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THE WEIZMANN INSTITUTE OF SCIENCE  
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE  
Algebraic Geometry and Representation Theory Seminar

on Wednesday, Oct 06, 2021 at 16:30

Zoom at: <https://weizmann.zoom.us/j/98304397425>

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Finite multiplicities beyond spherical pairs

Abstract:

Let  $G$  be a real reductive algebraic group, and let  $H$  be an algebraic subgroup of  $G$ . It is known that the action of  $G$  on the space of functions on  $G/H$  is "tame" if this space is spherical. In particular, the multiplicities of the space of Schwartz functions on  $G/H$  are finite in this case. I will talk about a recent joint work with A. Aizenbud in which we formulate and analyze a generalization of sphericity that implies finite multiplicities in the Schwartz space of  $G/H$  for small enough irreducible smooth representations of  $G$ . In more detail, for every  $G$ -space  $X$ , and every closed  $G$ -invariant subset  $S$  of the nilpotent cone of the Lie algebra of  $G$ , we define when  $X$  is  $S$ -spherical, by means of a geometric condition involving dimensions of fibers of the moment map. We then show that if  $X$  is  $S$ -spherical, then every representation with annihilator variety lying in  $S$  has (at most) finite multiplicities in the Schwartz space of  $X$ . For the case when  $S$  is the closure of the Richardson orbit of a parabolic subgroup  $P$  of  $G$ , we show that the condition is equivalent to  $P$  having finitely many orbits on  $X$ . We give applications of our results to branching problems. Our main tool in bounding the multiplicity is the theory of holonomic  $D$ -modules. After formulating our main results, I will briefly recall the necessary aspects of this theory and sketch our proofs. The talk is based on arXiv:2109.00204.