Finite multiplicities beyond spherical pairs

Abstract:

Let $G$ be a real reductive algebraic group, and let $H$ be an algebraic subgroup of $G$. It is known that the action of $G$ on the space of functions on $G/H$ is “tame” if this space is spherical. In particular, the multiplicities of the space of Schwartz functions on $G/H$ are finite in this case. I will talk about a recent joint work with A. Aizenbud in which we formulate and analyze a generalization of sphericity that implies finite multiplicities in the Schwartz space of $G/H$ for small enough irreducible smooth representations of $G$. In more detail, for every $G$-space $X$, and every closed $G$-invariant subset $S$ of the nilpotent cone of the Lie algebra of $G$, we define when $X$ is $S$-spherical, by means of a geometric condition involving dimensions of fibers of the moment map. We then show that if $X$ is $S$-spherical, then every representation with annihilator variety lying in $S$ has (at most) finite multiplicities in the Schwartz space of $X$. For the case when $S$ is the closure of the Richardson orbit of a parabolic subgroup $P$ of $G$, we show that the condition is equivalent to $P$ having finitely many orbits on $X$. We give applications of our results to branching problems. Our main tool in bounding the multiplicity is the theory of holonomic D-modules. After formulating our main results, I will briefly recall the necessary aspects of this theory and sketch our proofs. The talk is based on arXiv:2109.00204.