מכון ויצמן למדע

THE WEIZMANN INSTITUTE OF SCIENCE

FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Algebraic Geometry and Representation Theory Seminar

Room 261 ,Ziskind Building
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at 11:00

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Derived Categories of Bimodules

Abstract:

Homological algebra plays a major role in noncommutative ring theory. One important homological construct related to a noncommutative ring A is the dualizing complex, which is a special kind of complex of A-bimodules. When A is a ring containing a central field K, this concept is well-understood now. However, little is known about dualizing complexes when the ring A does not contain a central field (I shall refer to this as the noncommutative arithmetic setting). The main technical issue is finding the correct derived category of A-bimodules.

In this talk I will propose a promising definition of the derived category of A-bimodules in the noncommutative arithmetic setting. Here A is a (possibly) noncommutative ring, central over a commutative base ring K (e.g. K = Z). The idea is to resolve A: we choose a DG (differential graded) ring A', central and flat over K, with a DG ring quasi-isomorphism A' -> A. Such resolutions exist. The enveloping DG ring A'^{en} is the tensor product over K of A' and its opposite. Our candidate for the "derived category of A-bimodules" is the category D(A'^{en}), the derived category of DG A'^{en}-modules. A recent theorem says that the category D(A'^{en}) is independent of the resolution A', up to a canonical equivalence. This justifies our definition.

Working within D(A'^{en}), it is not hard to define dualizing complexes over A, and to prove all their expected properties (like when K is a field). We can also talk about rigid dualizing complexes in the noncommutative arithmetic setting.

What is noticeably missing is a result about existence of rigid dualizing complexes. When the K is a field, Van den Bergh had discovered a powerful existence result for rigid dualizing complexes. We are now trying to extend Van den Bergh's method to the noncommutative arithmetic setting. This is work in progress, joint with Rishi Vyas.

In this talk I will explain, in broad strokes, what are DG rings, DG modules, and the associated derived categories and derived functors. Also, I will try to go into the details of a few results and examples, to give the flavor of this material.

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