Abstract:

The question of determining if a given algebraic variety is rational is a notoriously difficult problem in algebraic geometry, and attempts to solve rationality problems have often produced powerful new techniques. A well-known open rationality problem is the determination of a criterion for when a cubic hypersurface of five-dimensional projective space is rational. After discussing the history of this problem, I will introduce the two conjectural rationality criteria that have been put forth and then discuss a package of tools I have developed with my collaborators to bring these two conjectures together. Our theory of Relative Bridgeland Stability has a number of other beautiful consequences such as a new proof of the integral Hodge Conjecture for Cubic Fourfolds and the construction of full-dimensional families of projective HyperKahler manifolds. Time permitting I'll discuss applications of the theory of relative stability conditions to problems other than cubic fourfolds.