The classical Berry-Esseen theorem provides optimal rates in the central limit theorem (CLT) for partial sums of iid random variables. Since then there have been many extensions to "weakly dependent" (aka mixing) random variables. A related question is the accuracy of approximation by Gaussians in $L^p$ (after coupling). The validity of such optimal $L^p$ rates was an open problem by E. Rio, which was recently (2018) solved by S. Bobkov for independent random variables. In the talk I will present recent results concerning optimal CLT rates in $L^p$ for a variety of weakly dependent random variables like (inhomogeneous) Markov chains, products of random matrices and partially hyperbolic dynamical systems. We will also discuss relations with almost sure rates of approximation by Gaussian random variables (i.e. rates in the almost sure invariance principle). Edgeworth expansions provide better than optimal CLT rates, with appropriate correction terms. Our proofs require non-uniform versions of such expansions for weakly dependent random variables, which are of independent interest and have several other applications. In particular, we obtain a non-uniform Berry-Esseen theorem for weakly dependent random variables.