Expansion, Codes and Optimization at the Frontiers of TCS

Abstract:

Expansion is a manifestation of (pseudo)randomness with expander graphs and their higher-dimensional analogues being concrete examples. Coding theory is the study of properties and algorithms of codes, which are, in particular, important for protecting data and communication against errors. Optimization, especially convex programs, underlies much of our understanding of efficient computation. Not only these are fundamental areas in their own right, but they also enjoy great synergy with mutually nourishing interactions. This synergy has been instrumental in advancing our understanding at the frontiers of almost optimal codes and expansion. Expansion is the key ingredient in the breakthrough construction of explicit almost optimal binary codes of Ta-Shma'17 (approximately matching the random parameters known since the 1950s!). Using optimization and exploring expansion properties, we give efficient decoding algorithms for these codes. Our first decoder is based on the Sum-of-Squares SDP hierarchy and builds on our earlier approximation algorithm for constraint satisfaction problems (CSPs) on high-dimensional expanders (HDXs). It gives the first polynomial time decoder for explicit almost optimal binary codes. Our second decoding algorithm is based on generalizations of weak regularity to expanding hypergraphs, and it runs in near-linear time. Regarding almost optimal expansion, we show how to efficiently transform an arbitrary bounded degree expander into an almost optimal (i.e., almost Ramanujan) one in a way that preserves its structural properties thereby having many applications. In particular, we obtain almost optimal Cayley expanders from any expanding group. This is done by generalizing the breakthrough techniques of Ta-Shma'17 from scalar to operators, and it is an example of coding theory nourishing back the study of expansion. Surprisingly, more precise parameters defining optimal binary codes are not known! This longstanding question is intimately connected to optimization from which the best bounds were derived in the 1970s. We now have a provably complete hierarchy of linear programs for the important class of linear codes. This gives a provably sufficient avenue to make progress on this problem. In this talk, I will give an account of these developments and these synergetic interactions. I will also point to some future research directions. Expansion, codes and optimization are vibrant areas of TCS with several intriguing questions of their own, many interactions and great potential for even further interactions with other areas. (This presentation is based on joint work with many collaborators. Please, see my webpage for details.)