Algorithmic Applications of Hypergraph and Partition Containers

Abstract:

We present a general method to convert algorithms into faster algorithms for almost-regular input instances. Informally, an almost-regular input is an input in which the maximum degree is larger than the average degree by at most a constant factor. This family of inputs vastly generalizes several families of inputs for which we commonly have improved algorithms, including bounded-degree inputs and random inputs. It also generalizes families of inputs for which we don't usually have faster algorithms, including regular inputs of arbitrarily high degree and all very dense inputs. We apply our method to achieve breakthroughs in exact algorithms for several central NP-Complete problems including k-SAT, Graph Coloring, and Maximum Independent Set. Our main tool is the first algorithmic application of the relatively new Hypergraph Container Method (Saxton and Thomason 2015, Balogh, Morris and Samotij 2015). This recent breakthrough, which generalizes an earlier version for graphs (Kleitman and Winston 1982, Sapozhenko 2001), has been used extensively in recent years in extremal combinatorics. An important component of our work is a new generalization of (hyper-)graph containers to Partition Containers.