Discontinuity of the phase transition for the planar random-cluster and Potts
models with $q > 4$

Abstract:

The random-cluster model is a dependent percolation model where the weight of a configuration is
proportional to $q$ to the power of the number of connected components. It is highly related to the
ferromagnetic $q$-Potts model, where every vertex is assigned one of $q$ colors, and monochromatic
neighbors are encouraged. Through non-rigorous means, Baxter showed that the phase transition is
first-order whenever $q > 4$ - i.e. there are multiple Gibbs measures at criticality. We provide a
rigorous proof of this claim. Like Baxter, our proof uses the correspondence between the above
models and the six-vertex model, which we analyze using the Bethe ansatz and transfer matrix
techniques. We also prove Baxter's formula for the correlation length of the models at criticality.
This is joint work with Hugo Duminil-Copin, Maxime Gangebin, Ioan Manolescu, and Vincent Tassion.