Computational Barriers in Continuous Optimization: Two Complexity Theories and One Tale of Symmetry

Abstract:

Since the modern formulation of mathematical optimization, researchers in the field have repeatedly expanded and re-defined the realm of tractable optimization problems. This endeavor has culminated in the well-known class of convex optimization problems with applications in a wide range of scientific fields. In this talk, I will present the traditional oracle-based complexity model, which has dominated (unstructured) continuous optimization for the past 40 years, and highlight some of its successes and failures in predicting the hardness of convex optimization problems. I will then introduce a novel structural-based model aimed at addressing major oracle-based complexity gaps. The new approach is intimately tied with approximation theory, and is proven to be particularly advantageous for characterizing the complexity of optimization methods in machine learning.

Studies in recent years have indicated that the realm of tractable optimization problems may be expanded once more -- this time into that of the nonconvex world. In the second part of the talk, I will present a novel symmetry-based approach to study nonconvex optimization landscapes, and use it to address optimization-related aspects of neural networks. The approach employs techniques, new to the field, from symmetry breaking and representation theory, and carries important implications for one’s ability to argue about statistical generalization through local curvature.