A determinantal point process governed by a locally trace class Hermitian contraction kernel on a measure space $E$ remains determinantal when conditioned on its configuration on an arbitrary measurable subset $B \subset E$. Moreover, the conditional kernel can be chosen canonically in a way that is "local" in a non-commutative sense, i.e. invariant under "restriction" to closed subspaces $L^2(B) \subset P \subset L^2(E)$.

Using the properties of the canonical conditional kernel we establish a conjecture of Lyons and Peres: if $K$ is a projection then almost surely all functions in its image can be recovered by sampling at the points of the process.