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The firefighter problem, Cayley graphs and the branching number of intermediate growth trees

Abstract:

In the fire-fighter model, a fire spreads on the vertices of a graph starting from some initial fire. Each turn all “non-protected” vertices neighbouring the fire catch fire and burn forever. The “player” can protect $f(n)$ vertices on his n'th turn and these vertices never catch fire. Given a graph G, The “asymptotic” fire-fighter problem asks for which $f(n)$ can the fire be contained in a finite set for any initial fire.

This is a quasi-isometry invariant of G, and is especially interesting for Cayley graphs.

We will first discuss the problem on Polynomial growth groups, where in joint work with Gady Kozma and Rangel Baldasso we show that for groups of growth $n^d$ the threshold is $\sim n^{d-2}$, answering a conjecture of Devline and Hartke.

We will then move to discuss the case of exponential growth groups where F. Lehner proved that the growth rate is also the order of the threshold for fire-fighting.

This leaves open the case of intermediate growth groups. In joint work with Shangjie Yang we get the correct threshold for a family of intermediate growth groups. To do so we introduce a notion of branching numbers for intermediate growth trees (IBN), which acts as the threshold for several random processes on such trees. We relate the IBN it to firefighting and find and analyze a good tree inside these groups.

If time permits we will also briefly discuss some other fire-fighting related problems.

Based on joint works with R. Baldasso, G. Kozma, M. Gerasimova and S. Yang.