Exponential decay of quotients of Ruelle operators

Abstract:

Ruelle's operator theorem states that the Ruelle operator $L$, which is a positive operator acting on Holder functions, is conjugated to $P+R$ where $R$ is a one-dimensional projection and the norm of $R$ is smaller than 1. This decomposition, also known as spectral gap, is of interest as it allows to characterise the underlying dynamical system through, e.g., central limit theorems or continuous response to perturbations. However, the conjugation depends on the existence of a positive eigenfunction of $L$, which might not exist in more general, fibred situations due to purely functorial reasons. A possibility to circumvent this problem is to consider quotients of operators of the form $f \mapsto \frac{L^m(f \cdot (1))}{L^{m+n}(1)}$. In fact, it is possible to provide reasonable conditions such that their dual operators contract the Wasserstein distance exponentially in $m$. The result gives rise, for example, to a law of the iterated logarithm for continued fractions with sequentially restricted entries or a topology on the set of equilibrium states for semigroups of expanding maps. This is joint work with Paulo Varandas and Xuan Zhang.