Abstract:

A general theme in geometry is the classification of algebraic/differential geometric structures which satisfy a positivity property. I will describe an "asymptotic" version of this theme that lies at the crossroads of algebraic geometry, differential geometry, and convexity. On the algebraic side, we introduce the class of asymptotically log Fano varieties and state a program towards classification in dimension 2, generalizing the classical efforts of the Italian School of Algebraic Geometry, Manin and Hitchin and making contact, somewhat surprisingly, with linear programming. On the differential side, we explain the resolution of the Calabi-Tian conjecture which implies such a classification classifies Kahler spaces with positive Ricci curvature away from a divisor and with conical singularities near the divisor, and relations to "small angle limits" in Riemannian geometry, stability, non-compact Ricci flat fibrations, Ricci solitons, and Kahler-Einstein edge metrics.