$.

For every $i \in \{1, \ldots, 6\}$, assuming the half-plane intersection of H is of type i, determine which types are possible for the half-plane intersection of the complementary collection H'.

Question 6

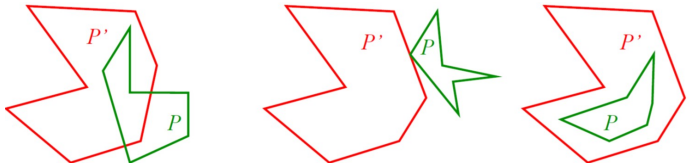
Describe the fastest algorithm you can for computing the area of a simple polygon P, given as a list of vertices $\langle v_1, \ldots, v_n \rangle$ in counterclockwise order of occurrence. Prove the correctness of your algorithm and analyze its time complexity.

Question 7

Consider the following problem.

Input: Two simple polygons $P = \langle p_1, \ldots, p_m \rangle$ and $P' = \langle p'_1, \ldots, p'_{m'} \rangle$, where $n = m + m'$.

Question: Do the areas of P and P' overlap?



(Note: Even touching in a single point is considered an overlap.)

Describe the most efficient algorithm you can for this problem.
Prove the correctness of your algorithm and analyze its complexity.