



THE WEIZMANN INSTITUTE OF SCIENCE  
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Seminar in Geometry and Topology

Room 1 ,Ziskind Building  
on Tuesday, Dec 25, 2018  
at 16:00

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Describing Blaschke products by their critical points

Abstract:

In this talk, I will discuss a question which originates in complex analysis but is really a problem in non-linear elliptic PDE. A finite Blaschke product is a proper holomorphic self-map of the unit disk, just like a polynomial is a proper holomorphic self-map of the complex plane. A celebrated theorem of Heins says that up to post-composition with a Möbius transformation, a finite Blaschke product is uniquely determined by the set of its critical points. Konstantin Dyakonov suggested that it may be interesting to extend this result to infinite degree. However, one must be a little careful since infinite Blaschke products may have identical critical sets. I will show that an infinite Blaschke product is uniquely determined by its "critical structure" and describe all possible critical structures which can occur. By Liouville's correspondence, this question is equivalent to studying nearly-maximal solutions of the Gauss curvature equation  $\Delta u = e^{2u}$ . This problem can then be solved using PDE techniques, using the method of sub- and super-solutions.