Almost Polynomial Hardness of Node-Disjoint Paths in Grids

Abstract:

In the classical Node-Disjoint Paths (NDP) problem, we are given an \( n \)-vertex graph \( G \), and a collection of pairs of its vertices, called demand pairs. The goal is to route as many of the demand pairs as possible, where to route a pair we need to select a path connecting it, so that all selected paths are disjoint in their vertices.

The best current algorithm for NDP achieves an \( O(\sqrt{n}) \)-approximation, while, until recently, the best negative result was a roughly \( \Omega(\sqrt{\log n}) \)-hardness of approximation. Recently, an improved \( 2^{\Omega(\sqrt{\log n})} \)-hardness of approximation for NDP was shown, even if the underlying graph is a subgraph of a grid graph, and all source vertices lie on the boundary of the grid. Unfortunately, this result does not extend to grid graphs.

The approximability of NDP in grids has remained a tantalizing open question, with the best upper bound of \( \tilde{O}(n^{1/4}) \), and the best lower bound of APX-hardness. In this talk we come close to resolving this question, by showing an almost polynomial hardness of approximation for NDP in grid graphs.

Our hardness proof performs a reduction from the 3COL(5) problem to NDP, using a new graph partitioning problem as a proxy. Unlike the more standard approach of employing Karp reductions to prove hardness of approximation, our proof is a Cook-type reduction, where, given an input instance of 3COL(5), we produce a large number of instances of NDP, and apply an approximation algorithm for NDP to each of them. The construction of each new instance of NDP crucially depends on the solutions to the previous instances that were found by the approximation algorithm.

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